Assignment #1

Instructions:

- For all questions, answer up to 4 decimal places.
- This assignment is due on **Tuesday, Mar 9, 2021 before class (11.00).**
- Write your answer in either digital or ordinary paper. For digital paper, export pages into a single PDF file. For ordinary paper, take photos of your writing and convert them into a single PDF file as well.
- There is no need to rewrite the question. Assign number item, ie. 1 a., clearly before your answer is sufficient.
- Submit your assignment into Moodle.
- Name your file as StudentID_Nickname (in Thai) such as 123456789_ไม้

1. Given this information

$n = 30$	$\sum_{i=1}^{n} X_i = 366$	$\sum_{i=1}^{n} Y_i = 631$	$\overline{X} = 12.20$	$\overline{Y} = 21.03$
$\sum_{i=1}^{n} (X_i)^2 = 5,564$	$\sum_{i=1}^{n} X_i Y_i = 7,524$	$\sum_{i=1}^{n} \left(X_i - \overline{X}\right)^2 = 1098.8$	$\sum_{i=1}^{n} (Y_i - \overline{Y})^2 = 882.97$	
$\sum_{i=1}^{n} (X_i - \overline{X})(Y_i - \overline{Y}) = -174.20$		$\sum_{i=1}^{n} \hat{u}_i^2 = 873.14$	$\Sigma x_i = \Sigma (x - \bar{x})$ ↳ small x	

Answer the following questions. Show your work.

a) From regression model: $Y_i = \beta_1 + \beta_2 X_i + u_i,\ u_i \sim NIID\left(0, \sigma^2\right)$ (normally, identically and independently distributed), find the estimators of β_1 and β_2 with OLS method and explain the meaning of the model.

b) Find r^2 and explain its meaning.

c) If $X_i = 5$, estimate the value of $\hat{Y}_i$ and explain its meaning.

d) Find the estimators of $\text{var}(u_i)$, $\text{var}\left(\hat{\beta}_1\right)$ and $\text{var}\left(\hat{\beta}_2\right)$

Two-tails test

e) Test the hypothesis whether coefficients are different from zero at 0.05 level of significance.

f) Test the hypothesis whether coefficients are less than zero at 0.01 level of significance.

One -tail test

1a.) $\hat{Y}_i = \hat{\beta}_1 + \hat{\beta}_2 X_i + u_i$

Find $\hat{\beta}_2$ from $\hat{\beta}_2 = \dfrac{\Sigma x_i y_i - \bar{Y}\Sigma x_i}{\Sigma x_i^2 - \bar{x}\Sigma x_i}$

$= \dfrac{7{,}524 - 21.03(366)}{5{,}564 - 12.20(366)}$

$\hat{\beta}_2 = -\dfrac{172.98}{1{,}098.8} = -0.1574$ #

Find $\hat{\beta}_1$ from $\hat{\beta}_1 = \bar{Y} - \hat{\beta}_2 \bar{X}$

$= 21.03 + 0.1574(12.20)$

$\hat{\beta}_1 = 22.9503$ #

1b.) Find $r^2 = 1 - \dfrac{\Sigma \hat{u}_i^2}{\Sigma(Y_i - \bar{Y})^2} \Rightarrow 1 - \dfrac{873.14}{882.97} \Rightarrow 0.0111$ # means that the data and the outcome have a lot of exogenous variables or doesn't related.

1c.) If $X_i = 5$ where $\hat{Y} = 22.9503 - 0.1574(5)$

$\hat{Y} = 22.1633$ #

1d.) $Var(u_i)$ or $\sigma^2 = \dfrac{\Sigma \hat{u}_i^2}{n-k} \Rightarrow \dfrac{873.14}{30-2} \Rightarrow 31.1836\%^2$ #

$Var(\hat{\beta}_1)$ or $\sigma^2_{\hat{\beta}_1} = \dfrac{\Sigma X_i^2}{n\Sigma x_i^2} \cdot \sigma^2 \Rightarrow \dfrac{5{,}564}{30(1{,}098.8)} \cdot 31.1836 \Rightarrow 5.2635\ \%^2$ #

$Var(\hat{\beta}_2)$ or $\sigma^2_{\hat{\beta}_2} = \dfrac{\sigma^2}{\Sigma x_i^2} \Rightarrow \dfrac{31.1836}{1{,}098.8} \Rightarrow 0.0284\ \%^2$ #

1e.) Hypothesis Testing (β_2) using T-test where $\alpha = 0.05$

step 1 : $H_0 : \beta_2 = 0$

$H_1 : \beta_2 \neq 0$

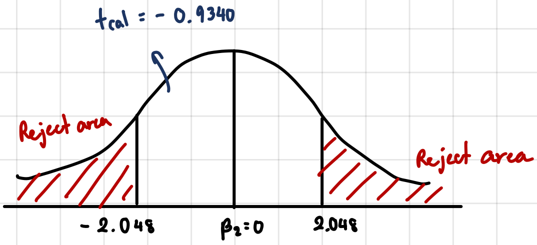
step 2 : $t_{\frac{\alpha}{2}} = t_{\frac{0.05}{2}} = t_{0.025}$

$d.f. = n-k = 30-2 = 28$

$t_{0.025}\ d.f.\ 28 = 2.048$ #

step 3 : $t_{cal} = \dfrac{\hat{\beta}_2 - \beta_2}{\sigma_{\hat{\beta}_2}} = \dfrac{-0.1574 - 0}{\sqrt{0.0284}}$

$t_{cal} = -0.9340$ #



∴ $T_{cal} = -0.9340$ fall into the acceptance region within the boundary of Confident Interval. We cannot reject Ho which is null hypothesis, at the confident Interval of 95% #

Hypothesis Testing (β_1) using T-test where $\alpha = 0.05$

step 1 : $H_0 : \beta_1 = 0$

$H_1 : \beta_1 \neq 0$

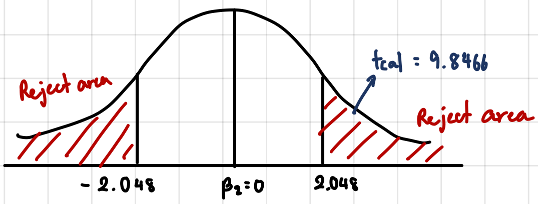
step 2 : $t_{\frac{\alpha}{2}} = t_{\frac{0.05}{2}} = t_{0.025}$

d.f. $= n-k = 30-2 = 28$

$t_{0.025}$ d.f. 28 $= 2.048$ #

step 3 : $t_{cal} = \dfrac{\hat{\beta}_1 - \beta_1}{\sigma_{\hat{\beta}_1}} = \dfrac{22.5903 - 0}{\sqrt{5.2635}}$

$t_{cal} = 9.8466$ #



∴ Tcal = 9.8466 fall into the rejection region within the boundary of Confident Interval. We reject Ho which is null hypothesis, at the confident Interval of 95% #

1f.) Hypothesis Testing (β_2) using T-test where $\alpha = 0.01$

step 1 : $H_0 : \beta_2 \geqslant 0$

$H_1 : \beta_2 < 0$

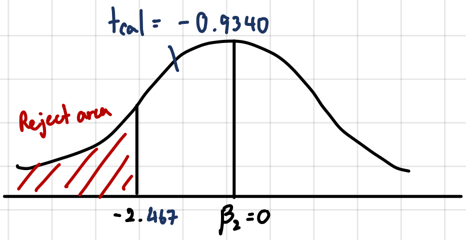
step 2 : One-tail test $= t_{0.01}$

d.f. $= n-k = 30-2 = 28$

$t_{0.025}$ d.f. 28 $= 2.467$ #

step 3 : $t_{cal} = \dfrac{\hat{\beta}_2 - \beta_2}{\sigma_{\hat{\beta}_2}} = \dfrac{-0.1574 - 0}{\sqrt{0.0284}}$

$t_{cal} = -0.9340$ #



Since we want to find whether coefficients are less than 0, so that mean in this case we will use 2.467 because we assume that less than -2.467 will be consider rejection region.

∴ Tcal = -0.9340 fall into the acceptance region within the boundary of Confident Interval. We cannot reject Ho which is null hypothesis, at the confident Interval of 99% #

Hypothesis Testing (β_2) using T-test where $\alpha = 0.01$

step 1 : $H_0 : \beta_2 \geq 0$

$H_1 : \beta_2 < 0$

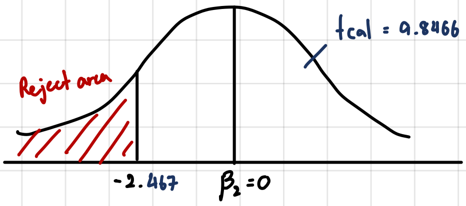
step 3 : $t_{cal} = \dfrac{\hat{\beta}_2 - \beta_2}{\sigma_{\hat{\beta}_2}} = \dfrac{22.5903 - 0}{\sqrt{5.2635}}$

$t_{cal} = 9.8466$ #

Step 2 : One-tail test $= t_{0.01}$

d.f. $= n-k = 30-2 = 28$

$t_{0.025}$ d.f. 28 $= 2.467$ #



Since we want to find whether coefficients are less than 0, so that mean in this case we will use 2.467 because we assume that less than −2.467 will be consider rejection region.

∴ Tcal = 9.8466 fall into the acceptance region within the boundary of Confident Interval. We cannot reject Ho which is null hypothesis, at the confident Interval of 99% #

cum. prob	$t_{.50}$	$t_{.75}$	$t_{.80}$	$t_{.85}$	$t_{.90}$	$t_{.95}$	$t_{.975}$	$t_{.99}$	$t_{.995}$	$t_{.999}$	$t_{.9995}$
one-tail	0.50	0.25	0.20	0.15	0.10	0.05	0.025	0.01	0.005	0.001	0.0005
two-tails	1.00	0.50	0.40	0.30	0.20	0.10	0.05	0.02	0.01	0.002	0.001
df											
1	0.000	1.000	1.376	1.963	3.078	6.314	12.71	31.82	63.66	318.31	636.62
2	0.000	0.816	1.061	1.386	1.886	2.920	4.303	6.965	9.925	22.327	31.599
3	0.000	0.765	0.978	1.250	1.638	2.353	3.182	4.541	5.841	10.215	12.924
4	0.000	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	0.000	0.727	0.920	1.156	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	0.000	0.718	0.906	1.134	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	0.000	0.711	0.896	1.119	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	0.000	0.706	0.889	1.108	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	0.000	0.703	0.883	1.100	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	0.000	0.700	0.879	1.093	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	0.000	0.697	0.876	1.088	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	0.000	0.695	0.873	1.083	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	0.000	0.694	0.870	1.079	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	0.000	0.692	0.868	1.076	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	0.000	0.691	0.866	1.074	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	0.000	0.690	0.865	1.071	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	0.000	0.689	0.863	1.069	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	0.000	0.688	0.862	1.067	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	0.000	0.688	0.861	1.066	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	0.000	0.687	0.860	1.064	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	0.000	0.686	0.859	1.063	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	0.000	0.686	0.858	1.061	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	0.000	0.685	0.858	1.060	1.319	1.714	2.069	2.500	2.807	3.485	3.768
24	0.000	0.685	0.857	1.059	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	0.000	0.684	0.856	1.058	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	0.000	0.684	0.856	1.058	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	0.000	0.684	0.855	1.057	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	0.000	0.683	0.855	1.056	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	0.000	0.683	0.854	1.055	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	0.000	0.683	0.854	1.055	1.310	1.697	2.042	2.457	2.750	3.385	3.646
40	0.000	0.681	0.851	1.050	1.303	1.684	2.021	2.423	2.704	3.307	3.551
60	0.000	0.679	0.848	1.045	1.296	1.671	2.000	2.390	2.660	3.232	3.460
80	0.000	0.678	0.846	1.043	1.292	1.664	1.990	2.374	2.639	3.195	3.416
100	0.000	0.677	0.845	1.042	1.290	1.660	1.984	2.364	2.626	3.174	3.390
1000	0.000	0.675	0.842	1.037	1.282	1.646	1.962	2.330	2.581	3.098	3.300
z	0.000	0.674	0.842	1.036	1.282	1.645	1.960	2.326	2.576	3.090	3.291
	0%	50%	60%	70%	80%	90%	95%	98%	99%	99.8%	99.9%
	Confidence Level										

2. Given that Y is market price of a car (USD) while X is how long a car aged (years), results of the regression are as follows.

$$\overset{\hat{\beta}_1}{} \quad \overset{\hat{\beta}_2}{}$$

$$\hat{Y}_i = 7{,}836 - 502.4X_i$$

$$\text{s.e.} \quad (52) \quad (411.8)$$

Given that u_i is normally, identically and independently distributed with zero mean and σ^2 variance, total number of observations is 11,

$\bar{X} = 7.45$,

$\hat{\sigma}^2 = 212{,}877$,

$\sum(X_i - \bar{X})^2 = 78.73$,

Answer the following questions. Show your work.

a) Does the sign of $\hat{\beta}_2$ make economic sense? Provide your explanation.

b) If you are a car expert and someone asks you to estimate how much his car will be **averagely** priced at when his car is 5 years old, how much is the market price range that you would estimate that you can make sure that for 95% of the time, market price will be within the specific range?

c) If you multiply all the X with 10, report the new SRF with the standard error resulted from the multiplication.

d) Calculate the elasticity of market price when a car is 10 years old.

$$\beta_2 = \frac{\text{relative change in } Y}{\text{relative change in } X}$$

2a.) $\hat{\beta}_2 = -502.4$ as it's negative regression. It makes sense as the increasing in the car aged will decrease the market price of that car.

2b.) $x_i = 5$ yrs From $\hat{Y} = 7{,}836 - 502.4X_i$ where $\hat{\sigma}^2_{\hat{\beta}_2} = 411.8$, CI = 95%, α = 5%

$$\hat{Y} = 7{,}836 - 502.4(5)$$
$$\hat{Y} = 5{,}324 \ \#$$

from formula $\text{Var}(\hat{Y}_0) = \hat{\sigma}^2\left[\frac{1}{n} + \frac{(x_0 - \bar{x})}{\Sigma(x_i - \bar{x})^2}\right]$

$$= 212{,}877\left[\frac{1}{11} + \frac{(5 - 7.45)^2}{78.73}\right]$$

$$= 35{,}582.5345 \ \#$$

Find $\hat{\sigma}_{\hat{Y}_0} = \sqrt{35{,}582.5345}$

$$= 188.6333 \ \#$$

Find Upper bound & Lower bound with CI = 95%, $\alpha = 0.05$

Formula : $\hat{Y}_0 \pm (t_{\frac{\alpha}{2}} \cdot \hat{\sigma}_{\hat{Y}_0})$ where $t_{\frac{0.05}{2}} = t_{0.025}$, d.f. = n - k = 11 - 2 = 9

$$= 5{,}324 \pm (2.262)(188.6333)$$

$$= 4{,}897.3115 \leqslant Y_0 \leqslant 5{,}750.6885 \text{ in USD} \ \#$$

∴ The market price for the car given aged 5 years with the confident Interval is between 4,897.3115 and 5,750.6885 USD #

2c.) Multiply x by 10, find new S.E. from multiplication

If $\hat{Y} = 7{,}836 - 502.4(10)X_i$

$$\hat{Y} = 7{,}836 - 5{,}024X_i \ \#$$

∴ 1 year aged of car will decrease the market price by 5,024 USD #

$\text{Var}(\beta_2) = \frac{\hat{\sigma}^2}{\Sigma x_i^2}$ where $x_i^2 = (x - \bar{x})^2$

$$= \frac{212{,}877}{78.73} \Rightarrow 2{,}703.8867 \ \#$$

$$\text{se}.(\beta_2) = 411.8(10)$$

$$= 4{,}118 \ \#$$

2d.) $x_i = 10$ yrs From $\hat{Y} = 7,836 - 502.4 X_i$.

$\hat{Y} = 7,836 - 502.4(10)$

$\hat{Y} = 2,812$ #

Find elasticity : $\frac{X}{Y} \cdot \frac{dY}{dX}$

$= \frac{10}{2,812} \cdot (-502.4)$

$= -1.7866$ # elastic

cum. prob	$t_{.50}$	$t_{.75}$	$t_{.80}$	$t_{.85}$	$t_{.90}$	$t_{.95}$	$t_{.975}$	$t_{.99}$	$t_{.995}$	$t_{.999}$	$t_{.9995}$
one-tail	**0.50**	**0.25**	**0.20**	**0.15**	**0.10**	**0.05**	**0.025**	**0.01**	**0.005**	**0.001**	**0.0005**
two-tails	**1.00**	**0.50**	**0.40**	**0.30**	**0.20**	**0.10**	**0.05**	**0.02**	**0.01**	**0.002**	**0.001**
df											
1	0.000	1.000	1.376	1.963	3.078	6.314	12.71	31.82	63.66	318.31	636.62
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20	0.000	0.687	0.860	1.064	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	0.000	0.686	0.859	1.063	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	0.000	0.686	0.858	1.061	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	0.000	0.685	0.858	1.060	1.319	1.714	2.069	2.500	2.807	3.485	3.768
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27	0.000	0.684	0.855	1.057	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	0.000	0.683	0.855	1.056	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	0.000	0.683	0.854	1.055	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	0.000	0.683	0.854	1.055	1.310	1.697	2.042	2.457	2.750	3.385	3.646
40	0.000	0.681	0.851	1.050	1.303	1.684	2.021	2.423	2.704	3.307	3.551
60	0.000	0.679	0.848	1.045	1.296	1.671	2.000	2.390	2.660	3.232	3.460
80	0.000	0.678	0.846	1.043	1.292	1.664	1.990	2.374	2.639	3.195	3.416
100	0.000	0.677	0.845	1.042	1.290	1.660	1.984	2.364	2.626	3.174	3.390
1000	0.000	0.675	0.842	1.037	1.282	1.646	1.962	2.330	2.581	3.098	3.300
z	0.000	0.674	0.842	1.036	1.282	1.645	1.960	2.326	2.576	3.090	3.291
	0%	50%	60%	70%	80%	90%	95%	98%	99%	99.8%	99.9%
	Confidence Level										