Interest Rate Futures

Chapter 6

Outline

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- Treasury bond quote & invoice prices
- Treasury bond futures
  - Conversion factors
  - Cheapest to deliver
  - Prices
- Eurodollar futures
- Forwards vs futures on interest rates

Outline

- Interest rate sensitivity of bond prices
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  - Duration of a portfolio
- Duration based hedging
  - Hedging a bond portfolio
  - Hedging a floating rate loan
  - Hedging difference a portfolio of assets and liabilities

Day Count Conventions

- Treasury bonds: actual/actual
  - Suppose that the bond face value is 100 and the coupon period is Mar.1 to Sept. 1. The coupon rate is 8%. The interest earned between Mar. 1 and Jul. 3 is: 4*124/184.
- Corporate and municipal bonds: 30/360
  - The number of days between Mar. 1 and Sept. 1 is 180 and the number of days between Mar.1 and Jul. 3 is (4*30+2)=122. The interest earned = 4*122/180.
- Money market instruments: actual/360
  - Interest rate earned = 4*124/360

Treasury bond

- Cash price or invoice price = quoted price + accrued interest
- Accrued interest = (last coupon date up to but excluding settlement date/last coupon date up to but excluding next coupon date) * coupon
- Trade date: date on which the transaction is executed
- Settlement date: next business day after the trade date

Treasury bonds

- Assume today is March 5, 2003 (Wed). Consider an 11% coupon bond that matures on July 10, 2009 with a quoted price of 95-16. The most recent coupon date is January 10, 2003 and the next coupon date is July 10, 2003.
- Settlement date is March 6
- Day before the settlement date is March 5
- Quoted price = 95+16/32=95.5
- Days since last coupon Jan10 – Mar5= 54
- Days in the coupon period = Jan10 – Jul10 = 181 days
- Invoice price = 5.5%* (54/181)+95.5 = 97.14

Concept check

- Assume today is March 7, 2003 (Fri). Consider an 11% coupon bond that matures on July 10, 2009 with a quoted price of 95-16. The most recent coupon date is January 10, 2003 and the next coupon date is July 10, 2003. Which is the settlement date and what is the invoice price?

G) Settlement date is the March 8, invoice price=97.20

Y) Settlement date is the March 7, invoice price=97.20

R) Settlement date is the March 10, invoice price=97.26

Invoice price

Day before settlement	3/5/2003	NCD	1/10/2003
Maturity date	7/10/2009	LCD	7/10/2003
Coupon	11	Accrual	54
Quote price	95.5	Basis	181

=COUPDAYBS(E15,E16,2,1)

=COUPDAYS(E15,E16,2,1)

Accrued interest	1.640884
Invoice price	97.140884

Treasury Bond Futures

- Example: The settlement price on February 4, 2004 for the June 2004 contract was 110-03 (110+3/32) for $100 face value. One contract (traded on the CBOT) involves the delivery of $100000 face value of government bonds that has more than 15 years to maturity on the first day of the delivery month.

Conversion Factor

- The Treasury long-term bond futures contract allow the short party to choose to deliver any bond that has a maturity of more than 15 years and that is not callable within 15 years
- The cash price applicable to the delivery is

  (quoted futures price*conversion factor)+accured interest

Conversion Factor

- Example: Suppose the quoted futures price is 90-00, conversion factor for the bond is 1.38 and the accrued interest on this bond at the time of delivery is $3 per $100 face value. The cash received by the short party is

(1.38*90)+3=127.20 per $100 face value

Conversion Factor

- The conversion factor for a bond is equal to the quoted price the bond would have per dollar of principal on the first day of the delivery month on the assumption that the interest rate for all maturities equals 6% per annum (semiannual compounding)
- The bond maturity at the times to the coupon payment dates are rounded down to nearest three months for the purpose of calculation
- If a bond has maturity of 20 years 2 months then conversion factor is calculated as if the bond had 20 years maturity. If a bond has maturity of 18 years 4 months, the conversion factor is calculated for a bond with 18 years 3 months maturity.

Conversion Factor calculation

- Consider a 10% coupon bond with 20 years and 2 months to maturity
- Calculate conversion factor based on 20 years to maturity and flat TIR at 6%

$$\sum_{i=1}^{40} \frac{5}{(1+0.06/2)^i} + \frac{100}{(1+0.06/2)^{40}} = 146.23$$

- The conversion factor is 146.23/100=1.4623

Conversion Factor calculation

- Consider a 8% coupon bond with 18 years and 4 months to maturity
- Calculate conversion factor based on 18 years and 3 months to maturity and flat TIR at 6%. At 3 months

$$4 + \sum_{i=1}^{36} \frac{4}{(1+0.06/2)^i} + \frac{100}{(1+0.06/2)^{36}} = 125.83$$

- Three months interest = sqrt(1.03)-1 = 1.4889%
- The price today = 125.83/(1.014899) = 123.99
- The conversion factor = 123.99/100=1.2399

Why Conversion Factor

- If we know for sure which bond is going to be delivered, then the futures price should be the cost of buying this bond today and holding it until the maturity of the futures contract
- But a range of bonds with very different coupons and maturities can be delivered
- The conversion factor adjusts for the difference in the price of bonds due to different maturities and coupons

Why Conversion Factor

- Given the best prediction of future interest rates when the futures contract matures, we can calculate the expected price of different bonds that may be delivered
- When the interest rate is different than 6% flat, the futures price will be set such that

  expected price – (futures price * conversion factor) =0
- Where we should assume that the bond with the lowest expected price will be delivered
- The price setting process carries a risk because as interest rate changes a different bond may become the cheapest to deliver

Cheapest to deliver bond

- Futures price =93-08 or 93.25
- Deliverable bonds

Quoted market price	conversion factor	cost of delivery
1. 99.50	1.0382	99.5-93.25*1.0382=2.69
2. 143.50	1.5188	143.5-93.25*1.5188 =1.87
3. 119.75	1.2615	119.75-93.25*1.2615=2.12

CBOT T-Bonds & T-Notes

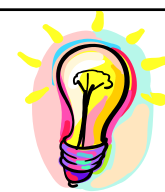
Factors that affect the futures price:

- Delivery can be made any time during the delivery month
- Any of a range of eligible bonds can be delivered
- The wild card play

The wild card play

- Trading at the CBOT Treasury bond futures contract stops at 2pm
- Treasury bonds continue trading until 4pm
- A trader with a short futures position has until 8pm to issue a notice of intention to deliver (to the clearing house)
- The invoice price is calculate on the basis of the settlement price that day. This is the price at which trading was conducted just before the closing bell at 2pm.
- The short party can deliver notice to deliver and buy the bonds at 3:45pm and buy the cheapest to deliver bonds for delivery at the 2pm futures price. If the bond price does not decline, then the short party can wait and try the next day.
- The wild card play is not free. This option is reflected in the futures price

Concept check



- How should the wild card play affect futures price?

G) Increase futures price

Y) Decrease futures price

R) Futures price remain the same

Determining Futures price

- An exact theoretical futures price for Treasury bond contracts is difficult to determine because the short party's option to deliver different bonds and choose different delivery dates.
- If we assume that the cheapest to deliver bond and the delivery date are known then a futures contract on bonds is a futures contract on an underlying asset that provide income, thus

$$F_0 = (S_0 - I)e^{rT}$$

Where I is the present value of coupons during the life of the futures

Bond futures price

- Assume that we know that the cheapest-to-deliver bond will be a 12% coupon bond with a conversion factor of 1.4 and that it will be delivered in 270 days. The last coupon date was 60 days ago and the next coupon date is in 122 days.
- Today' quoted price of this bond is 120.
- The interest rate is flat at 10% cont. rate

Bond futures price

- The cash price is 120+60/(60+122)*6=121.978
- The present value of the future coupon is
  6 exp(-0.1*122/365) =5.803

The futures price for this bond is

$$F_0 = (121.978 - 5.803)e^{rT} = 125.094$$

At delivery the bond will be paid accrued interest, so we subtract it now

125.094-6*(148/148+35) =120.242

Using the conversion factor

Futures price * 1.4 = 120.242

So, quoted futures price = 85.887

Eurodollar Futures

- The three-month Eurodollar futures is the most popular interest rate futures on the CME
- The three-month Eurodollar futures allows an investor to fix in an interest rate on $1M for a future three months period.
- The contracts have maturities in March, June, Sept, and Dec for up to 10 years.
- This means that in Oct of 2006 an investor can lock in an interest rate for three months as far as 2016
- Shorter maturity contracts trade for months other than the above.

Eurodollar Futures

- If Q is the quoted price of a Eurodollar futures contract, the value of one contract is 10,000[100-0.25(100-Q)]
- A Eurodollar futures contract is settled in cash
- When it expires (on the third Wednesday of the delivery month) Q is set equal to 100 minus the 90 day Eurodollar interest rate (actual/360) and all contracts are closed out

Eurodollar Futures

- When the third Wednesday of the delivery month is reached the contract is settle in cash.
- The settlement cash is

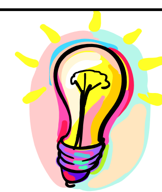
  10,000[100-0.25(100-Q)]
- If the 3-month Eurodollar rate on the third Wednesday is 3% then the final settlement is

  10,000[100-0.25(100-97)]=992,500

Eurodollar Futures

- The effect of futures price change
- A change of one basis point or 0.01 in a Eurodollar futures quote corresponds to a contract price change of $25
- Assume settlement price is 98.84 futures contract price=10K(100-0.25(100-98.84)=997,100
- If price changes by 1pbs futures contract price =10K(100-0.25(100-98.85) =997,125

Concept check question



If you believe (predict) that interest rates are going to increase (decrease) for the next two years, should you long or short 3 month-Eurodollar futures maturing in August 2008?

G) Long (short)

R) Short (long)

Forward Rates and Eurodollar Futures

- Apart from the daily settlement, the Eurodollar futures is similar to the forward rate agreement contract
- For short maturities (up to one year) the Eurodollar futures interest rates can be assumed to be the same as the corresponding forward interest rates
- For longer maturities forward rates differ from futures rates because futures contract have daily settlement.

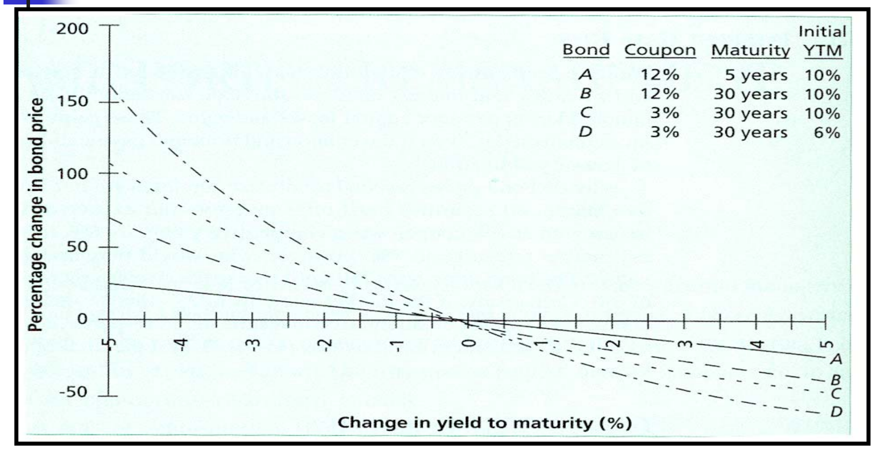
Forward Rates and Eurodollar Futures

A "convexity adjustment" often made is

$$\text{Forward rate} = \text{Futures rate} - \frac{1}{2}\sigma^2 t_1 t_2$$

where t_1 is the time to maturity of the futures contract, t_2 is the maturity of the rate underlying the futures contract (90 days later than t_1) and σ is the standard deviation of the short rate changes per year (typically σ is about 0.012)

How interest rate affects bond price



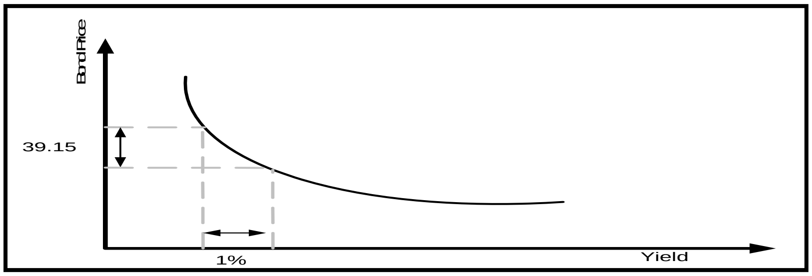
How interest rate affects bond price

Characteristics of option-free bonds

- Price and yield have negative relations
- Bond price change is higher when interest rates decrease (convex relation)
- A longer-maturity bond is more sensitive to interest rate changes, everything else equal
- A lower-coupon bond is more sensitive to interest rate changes, everything else equal

How interest rate affects bond price

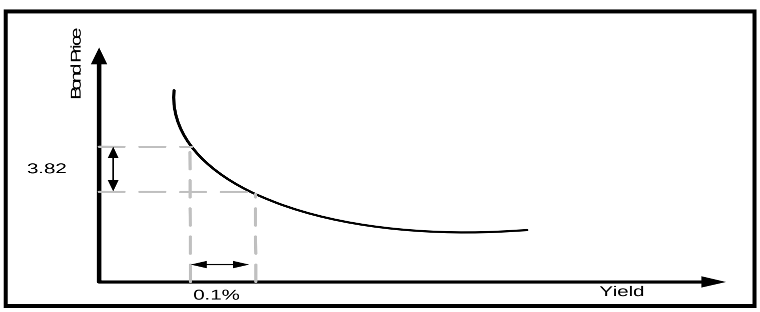
- What is the price change of a 5-year zero-coupon bond when the yield (flat yield curve) changes from 5% to 4%?



P(5%)= 781.2; P(4%) = 820.35; Price change = $39.15

How interest rate affects bond price

- What is the price change of a 5-year zero-coupon bond when the yield changes from 5% to 4.9%?



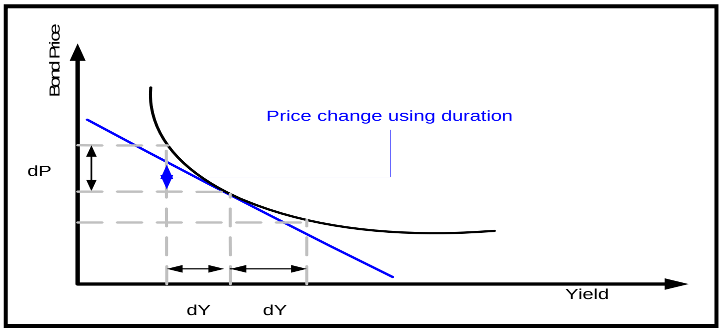
P(5%)= 781.2; P(4.9%) = 785.02; Price change = $3.82

Duration

- Duration is a measure of how sensitive a fixed income security such as a bond is to changes in the interest rate.
- The duration measure is derived under the assumption that the interest rate change is the same for all spot rates. That is, that the term-structure of interest rates moves in parallel.
- To obtain duration, for a portfolio of known cash flows such as a bond, we take the derivative of the bond price with respect to the interest rate

Duration

- Approximates price change using the first derivative of a bond's price



$$dP = \frac{dP}{dy} dy + error$$

Duration (for cont. comp rates)

- Bond price calculated as the sum of discount cash flows
- Duration of a bond that provides cash flow c_i at time t_i is

$$B = \sum_{i=1}^{n} \left[c_i e^{-yt_i} \right] \rightarrow \frac{\partial B}{\partial y} = \sum_{i=1}^{n} -t_i \left[c_i e^{-yt_i} \right]$$

$$\frac{1}{B} \frac{\partial B}{\partial y} = \frac{1}{B} \sum_{i=1}^{n} -t_i \left[c_i e^{-yt_i} \right]$$

where B is its price and y is its yield (continuously compounded)

- Define duration as $D = \sum_{i=1}^{n} t_i \left[c_i e^{-yt_i} \right]$

Duration (for cont. comp rates)

- When consider delta y to be a small change in the interest rates and delta B to be the corresponding change in the bond price, this leads to

$$\frac{\Delta B}{B} = -D \Delta y$$

Duration (for cont. comp rates)

- The duration of a bond portfolio equals the weighted average of the duration of each security
- If the securities are short then it is viewed as a liability

Duration Matching

- This involves hedging against interest rate risk by matching the dollar durations of assets and liabilities
- It provides protection against small parallel shifts in the zero curve (in this case cont. comp.)

Duration-Based Hedge Ratio

Consider the situation where a position in an interest rate dependent asset such as a bond portfolio is being hedged using interest rate futures

F_C	Contract Price for Interest Rate Futures
D_F	Duration of Asset Underlying Futures at Maturity
P	Value of portfolio being Hedged
D_F	Duration of Portfolio at Hedge Maturity

Duration-Based Hedge Ratio

- If we assume that delta y is the same for all maturities. That is, the term structure of interest rates shift in parallel, then it is approximately true that

$$\Delta P = -PD_P\Delta y$$

- And that

$$\Delta F_C = -F_C D_F \Delta y$$

Duration-Based Hedge Ratio

- Let N be the number of futures contract needed
- Equating the two price changes, we have that

$$-PD_p \Delta y = -NF_C D_F \Delta y \rightarrow N = \frac{PD_p}{F_C D_F}$$

Duration-Based Hedge Ratio

- It is Aug 2 and a fund manager has 10M invested in government bonds and is concerned that interest rates are expected to be highly volatile over the next three months. He decides to use the Dec Treasury bond futures to hedge
- The current futures price is 93-02 or 93.0625. Each contract is for delivery of $100000 face value, thus the contract price is 93,062.5

Duration-Based Hedge Ratio

- The contract price is 93,062.5
- The duration of the bond portfolio in 3 months is 6.8 years. The cheapest-to-deliver bond is expected to be a 20-year 12% (per annum) coupon bond. The yield on this bond is currently 8.8% and the duration will be 9.2 years at maturity of the futures contract.
- The number of contracts needed to hedge this bond portfolio is

$$N = \frac{PD_p}{F_C D_F} \rightarrow N = \frac{10M * 6.8}{93062.5 * 9.2} = 79.42$$

Duration-Based Hedge Ratio

- Suppose that during the period from Aug 2 to Nov 2 interest rates decline rapidly and the value of the bond portfolio increases from $10M to 10.45M. Suppose further that on Nov 2 the bond futures price is 98-16 or 98.5
- The total loss on the Treasury bond futures is

  79*(98500.00-93062.5)=429,562.5
- The net change in the value of the portfolio's manager total position is

  450000.00-429562.5=20,437.5

Hedging floating rate loans

- Suppose today is April 29.
- A company just borrowed 15M for three months. The interest rate for each one month in the three months period is the one-month LIBOR+1%

	May 1	June 1	July 1	August 1
1-month 9%				
LIBOR+1				
Payment		9%/12*15M	Jun rate*15M	July rate*15M

Hedging floating rate loans

	May 1	June 1	July 1	August 1
1-month 9%				
LIBOR+1				
Payment		9%/12*15M	Jun rate*15M	July rate*15M
3-month			Jun rate*15M	Sept future rate*15M
LIBOR				

Strategy

Short June and September Eurodollar futures contracts

Duration-Based Hedge Ratio

- The one month rate has a duration of 0.833 and the three month duration of 0.25
- Suppose the quote price of this contract is 91.88
- The contract price is

  10K*(100-0.25(100-91.88)) = 979,700
- The number of contracts needed is

$$N = \frac{PD_p}{F_C D_F} \rightarrow N = \frac{15M * 0.08333}{979,700 * 0.25} = 5.1$$

Hedging mismatch in assets and liabilities

- Consider a small bank. In general the assets of an un-hedged bank are longer-term than the liabilities
- To hedge interest rate risk, first we find the overall duration of a bank
- Treat assets as long positions and liabilities as short position. Suppose the bank has 15M in assets with duration (from cont. comp.) of 15 years and has 10M of liabilities with duration of 8 years. The share holders equity's (SHE) is 5M and the duration of the SHE is

$$D_E = \frac{AD_A - LD_L}{A - L} = \frac{15M * 15 - 10 * 8}{5M} = 29$$

Hedging mismatch in assets and liabilities

- Suppose the bank wants to hedge interest rate risk for the next 9 months, it can use a bond futures that matures in 10 months. Assume the back uses futures on a 8% 25-year coupon bond which has a duration of 21 years. The futures price is currently 93.5. Thus the futures contract price is 93,500 (delivery of 100K face value).
- The bank needs to ________

$$N = \frac{5M * 29}{93500 * 21} = 73.84 \quad \text{contracts}$$

Concept check question

What is the nature of the duration of a pension fund?

G) Duration of a pension fund is positive

Y) Duration of a pension fund is negative

R) Close to zero

Concept check question

If you want to use a futures on bond to hedge interest rate risks of a pension fund, would you long or short the futures contract?

G) Long

R) Short

Duration

- When the yield y is expressed with compounding m times per year

$$\Delta B = -\frac{BD\Delta y}{1 + y/m}$$

- D is Macaulay duration and the expression

$$\frac{D}{1 + y/m}$$

is referred to as the “modified duration”

When yield is semiannual

r equal the half year rate or half of the semiannual rate

Bond price given a yield of r,

$$B = \sum_{t=1}^{n} \frac{C_t}{(1+r)^t}$$

Change:

$$\frac{dB}{dr} = -\frac{1}{1+r} \sum_{t=1}^{n} \frac{tC_t}{(1+r)^t}$$

C_t = cash flow at time t; n = # of semiannual periods

$$\frac{dB}{dr}\frac{1}{B} = -\frac{1}{1+r}\left[\frac{1}{B} \sum_{t=1}^{n} \frac{tC_t}{(1+r)^t}\right]$$

Duration

Define Macaulay duration, D_{mac} as:

$$D_{mac} = \frac{1}{B} \sum_{t=1}^{n} \frac{tC_t}{(1+r)^t}$$

Define Modified duration, D, as

$$D = \frac{1}{1+r}\left[\frac{1}{B} \sum_{t=1}^{n} \frac{tC_t}{(1+r)^t}\right] = \frac{1}{1+r} D_{mac}$$

We have:

$$\frac{dB}{dr}\frac{1}{B} = -\frac{1}{1+r} D_{mac} = -D$$

Duration

Percentage price change for dy change in interest rate is:

$$\frac{dB}{B} = -\frac{1}{1+r} D_{mac} dr = -D dr$$

Caveats

- Duration hedging is not a complete hedging, it is the first approximation of the sensitivity of the portfolio to interest rates.
- Duration hedging only works when the TIR move is parallel
- Hedging can be improved by matching $convexity as well as $duration.
- As interest rate changes, so do duration and convexity
- How often to rebalance your hedge portfolio depends on cost of hedging

Summary

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- Calculating duration
- Duration based hedging