

Practice problem set 5

EE320 Semester 2/2015

Chapter 7: Multivariate calculus: basis derivative and applications

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Question 1: Determine Z_x , and Z_y

- a. $Z = (x^2y + 2)^2$
- b. $Z = x^{1/2}y^{1/3}$
- c. $Z = xy - \ln(xy)$

Question 2:

Consider a function $Z = f(x, y) = x^3 + 3x^2y + 6xy^2 - y^3$

- a. Evaluate the gradient matrix when $x = 0$ and $y = 2$
- b. Derive the Hessian matrix.
- c. Evaluate the value of the Hessian matrix where $x = 2$ and $y = 3$

Question 3:

- a. Given that $Z = \frac{x^3 - y^3}{x^2y^2}$, show that $x\frac{\partial Z}{\partial x} + y\frac{\partial Z}{\partial y} = -Z$
- b. Given that $Z = \frac{x-y}{x+y}$, find $x\frac{\partial Z}{\partial x} + y\frac{\partial Z}{\partial y} = ?$
- c. If $z = 2x^2y + 3xy + y^2$ where $x = r^2 + 2rs$ and $y = 2r - 4s$, then by means of the chain rule, (c.1a) find $\partial z / \partial s$ and $\partial z / \partial r$; (c.2) evaluate when $r = 1$ and $s = 0$
- d. For $2x^2 + 3y^2 + 2z^2 = 16$, evaluate $\partial z / \partial y$ when $x = 1, y = 2, z = -1$.
- e. Given that $\ln(x + y + z) + xyz = ze^{x+y+z}$, evaluate $\partial z / \partial x$ when $x = 0, y = 1, z = 0$.

Question 4

Suppose that the demand of a product is given by,

$$Q_x = 100 - 4P_x - \frac{50}{\sqrt{P_y}} + 0.5I^2.$$

where P_x is the price of good X, P_y is the price of good Y, and I is the level of income.

Consider the following problems

- What is the relationship between good x and good y? Are they substitute product/complementary product? Show your results using the partial derivative
- Is the product X considered an inferior product?
- What is the level of quantity demanded if $P_x = 10$, $P_y = 25$ and $I = 10$?
- Calculate the own-price elasticity of demand, and evaluate the value when $P_x = 10$, $P_y = 25$ and $I = 10$.
- Calculate the cross-price elasticity of demand when $P_x = 10$, $P_y = 25$ and $I = 10$.
- Calculate income elasticity of demand when $P_x = 10$, $P_y = 25$ and $I = 10$. Is the product a necessary or luxurious product?

Question 5: Consider a simple market model where

Demand: $q = -p + \sqrt{I}$

Supply: $q = 3p - w^2$

where q is quantity of output, p is price, I is the level of income, and w is the price of factor input

- Derive the solution of all the endogenous variables.
- Use the partial derivative to show that, under the equilibrium, an increase in W causes a decrease in quantity, but an increase in price.
- Predict the impact of an increase in income on equilibrium quantity and price.
- Suppose initial wage and income are \$2 and \$4 respectively. Find the approximate changes in equilibrium price and equilibrium quantity if income and wage both equally increase by \$1.

Question 6: Find the total differentials for the following functions:

- a. $z = 4x^3 - 13xy - 6y^5$
- b. $z = (2x^2 - y)(3x - 4y^3)$
- c. $z = 8x^{\frac{1}{2}}y^{\frac{1}{4}}w^{\frac{1}{4}}$

Question 7: Suppose that Mr. A's utility depends on two commodities: x_1 and x_2 . His utility function is given by

$$U = 25x_1 + 27x_2 - 3x_1^2 - 7x_1x_2 - 4x_2^2$$

- (a) Determine the marginal utility function of each commodity.
- (b) Find the marginal rate of substitution (MRS) between the two commodities when $x_1 = 2$ and $x_2 = 1$.

Question 8: Given the equation for the production isoquant

$$18[0.2K^{-0.4} + 0.8L^{0.4}]^{-2.5} = 1936$$

Use the implicit function rule to find the marginal rate of technical substitution (MRTS) of L for K.

Question 9: The demand for money, M , in the United States for the period 1929-1952 has been estimated as

$$M = 0.14Y + 76.03(r - 2)^{-0.84}, \quad (r > 2)$$

where Y is the annual national income, and r is the interest rate measured in percent per year. Find $\frac{\partial M}{\partial Y}$ and $\frac{\partial M}{\partial r}$ and discuss their signs.

Question 10:

A joint-cost function is defined implicitly by the equation

$$c + \sqrt{c} = 12 + q_A \sqrt{9 + q_B^2}$$

where c is denoted as the total cost for producing q_A and q_B units of product A and B, respectively.

- If $q_A = 6$ and $q_B = 4$, find the corresponding value of c .
- Determine the marginal costs with respect to q_A and q_B , when $q_A = 6$ and $q_B = 4$.

Question 11:

Suppose that a firm produces $Q = f(L) = L^{1/2}$ units of commodity using L units of labor. Assume that $f'(L) > 0$ and $f''(L) < 0$, so f is strictly increasing and strictly concave. If the firm gets P baht per unit produced and pays w baht for a unit of labor, write down the profit function, and find the first-order condition for profit maximization at $L^* > 0$.

Question 12:

Let the demand for a commodity be

$$\begin{aligned} Q_d &= D(P, Y_0, t_0), & \frac{\partial D}{\partial P} < 0; \frac{\partial D}{\partial Y_0} > 0; \frac{\partial D}{\partial t_0} < 0 \\ Q_s &= S(P, T_0), & \frac{\partial S}{\partial P} > 0; \frac{\partial S}{\partial T_0} < 0 \end{aligned}$$

where P is the price, Y_0 is the consumer's income, t_0 is the taste for the commodity and T_0 is the tax on the commodity. All derivatives are continuous. Use implicit function rule to find $\frac{\partial P^*}{\partial t_0}$, $\frac{\partial P^*}{\partial T_0}$, $\frac{\partial Q^*}{\partial Y_0}$, and $\frac{\partial Q^*}{\partial T_0}$. Discuss their economic implications.

Question 13: IS-LM

A nonlinear generalization of the IS-LM model is the system

$$Y = C + I + G \quad (\text{national accounting identity}),$$

$$\begin{aligned}
C &= C(Y_d) && \text{(consumption function),} \\
Y_d &= Y - T && \text{(disposable income)} \\
I &= I(r) && \text{(investment function),} \\
T &= T_0 && \text{(Exogenous tax rule)} \\
G &= G_0 && \text{(Exogenous government spending rule)} \\
M_s &= M(Y, r) && \text{(money market equilibrium),}
\end{aligned}$$

where C , I and M satisfy the following conditions:

$$0 < C'(*) < 1, \quad I'(r) < 0, \quad M_Y(Y, r) > 0 \quad \text{and} \quad M_r(Y, r) < 0.$$

Use the implicit function theorem to find $\frac{\partial Y^*}{\partial T_0}$, $\frac{\partial r^*}{\partial T_0}$, $\frac{\partial Y^*}{\partial G_0}$, $\frac{\partial r^*}{\partial G_0}$, $\frac{\partial Y^*}{\partial M_{s0}}$, $\frac{\partial r^*}{\partial M_{s0}}$. Discuss their economic implications.

Question 14:

Suppose a production function is given by $P = \frac{kl}{k+l}$

- Determine the marginal productivity functions.
- Show that when $k = l$, the marginal productivities are equal.

Question 15:

Suppose the demand equations for related products A and B are

$$q_A = e^{-(p_A + p_B)} \quad \text{and} \quad q_B = \frac{16}{p_A^2 p_B^2}$$

where q_A and q_B are the number of units of A and B demanded when the unit prices (in thousands of dollars) are p_A and p_B , respectively.

- Classify A and B as competitive, complementary, or neither.
- Calculate elasticity of demand for good A and good B, respectively. (Get me all whatsoever you can do for the elasticities.)

- c. If the unit prices of A and B are \$1000 and \$2000, respectively, estimate the change in the demand for A when the price of B is decreased by \$20 and the price of A is held constant.

Question 16:

The demand equations for related products A and B are given by

$$q_A = \frac{30 \sqrt{p_B}}{p_A^{2/3}} \quad \text{and} \quad q_B = \frac{50 p_A}{p_B^{1/3}}$$

Where q_A and q_B are the quantities of A and B demanded and p_A and p_B are the corresponding prices (in dollars) per unit.

- Find the values of the two marginal demands for product A when $p_A = 8$ and $p_B = 64$
- If p_B were reduced to 60 from 64, with p_A fixed at 8, use part (a) to estimate the corresponding change in demand for product A.

Question 17:

A manufacturer's joint-cost function for producing q_A units of product A and q_B units of product B is given by

$$c = \frac{q_A^2 (q_B^3 + q_A)^{1/2}}{17} + q_A q_B^{1/3} + 600$$

Where c is in dollars.

- Find the marginal-cost functions with respect to q_A and q_B .
- Evaluate the marginal-cost function with respect to q_A when $q_A = 17$ and $q_B = 8$.
- Use your answer from part (b) to estimate the change in cost if production of product A is decreased from 17 to 16 units, while production of product B is held constant at 8 units.

Question 18:

Suppose the cost c of producing q_A units of product A and q_B units of product B is given by

$$c = (3q_A^2 + q_B^3 + 4)^{1/3}$$

And the coupled demand functions for the products are given by

$$q_A = 10 - p_A + p_B^2$$

and

$$q_B = 20 + p_A - 11p_B$$

Use a chain rule to evaluate $\frac{\partial c}{\partial p_A}$ and $\frac{\partial c}{\partial p_B}$ when $p_A = 25$ and $p_B = 4$