

1.a) Based on the regression results provided, write out the estimated coefficients in the form of regression equation (1.1). Interpret the estimated coefficients associated with  $educ_i$ . Based on Model (1.1), test whether education has an impact on logarithm of hourly wage. Show your work. (Use  $\alpha = 0.05$ )

①  $\log cwage_i = 0.4436 + 0.0708 educ_i + 0.03993 exper_i - 0.000598 expersq + 0.1925 union_i - 0.44216 female_i$ ;

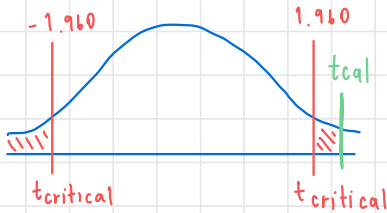
② if years of school increase by 1 year, logarithm of hourly wage will increase by 0.0708 dollar.

③ •  $H_0: \beta_2 = 0$ ; null hypothesis  
 $H_a: \beta_2 \neq 0$

$$\begin{aligned} \bullet t_{cal} &= \frac{\hat{\beta}_2 - \beta_2}{se \hat{\beta}_2} \\ &= \frac{0.07085 - 0}{0.00529} \\ &= 13.5468 // \end{aligned}$$

•  $\alpha = 0.05$     d.f = 1254

$t_{critical} = 1.960$



$\therefore$  We can reject null hypothesis and we can make sure 95% that education impact on logarithm of wage.

1.b) What is the overall significance of the regression from Model (1.2)? What test do you use?  
(Use  $\alpha = 0.05$ )

used : F-test

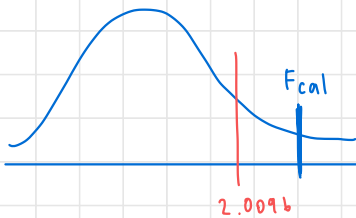
$$H_0 : \beta_1 = \beta_2 = \beta_3 = \beta_4 = \beta_5 = \beta_6 = \beta_7 = \beta_8 = 0$$

$H_a$  ; otherwise

$$F_{cal} : \frac{E_{ss}/u-1}{R_{ss}/n-u} = \frac{168.69715/7}{276.292816/1252} = 109.5496$$

$$\alpha : 0.05$$

$$F_{crit}(7, 1252) : 2.0096$$



$\therefore$  We can reject null hypothesis and we can make sure 95% that overall variables are significant in 1.2 model.

1.c) If we are interested in testing whether "physical attractiveness" has an impact on logarithm of hourly wage at all, what kind of null/alternative hypothesis would we be testing? Perform the test and discuss your finding. (Use  $\alpha = 0.05$ )

: F-test ; marginal contribution

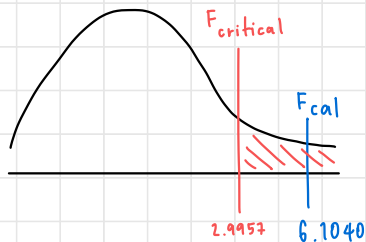
$H_0$  : physical attractiveness has no impact on logarithm of hourly wage ;

$H_a$  : otherwise

$$\begin{aligned} F_{cal} &= \frac{Ess_{1,2} - Ess_{1,1}}{2} \\ &= \frac{Rss_{1,2} / n - u_{1,2}}{(276.282916) / 1260 - 8} \\ &= \frac{1.9429}{0.22} = 6.1040 \end{aligned}$$

$$\alpha = 0.05$$

$$F_{cri}(2, 1252) = 2.9957$$



$\therefore$  We can reject  $H_0$

1.d) Is there convincing evidence that women with above average looks earn more than women with average looks? Explain.

$$\begin{aligned}
 \text{a5 (1.2)} \quad \log(\text{wage}_i) &= \beta_1 + \beta_2 \text{educ}_i + \beta_3 \text{exper}_i + \beta_4 \text{expersq}_i + \beta_5 \text{union}_i + \beta_6 \text{female}_i \\
 &+ \beta_7 \underbrace{\text{belavg}_i}_{\text{below}} + \beta_8 \underbrace{\text{abvavg}_i}_{\text{above}} + u_i
 \end{aligned}$$

• woman, average looks  $\rightarrow \text{belavg}_i = 0 \quad \text{abvavg}_i = 0$

$$\log(\text{wage}) = 0.4797 + \beta_2 \text{educ}_i + \beta_3 \text{exper}_i + \beta_4 \text{expersq}_i + \beta_5 \text{union}_i - 0.438235(1) + 0 + 0$$

$$\log(\text{wage}) = 0.041465 + \beta_3 \text{exper}_i + \beta_4 \text{expersq}_i + \beta_5 \text{union}_i - \text{woman with average looks}$$

• woman (1) above average (1)

$$\log(\text{wage}) = 0.4797 + \beta_2 \text{educ}_i + \beta_3 \text{exper}_i + \beta_4 \text{expersq}_i + \beta_5 \text{union}_i - 0.438235(1) + 0.0070104(1)$$

$$\log(\text{wage}) = 0.49071 + \beta_2 \text{educ}_i + \beta_3 \text{exper}_i + \beta_4 \text{expersq}_i + \beta_5 \text{union}_i - \text{woman with above average looks}$$

• Explain: the intercept of 2 groups are different, which woman above rate looks have higher value of intercept.

2.a) Do all the signs for each coefficient make economic sense? Explain.

$$\begin{aligned}
 \widehat{\text{hexp}}_i &= \beta_1 + \beta_2 \text{area}_i + \beta_3 \text{child}_i + \hat{u}_i \\
 &(43.83) \quad (-15.8) \quad (6.82)
 \end{aligned}$$

•  $\beta_2$  is negative value, this means not living in municipality have lower expense by 2,835, this make economic sense because people who living in municipality area are likely to have less expense due to lower income and lower cost of living.

•  $\beta_3$  is positive, this means the more children family have, the more expense in that house. This is obvious that more people consume food and money for living

2.b) Test each parameter separately if they are significantly different from zero or not. (Use  $\alpha = 0.01$ )

$$\widehat{hhexp}_i = 9,693 - 2,742area_i + 910child_i - 64(area_i * child_i) + \hat{u}_i$$

(34.38) (-6.55) (5.17) (-0.25)

$H_0 : \beta_2 = 0$  - null hypothesis

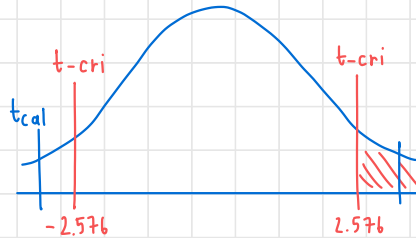
$H_1 : \beta_2 \neq 0$

$$\alpha : 0.01 \rightarrow \frac{\alpha}{2} = 0.005$$

$$t_{cri} = 2.576$$

$$df = 14908 - 3$$

$$= 14905$$



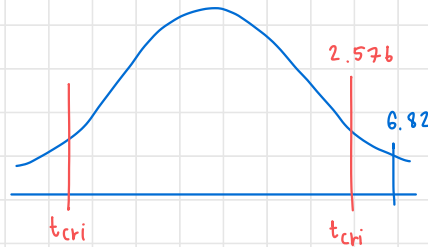
$\therefore$  We can reject  $H_0$

$$\alpha = 0.01 \rightarrow \frac{\alpha}{2} = 0.005$$

$$t_{cri} = 2.576$$

$$df = 14908 - 3$$

$$= 14905$$



$\therefore$  We can reject  $H_0$

2.c) Find the expected value of a household expenditure not living in a municipal area with 3 children aged under 15.

• HH expenditure municipal area = 1, no. children = 3

$$\widehat{hhexp}_i = 9,736 - 2,835area_i + 881child_i + \hat{u}_i$$

$$\widehat{hhexp}_i = 9,736 - 2,835 + 881(3)$$

$$= 9,544 //$$

2.d) When an interaction term is included in this model, the result becomes with **t value in parentheses**.

$$\widehat{hhexp}_i = 9,693 - 2,742\text{area}_i + 910\text{child}_i - 64(\text{area}_i * \text{child}_i) + \hat{u}_i$$

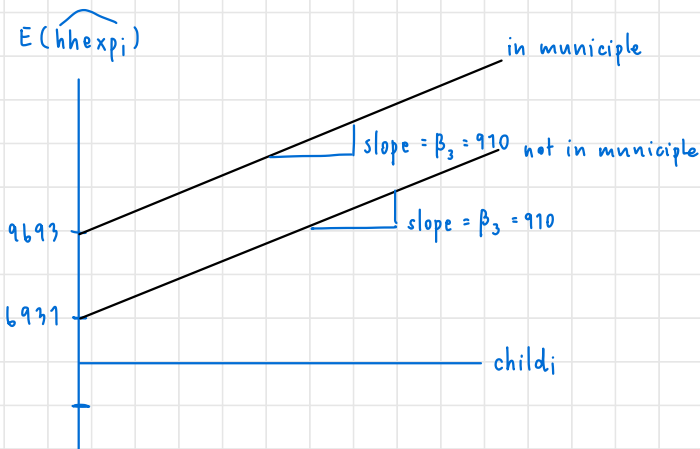
(34.38)    (-6.55)    (5.17)    (-0.25)

Draw a diagram for this model displaying sampled regression functions (SRF) with expected value of household expenditure on the vertical axis and number of children on the horizontal axis, **taking only significant parameter(s) into account**. **Indicate the intercept and slope for each SRF where applicable**. **Testing of significance can be shortened**.

$$\widehat{hhexp}_i = 9,693 - 2,742\text{area}_i + 910\text{child}_i - 64(\text{area}_i * \text{child}_i) + \hat{u}_i$$

(34.38)    (-6.55)    (5.17)    (-0.25)

|                             |                          |                          |
|-----------------------------|--------------------------|--------------------------|
| $H_0; \hat{\beta}_2 = 0$    | $H_0; \beta_3 = 0$       | $H_0; \beta_4 = 0$       |
| $H_a; \hat{\beta}_2 \neq 0$ | $H_a; \beta_3 \neq 0$    | $H_a; \beta_4 \neq 0$    |
| $t_{\text{cal}} = -6.55$    | $t_{\text{cal}} = 5.17$  | $t_{\text{cal}} = -0.25$ |
| $t_{\text{cri}} = 2.576$    | $t_{\text{cri}} = 2.576$ | $t_{\text{cri}} = 2.576$ |
| $\alpha = 0.01$             |                          |                          |
| $\frac{\alpha}{2} = 0.005$  | can reject $H_0$         | cannot reject $H_0$      |
| can reject $H_0$            |                          |                          |



$\text{mun} = 1$      $\text{other} = 0$

$$\widehat{hhexp}_i = 9,693 - 2,742\text{area}_i + 910\text{child}_i - 64(\text{area}_i * \text{child}_i) + \hat{u}_i$$

$$\widehat{hhexp}_i = 9693 - 2742(0) + 910(0) = 9693; \text{ base case : in municipality}$$

$$\widehat{hhexp}_i = 9693 - 2742(1) + 910(0) = 6951; \text{ no kid : not in municipality}$$

$$\widehat{hhexp}_i = 9693 - 2742(0) + 910(1) = 10603 \text{ or } (\beta_1 + \beta_3 \text{ child}_i)$$

$$\widehat{hhexp}_i = 9693 - 2742(1) + 910(1) = 7797$$

$$= (\beta_1 + \beta_2) + (\beta_3) \text{ child}_i$$

3.a) A VIF and tolerance table (postestimation) is given below

| Variable | VIF   | 1/VIF    |
|----------|-------|----------|
| 2.sex    | 1.02  | 0.979129 |
| age      | 50.61 | 0.019759 |
| agesq    | 50.68 | 0.019731 |
| weekot   | 1.01  | 0.985618 |

Mean VIF | 25.83

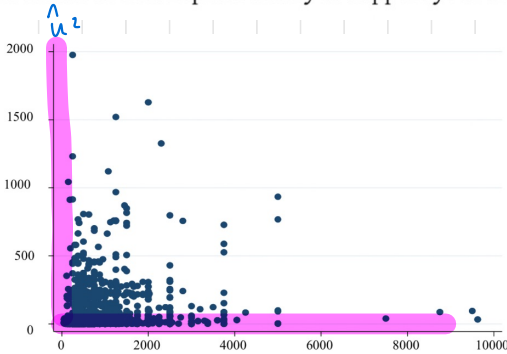
Given that you are exploring multicollinearity assumption, which pair of variables that you suspect they might be linearly correlated? Provide clear explanation what criteria (ion) that you rely on making that judgement.

: Age and ageeq are suspected to be linearly correlation.  
because the VIF value of these 2 variables are exceed 10 and TOL value or  $\frac{1}{VIF}$  are nearly 0, this means  $r^2$  between these 2 independences are high.

3.b) From (3.a), do you consider removing one of the variables from the model? Why or why not and which one that you choose to remove, if that is the case?

: No, we don't have other data to make sure that which variable should be eliminated.

3.c) The graph provided below is a scatter plot between  $\hat{u}_i^2$  (vertical axis) and  $weekot_i$  (horizontal axis). Using the graphical method, do you conclude that heteroscedasticity is present in this model or not. Explain clearly to support your answer.



: Yes, heteroscedas is detected in this model because as weekot increase, there are increase in  $\hat{u}_i^2$  as well.

3.d) An auxiliary model here is estimated and the result is given in the table below.

$$\hat{u}_i^2 = \beta_1 + \beta_2 \text{sex}_i + \beta_3 \text{age}_i + \beta_4 \text{agesq}_i + \beta_5 \text{weekot}_i + v_i$$

| Source   | SS         | df    | MS         | Number of obs | = | 2,032  |
|----------|------------|-------|------------|---------------|---|--------|
| Model    | 829063.863 | 4     | 207265.966 | F(4, 2027)    | = | 9.52   |
| Residual | 44148135   | 2,027 | 21780.037  | Prob > F      | = | 0.0000 |
|          |            |       |            | R-squared     | = | 0.0184 |
|          |            |       |            | Adj R-squared | = | 0.0165 |
| Total    | 44977198.8 | 2,031 | 22145.3465 | Root MSE      | = | 147.58 |

| uhat2  | Coef.     | Std. Err. | t     | P> t  | [95% Conf. Interval] |          |
|--------|-----------|-----------|-------|-------|----------------------|----------|
| 2.sex  | -5.648899 | 6.630832  | -0.85 | 0.394 | -18.65286            | 7.355058 |
| age    | -2.490434 | 2.37094   | -1.05 | 0.294 | -7.140168            | 2.1593   |
| age2   | .044175   | .0301279  | 1.47  | 0.143 | -.0149098            | .1032599 |
| weekot | .0229916  | .0043502  | 5.29  | 0.000 | .0144603             | .0315229 |
| _cons  | 83.8484   | 44.4418   | 1.89  | 0.059 | -3.307973            | 171.0048 |

From the table, setup the hypotheses and perform the Breusch-Pagan test to check that heteroscedasticity is present in the original model or not.

$H_0$  : the model is Homoscedasticity

$H_a$  : otherwise

$$F_{cal} = \frac{R^2_{\hat{u}_i^2} / (n)}{(1 - R^2_{\hat{u}_i^2}) / (n - 1)} = \frac{(0.0184) / 5}{(1 - 0.0184) / (2032 - 5 - 1)} = \frac{0.00368}{0.00048} = 7.6667$$

$$F_{cri}(5, 2026) = 2.2141$$

$$\alpha = 0.05$$

$\therefore F_{cal} > F_{cri}$

We can reject  $H_0$ , this means that 95 times of observation, Heteroscedast is present.