CH9. Two-Period Model: the Consumption-Savings Decision

EE312 (for Section 046402 - Sicha)

Read: Williamson Ch. 9

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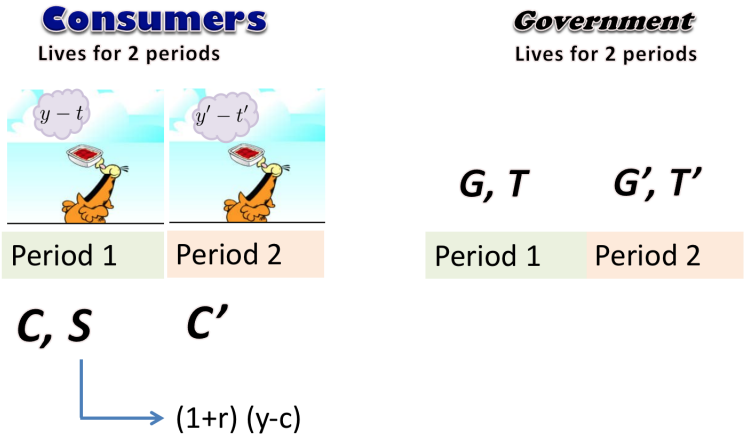
2 Overview



3 Two-Period Model

- Macroeconomics studies how key economic variable evolve over time
- The simplest way to think about interemporal decsion is in a two-period model
  - the first period is current period (or today)
  - the second period represents the future (or tomorrow)
- The consumer makes **intertemporal choice** between current consumption and future consumption.
  - Saving (lending) and dissaving (borrowing).
  - **The real interest rate** is the relative price of future consumption in terms of current consumption.
- Key trade-off : consuming today or consuming tomorrow or the consumption-saving decision
- Decisions to be affected by changes in the real interest rate and in current and future incomes.

4 Consumers

- Assume the consumer receives exogenous income.
- To keep this simple, we will omit labour/leisure choice.

4.1 The consumer's budget constraint:

c = current consumptions
s = current saving
y = current income
t = current lump-sum taxes

4.1.1 Current budget constrant

- The current disposable income $(y - t)$ equals consumption plus saving $(c + s)$.

$$c + s = y - t$$

- Assuming bonds directly traded in the credit market.Assume one single real interest rate (r) for borrowers and lenders.
- A bond is a promise to pay $1 + r$ units of consumption goods tomorrow in exchange for 1 unit of consumption goods today.
- r is the real interest rate on bonds.
- Consumer can exchange one unit of current consumption for (1+r) units of future consumption in the credit market.
  - Consumer can exchange (1+r) unit of future consumption for one unit of current consumption in the credit market.
- Consumer can exchange 1 unit of future consumption for ..... unit of current consumption in the credit market.
- **The relative price of future consumption in terms of current consumption is** $\frac{1}{(1+r)}$.
- If s > 0, the consumer is saving. The consumer is a **lender** in the credit market.
- If s < 0, the consumer is disaving. The consumer is a **borrower** in the credit market.

4.1.2 Future budget constraint

- The consumer's future budget constraint:
- If s < 0, the consumer pays the interest and principal on loan.

$$c' = y' - t' + (1 + r)s \tag{1}$$

- consumption must equal disposable income in the future period $(y' - t')$ plus gross return on savings.

4.1.3 Lifetime budget constraint :

- use $c' = y' - t' + (1+r)s$ and $c + s = y - t$

$$\begin{aligned} c' &= y' - t' + (1+r)s \\ s &= \frac{c' - y' + t'}{(1+r)} \end{aligned}$$

since $c + s = y - t$

$$\begin{aligned} c + \frac{c' - y' + t'}{(1+r)} &= y - t, \\ c + \frac{c'}{(1+r)} &= y + \frac{y'}{(1+r)} - t - \frac{t'}{(1+r)}. \end{aligned}$$

$$\begin{array}{ccc} \text{The PV of lifetime} & = & \text{PV of lifetime income} \\ \text{consumption} & & \text{minus PV of lifetime taxes.} \end{array}$$

$$c + \frac{c'}{(1+r)} = \left(y + \frac{y'}{(1+r)} \right) - \left(t + \frac{t'}{(1+r)} \right) \tag{2}$$

- The PV of lifetime consumption equals PV of lifetime income minus PV of lifetime taxes.
- The lifetime disposable income is the same as **lifetime wealth (we).**

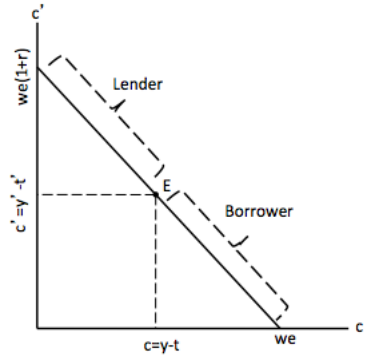
$$\begin{aligned} we &= y + \frac{y'}{(1+r)} - t - \frac{t'}{(1+r)}. \\ c + \frac{c'}{(1+r)} &= we \end{aligned}$$

$$c' = -(1+r)c + we(1+r) \tag{3}$$

- $we(1+r)$ = what could be consumed in the future period if the consumer saved all of his or her current-disposable income and consume lifetime wealth (after earning the real interest rate r on savings) in the future period.
- $(1+r)c$ = future value of current consumption
- plot c' (future consumption) against c (current consumption)

Life time budget constraint : $c' = -(1+r)c + we(1+r)$

- E = endowment point where s = 0;
- c = y - t;
- $c' = y' - t'$.
- Above E, the consumer is a lender.
- Below E, a borrower.

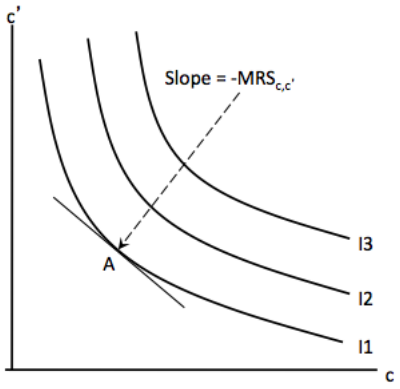


4.2 The consumer's preferences

- A **consumption bundle** is a combination of current and future consumptions.
- Properties of consumer preference:
  - More is preferred to less.
  - **Diversity** in the consumption bundle is preferred (consumption smoothing).
  - Current and future consumptions are **normal goods**.

The consumer 's indifference curves

- The ICs are downward-sloped and convex.
- Slope = -$MRS_{c,c'}$ = the marginal rate of substitution of c for c'.
- $MRS_{c,c'}$ is falling as c increases .



4.3 Consumer optimization

- The consumer choses c, c' to maximize utility subject to the life time budget constraint.
- Consumer choses a consumption bundle on the BC. The indifference curve is tangent to the life time budget constraint.
- The optimization condition:

$$\frac{MU_c}{MU_{c'}} = MRS_{c,c'} = (1+r)$$

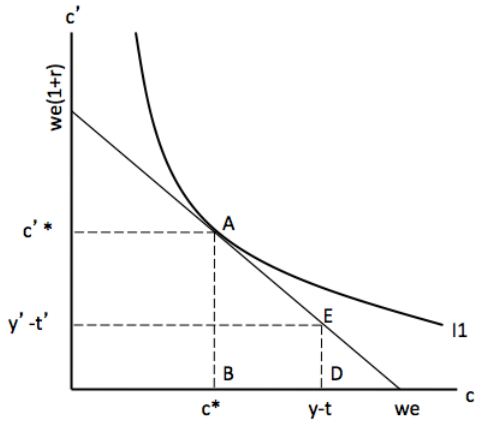
- The consumer is optimizing where the marginal rate of substitution of c for c' equals the relative price of c in terms of c'.

– The willingness to trade c for c' equals the market rate of trading c for c'.

- The optimized consumption bundle is (c, c') = (c*, c'^*).

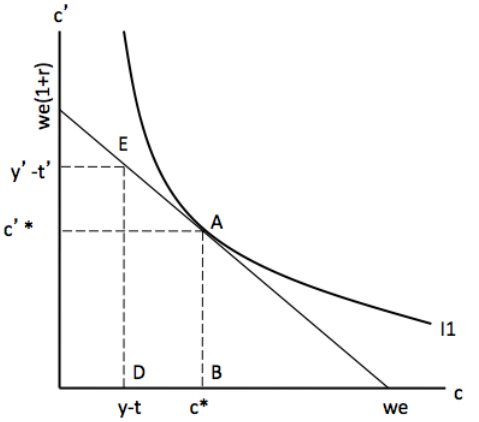
4.3.1 The consumer is a lender.

- At A, (c, c') = (c*, c'^*) and c*< (y - t).
- s = y – t – c* = BD > 0.



4.3.2 The consumer is a borrower.

- At A, (c, c') = (c*, c'^*) and c*> (y - t).
- s = y – t – c* = BD < 0.



4.3.3 An increase in current income

- An increase in **current income** results in an increase in **lifetime wealth**.
- **A pure income effect**.
- The budget line shifts horizontally to the right

$$
\begin{aligned}
we_1 &= y_1 + \frac{y'}{(1+r)} - t - \frac{t'}{(1+r)} \\
we_2 &= y_2 + \frac{y'}{(1+r)} - t - \frac{t'}{(1+r)} \\
\Delta we &= we_2 - we_1 \\
&= y_2 - y_1
\end{aligned}
$$

- Both current and future consumptions increase (normal goods).
  - The increase in c is **smaller** than the increase in y.
  - Saving increases; hence, c* increases.
  - The consumer prefers diversity in the consumption bundle — **consumption smoothing**.

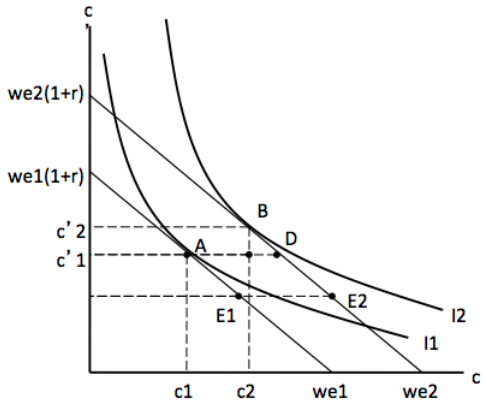
$$\Delta s \quad = \quad \Delta y - \Delta t - \Delta c$$

and because $\Delta t = 0$, and $\Delta y > \Delta c > 0$,

$$\Delta s > 0.$$

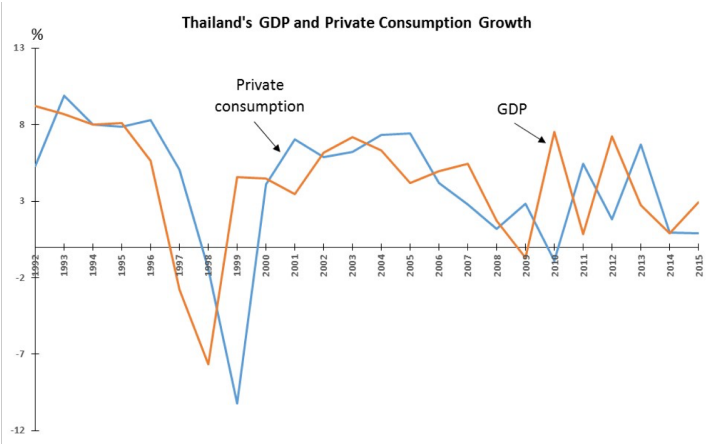
Increase in current y for a lender.

- Both c and c' increase (A to B).
- $\Delta c = c_1 c_2 < \Delta Y = AD$
- $\Delta s > 0$
- s = y – t – c*
- $\Delta s = \Delta y - \Delta t - \Delta c$ = BD > 0.



Consumption smoothing

- **An increase in current income causes an increase in consumption in both periods and an increase in savings**.
- This behaviour arises because of the consumer's desire to **smooth consumption over time**.
- The theory **predicts aggregate consumption is less volatile than aggregate output.**
- Theory is qualitatively consistent with data
  - Aggregate consumption is less variable than GDP.
  - Consumption of **nondurables and services** is even less variable.
  - Consumption of **durables** is more volatile.
    - Durable consumption is more like investment.
    - Returns of service flow from durable goods.
- While consumption is less volatile than GDP, it is still not smooth enough to be in line with the theory.



4.3.4 Increase in future income

An increase in future income (y')

- An increase in **future income** results in an increase in **lifetime wealth**.
- The budget line shifts vertically to the top.

$$
\begin{aligned}
we_1 &= y + \frac{y_1'}{(1+r)} - t - \frac{t'}{(1+r)} \\
we_2 &= y + \frac{y_2'}{(1+r)} - t - \frac{t'}{(1+r)} \\
\Delta we &= we_2 - we_1 \\
&= \frac{y_2' - y_1'}{1+r}
\end{aligned}
$$

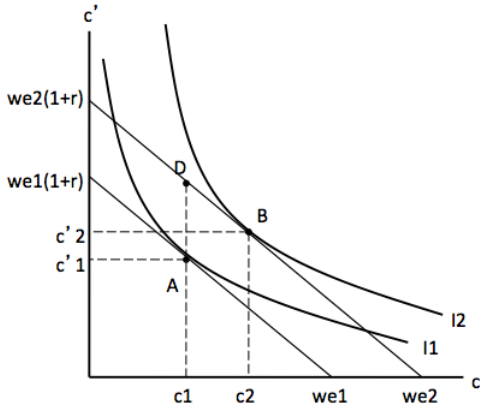
- Future consumption increases, but by a lesser amount than the increase in future income.
  - **Saving decreases**; **current consumption increases**.
- The increase in future income is smoothened into increases in both future and current consumption.

$$\Delta s = \Delta y - \Delta t - \Delta c$$

and because $\Delta t = 0$, and $\Delta y = 0$,

$$\Delta c > 0,\ \Delta s < 0.$$

- Both c and c' increase (A to B).
- $\Delta c' = c'_1 c'_2$
- $\Delta c' < \Delta y' = AD$
- $\Delta s < 0$

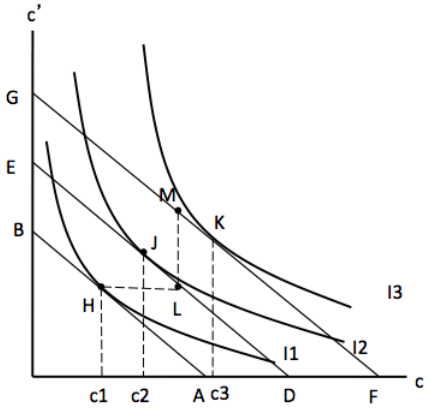


4.3.5 Temporary and permanent increases in income

- Consumer's response will be different when they face temporary or permanent changes in income
- **A permanent increase** in income has a larger positive effect on lifetime wealth and current income than a **temporary increase**.
  - **Milton Friedman's 'permanent income hypothesis':** the level of current consumption depends on the level of permanent income'.
- The consumer will tend to save most of the **temporary increase** in income.
- Temporary changes in income yield small changes in permanent income, hence will have small effects on current consumption.
- Permanent increase: increase both y and y'

Temporary versus Permanent Δy

- HJ = effect of temporary rise in y.
- HK = effect of permanent rise in y.



- **A temporary increase in y** = HL: the budget line shifts from AB to ED.
  - The consumption bundle rises from H to J.
  - Current consumption rises less than current income; saving increases — **consumption smoothing.**

- **A permanent increase in y** = $y_2 - y_1 = y_2' - y_1'$: the budget line shifts from AB to GF.
  - $y_2 - y_1$= HL = $y_2' - y_1'$= LM.
  - The consumption bundle rises from H to K.
  - Larger effect on current consumption: c1c3

4.3.6 Effect of a tax cut?

- The effect of the government's tax cut on consumption depends on whether the cut is temporary or permanent.
- **If temporary**, the increase in consumption will be small; most of the increased income is saved.
- **If permanent**, the increase in consumption will be large.

4.3.7 An increase in the real interest rate

4.3.8 An increase in the real interest rate and the budget line

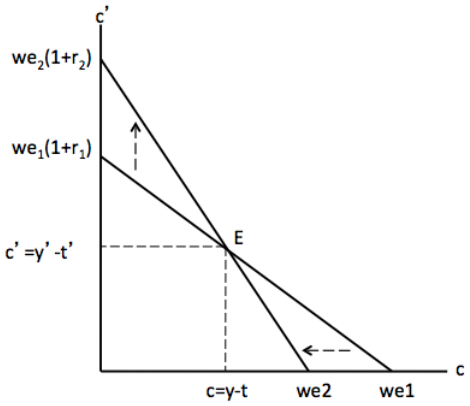
- $\frac{1}{1+r}$ is the relative price of future consumption in terms of current consumption.
- The slope of the budget line is -(1+r).
- A change in the real interest rate causes the budget line to rotate.
- Changes in intertemporal decision between current and future consumptions.
- Analysis of the substitution effect and income effect.

$$\begin{aligned} we &= y + \frac{y'}{(1+r)} - t - \frac{t'}{(1+r)} \\ we(1+r) &= y(1+r) + y' - t(1+r) - t' \end{aligned}$$

- The budget line rotates upwards as the real interest rate increases.
- 'we' decreases (horizontal intercept).
- we(1+r) increases if r increases (vertical intercept).
- The endowment point (E) remains the same (no change in the initial endowment of y - t, y'- t').
- E is the pivot point.

An increase in the real interest rates

- The increase in the real interest rate rotates the budget line upwards with the same E.



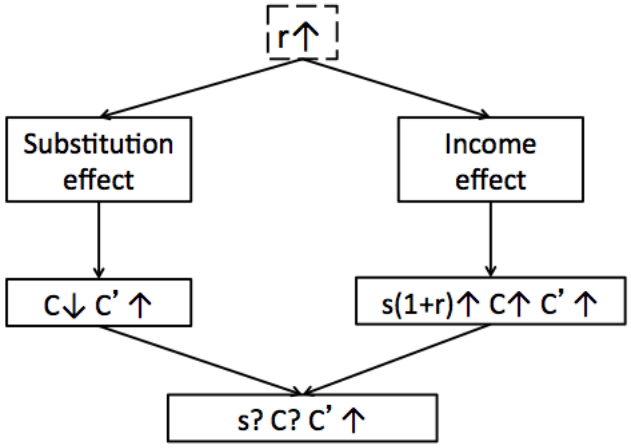
4.3.8.1 The dual effects of a higher r An increase in the real interest rate (r) causes a change in the relative price of current and future consumptions.

- **Future consumption** becomes cheaper; **current consumption** becomes more expensive.
- Higher return on savings; less sacrifice in current consumption is needed for given future consumption.
- **Lender:** cheaper future consumption.
- **Borrower:** more expensive current consumption

4.3.8.2 Effect of higher r on the lender

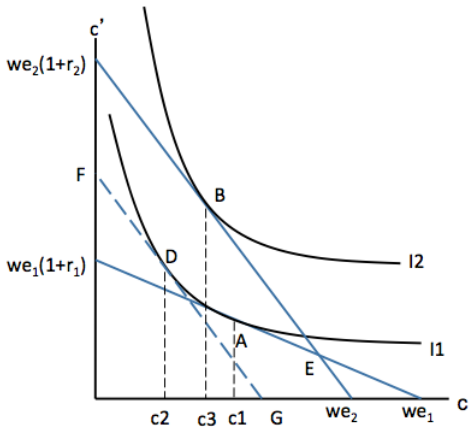
- Future consumption becomes cheaper in terms of current consumption.
- **The substitution effect**: more future consumption for less current consumption.
- **The income effect**: given savings yield more future income — higher current and future consumptions.
- Higher future consumption; unclear current consumption and savings, given y and y'

Effect of higher r on the lender



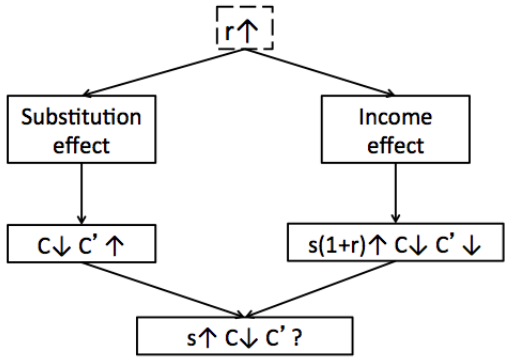
Stronger substitution effect

- AD = substitution effect; lower c.
- DB = income effect; higher c.
- AD > DB; lower c at c3, assuming **a lender**.



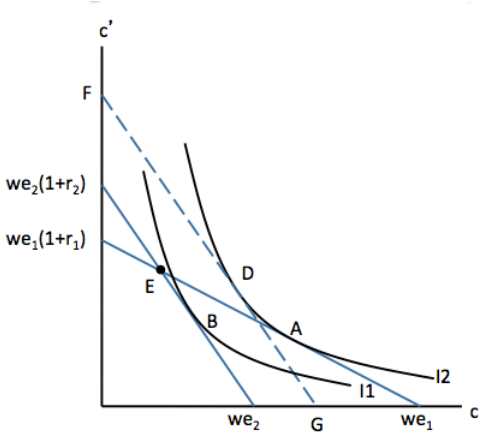
4.3.8.3 An increase in r for a borrower

- Current consumption becomes more expensive in terms of future consumption.
  - **The substitution effect**: more future consumption and less current consumption.
  - **The income effect: loans** for future consumption become more expensive — reduced current and future consumptions.
- Current consumption decreases while saving increases; unclear future consumption.



An increase in r for a borrower

- AD = substitution effect; lower c and higher c'.
- DB = income effect; lower c and c'.
- Net effect: lower c; unclear c'.



4.3.8.4 Conclusions on effects of Δr

- A higher real interest rate (r) has an intertemporal substitution effect.
  - Future consumption is substituted for current consumption — saving increases.
- Positive income effect for lenders but negative income effect for borrowers.
- No certainty that current consumption will fall if the real interest rate rises.

5 The government sector

- G = current government purchase of goods.
- G'= future government purchase of goods.
- T = current taxes collected by the government.
- N = number of consumers, each paying the current tax of t; so T = Nt.
- T'= future taxes; and T' = Nt'.
- Government borrows by issuing bonds (B) at the real interest rate of r.

5.1 Government's current budget constraint

- Government spending is financed by taxes and bond issue in each period.
- Government's current budget constraint: G = T + B or G – T = B.
  - B > 0 government is a borrower;
  - B < 0 government is a lender.
- Government's future budget constraint: $G' + (1+r)B = T'$
- Government present-value budget constraint : The present value of government purchases must equal the present value of taxes.

$$
\begin{aligned}
G &= T + B \\
G' + (1+r)B &= T' \\
\text{total government outlays in the future} &= \text{future taxes} \\
B &= \frac{T' - G'}{1+r} \\
G + \frac{G'}{1+r} &= T + \frac{T'}{1+r}
\end{aligned}
$$

5.2 Competitive equilibrium

- Consumers and government interact in the credit market.
  - Trading of future consumption goods for current consumption goods through the interest rate.
- Equilibrium condition:
  - Each consumer optimizes current and future consumptions and saving, given r.
  - Government budget constraint holds.
  - The credit market clears.

The credit market clears.

- Private savings (sp) equals government borrowing (B) or $S^p = B$
  - National saving is the sum of private savings and government savings; $S = S^p + S^g$.

$$\begin{aligned} S &= S^p + S^g \\ S^p &= B \\ S^g &= -B \\ S &= B - B = 0 \end{aligned}$$

The income-expenditure identity

- The credit-market clearing implies that the income-expenditure identity holds.

$$\begin{aligned} Y &= C + G \\ S^P &= Y - C - T \\ B &= G - T \\ Y - C - T &= G - T \\ Y &= C + G \end{aligned}$$

5.3 The Ricardian Equivalence

- **A change in current taxes** with an equal and opposite change in the present value of future taxes has **no effect on the real interest rate and the consumption of individual consumers.**
  - Assume equilibrium in the credit market, given r.
  - Current and future government spending are held constant.
  - Consumers' life-time budget constraint and government's present-value budget constraint.

5.3.1 Algebraic formulation

$$G + \frac{G'}{1+r} = Nt + \frac{Nt'}{1+r}$$

$$t + \frac{t'}{1+r} = \frac{1}{N}\left[G + \frac{G'}{1+r}\right] \quad \text{....(eq.9.25)}$$

$$\text{From } c + \frac{c'}{1+r} = y + \frac{y'}{1+r} - \left[t + \frac{t'}{1+r}\right]$$

$$c + \frac{c'}{1+r} = y + \frac{y'}{1+r} - \frac{1}{N}\left[G + \frac{G'}{1+r}\right] \quad \text{....(eq.9.26)}$$

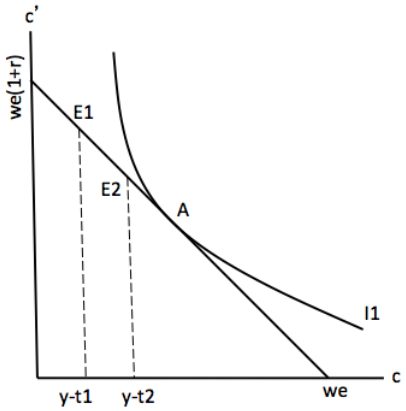
$$t + \frac{t'}{1+r} = \frac{1}{N}\left[G + \frac{G'}{1+r}\right] \quad \text{....(eq.9.25)}$$

$$c + \frac{c'}{1+r} = y + \frac{y'}{1+r} - \frac{1}{N}\left[G + \frac{G'}{1+r}\right] \quad \text{....(eq.9.26)}$$

- The change in current taxes (Δt) is matched by $-(1+r)\Delta t$ so that equation 9.25 holds.
  - Equation 9.26 remains unchanged, given r (as y, y', G, G' and N are the same).
  - And Y = C + G; the credit market clears.
  - No welfare change for consumers.

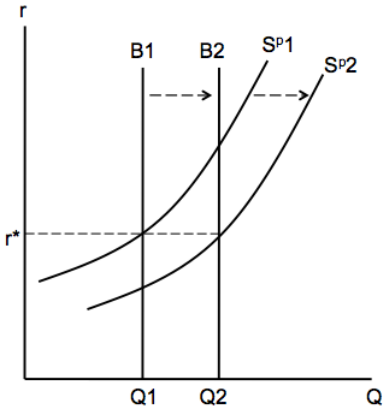
5.3.2 A current tax cut for a borrower

- A current tax cut equals a future tax increase.
- Lifetime wealth and consumption bundle (A) are the same.
- Only the endowment point changes from E1 to E2.



5.3.3 Unchanged credit market

- Private saving and government borrowing increase by the same amount.
- Equilibrium r is the same.



5.3.4 A tax cut is not a free lunch!

- current tax cut gives all consumers higher current disposable income.
- But consumers must bear higher future taxes by the same amount.
- No welfare gain for consumers!

5.3.5 Ricardian equivalence assumptions

- A tax change affects every consumer by the same amount so that the present-value tax burden is unchanged.
  - If some consumers receive higher tax cuts, then their lifetime wealth and consumption choices (and the real interest rate) change.
- Future tax burdens may be shared unequally.
- Government can redistribute wealth among income classes through tax policy!
- Government debt is paid off during the lifetimes of current consumers.
  - But the government can postpone debt payment (and future taxes) to next generations.
  - The old receive tax cuts and higher disposable incomes; the young pay higher future taxes.
- The government can redistribute wealth between generations!
  - The effect of the social security programs.
- The tax is lump-sum.
  - All taxes cause distortions in the relative prices and consumption choices.
  - Welfare loss is greater than tax revenues.
- **Perfect credit markets:** consumers can borrow and lend as much at the same interest rate.
  - But consumers have limits on borrowing.
  - The borrowing rate is higher than the lending rate.
  - Government borrows at a lower rate.
  - Credit-constrained consumers benefit from a tax cut.

TIM HARFORD
THE UNDERCOVER ECONOMIST

Why a tax cut just isn't fair on teenagers

Alistair Darling did something rather strange recently, to baffling applause from his own backbenchers, and cries of "bribery" from the opposition: he announced a tax on teenagers.

Darling's plan – for those who missed it – is to cut income taxes temporarily for all but the most prosperous taxpayers. The apparent windfall is £120 a head. A similar plan is already in place in the US, where a temporary "tax rebate" began to arrive in the bank accounts of a grateful nation about a month ago.

But there is no such thing as a free lunch: since neither the UK nor US governments plans to alter its spending plans, these tax holidays will be funded by government borrowing – borrowing that must eventually be repaid. That will require taxes to go up in the future, or not to fall when they otherwise might.

Who should celebrate? Not the typical taxpayer, that is for sure. The tax cut makes no difference to her. If she – assume she is British – had wanted an extra £120 right now, she could already have it in her pocket, either by withdrawing it from savings or by borrowing the money. If she did that, of course, she would later have to repay £120 plus interest. But that is exactly what Darling's successor as chancellor will require of her. To look at it another way, the rational taxpayer should save the £120 windfall now, keeping it to pay the higher taxes that are surely on the horizon.

But whichever way you look at it, the US and UK governments are handing their citizens borrowed cash – and the citizens themselves are liable for the debt. If my bank manager arranged a surprise loan in my name and handed me the cash, I might feel pampered or put-upon depending on whether I was planning to take out the loan myself anyway. Either way, I doubt I would feel any richer.

Of course, some people should count themselves wealthier after the tax cut. Anyone expecting to die without making a bequest should be pleased: if the Grim Reaper knocks on the door before the taxman does, he can spend the tax rebate now and leave the bill for some other sucker.

Who will be the fall guy? We don't know for sure, because we can't say who a future government will tax. But an obvious candidate would be today's teenagers, very few of whom are paying income tax right now, but most of whom will pay it in the next few years. Their best hope is that their grandparents add the tax windfall to their bequests rather than blowing the money on a weekend in the sun.

The idea that a debt-funded tax cut makes little difference to anybody is called "Ricardian equivalence", after David Ricardo, one of the founders of modern economics. The equivalence is between government taxes and government borrowing. However government spending is funded, it generates a bill that will fall due sooner or later. Far-sighted taxpayers will immediately take note.

Clearly, there are reasons for some taxpayers to care whether taxes arrive today or later on with interest. Even so, these tax gimmicks matter much less than we might think. It is current government spending, not current government taxation, that is the real measure of a government's size.

Empirical economists are still arguing over whether Ricardian equivalence holds good, but one study by Matthew Shapiro and Joel Slemrod concluded that most US citizens used a 2001 tax windfall to pay off their debts, leaving more money available to pay future taxes – Ricardian equivalence in action.

That suggests that as consumers and taxpayers, we aren't fooled by fiscal sleight of hand. Are we fooled as voters? Alistair Darling obviously hopes so.

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