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1. Two individuals agree at date 0 to a forward contract that matures at date 2. The contract is written on an underlying asset that pays a dividend at date 1 equal to D_1. Let f_2 be the date 2 random payoff (profit) to the individual who is the long party in the forward contract. Also let m_{0i} be the stochastic discount factor over the period from dates 0 to i where $i = 1, 2$, and let $E_0[\cdot]$ be the expectations operator at date 0. What is the value of $E_0[m_{02} f_2]$? Explain your answer.

P = current price

From: $P_{jt} = E_t \left[\sum_{j=1}^{\infty} m_{t,t+j} \, d_{j,t+j} \right]$

using stochastic discount factor

$P_0 = E_0[m_{01} D_1] + E_0[m_{02} P_2] = D_0 + E_0[m_{02} P_2]$

Payoff to the long party

$f_2 = P_2 - F_{02}$ → long forward position, F_{02} = Borrowing at date 0 and Repay at date 2.

we know that

$E_0[m_{02} f_2] = E_0[m_{02}(P_2 - F_{02})] = E[m_{02} P_2] - E[m_{02} F_{02}]$

note that: $P_0 = E_0[m_{01} D_1] + E_0[m_{02} P_2] = D_0 + E_0[m_{02} P_2]$

and $E[m_{02} F_{02}] = E[m_{02}] F_{02} = R_f^{-2} F_{02}$

and Then,

$E_0[m_{02} f_2] = E[m_{02} P_2] - E[m_{02} F_{02}]$

$= P_0 - D_0 - R_f^{-2} F_{02}$ ※

∴ It implies that forward price satisfies $F_{02} = R_f^{2}(P_0 - D_0)$

↳ $E[m_{02} F_{02}] = 0$

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2. Assume that there is an economy populated by infinitely-lived representative individuals who maximize the lifetime utility function

$$E_0\left[\sum_{t=0}^{\infty} -\delta^t e^{-ac_t}\right]$$

where c_t is consumption at date t and $a > 0$, $0 < \delta < 1$. The economy is a Lucas (1978) endowment economy having multiple risky assets paying date t dividends that total d_t per capita. Write down an expression for the equilibrium per capita price of the market portfolio in terms of the assets' future dividends.

$$E_0\left[\sum_{t=0}^{\infty} -\delta^t e^{-ac_t}\right]$$

From Lucas model,

$$P_0 = E_0\left[\sum_{t=1}^{\infty} \frac{U_c(c_t,t)}{U_c(c_0,0)} d_t\right]$$

$$U(c_t,t) = -\delta^t e^{-ac_t} \rightarrow U_c(c_t,t) = a\delta^t e^{-ac_t} \quad ; \; c_t = d_t$$

Then

$$P_0 = E_0\left[\sum_{t=1}^{\infty} \frac{U_c(c_t,t)}{U_c(c_0,0)} d_t\right]$$

$$= E_0\left[\sum_{t=1}^{\infty} \frac{a\delta^t e^{-ac_t}}{a\delta^t e^{-ac_0}} d_t\right]$$

$$= E_0\left[\sum_{t=1}^{\infty} \delta^t e^{-a(d_t - d_0)} d_t\right] \quad ✗$$

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3. For the Lucas model with labor income, show that assumptions (6.25) and (6.26) lead to the pricing relationship (6.27) and (6.28).

$$\begin{aligned} \ln\left(C^*_{t+1}/C^*_t\right) &= \mu_c + \sigma_c \eta_{t+1} \\ \ln\left(d_{t+1}/d_t\right) &= \mu_d + \sigma_d \varepsilon_{t+1} \end{aligned} \tag{6.25}$$

$$\begin{pmatrix} \eta_t \\ \varepsilon_t \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}\right) \tag{6.26}$$

how?

$$P_t = d_t \frac{\delta e^{\alpha}}{1 - \delta e^{\alpha}}$$

where

$$\alpha \equiv \mu_d - (1-\gamma)\mu_c + \frac{1}{2}\left[(1-\gamma)^2 \sigma_c^2 + \sigma_d^2\right] - (1-\gamma)\rho\sigma_c\sigma_d \tag{6.28}$$

theoretically, we know that $P_t = E_t\left[\sum_{j=1}^{\infty} \delta^j \left[\frac{C^*_{t+j}}{C^*_t}\right]^{\gamma-1} d_{t+1}\right]$

From, $\ln\left(\frac{C^*_{t+1}}{C^*_t}\right) = \mu_c + \sigma_c \eta_{t+1}$

So, $\ln\left(\frac{C^*_{t+2}}{C^*_t}\right) = \ln\left(\frac{C^*_{t+2}}{C^*_{t+1}}\right) + \ln\left(\frac{C^*_{t+1}}{C^*_t}\right)$

$$= 2\mu_c + \sigma_c \eta_{t+1} + \sigma_c \eta_{t+2}$$

$$= 2\mu_c + \sum_{i=1}^{2} \sigma_c \eta_{t+i}$$

Then, we will get

$$\ln\left(\frac{C^*_{t+j}}{C^*_t}\right) = j\mu_c + \sigma_c \sum_{i=1}^{j} \mu_{t+i}$$

$$\ln\left(\frac{d_{t+j}}{d_t}\right) = j\mu_d + \sigma_d \sum_{i=1}^{j} \varepsilon_{t+j}$$

$$\frac{C^*_{t+j}}{C^*_t} = e^{j\mu_c + \sigma_c \sum_{i=1}^{j} \mu_{t+i}}$$

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↳ From $P_t = E_t\left[\sum_{j=1}^{\infty} \delta^j \left(\frac{C^A_{t+j}}{C^A_t}\right) d_{t+j}\right]$

$$\frac{P_t}{d_t} = E_t\left[\sum_{j=1}^{\infty} \delta^j \left(\frac{C^A_{t+1}}{C^A_t}\right) \frac{d_{t+1}}{d_t}\right]$$

$$\frac{P_t}{d_t} = E_t\left[\sum_{j=1}^{\infty} \delta^j e^{(\gamma-1)[j\mu_c + \sigma_c \sum_{i=1}^{j} \eta_{t+1}] + j\mu_d + \sigma_d \sum_{i=1}^{j} \varepsilon_{t+j}}\right]$$

take expectation E[•] inside

$$\frac{P_t}{d_t} = \sum_{j=1}^{\infty} \delta^j e^{(\gamma-1)j\mu_c + \frac{1}{2}(\gamma-1)^2\sigma_c^2 j + j\mu_d + \frac{1}{2}j\sigma_d^2 + 2(\gamma-1)\cdot\sigma_c\sigma_d\rho + \frac{1}{2}(2)(\gamma-1)\sigma_c\sigma_d\rho j}$$

$$\frac{P_t}{d_t} = \sum_{j=1}^{\infty} e^{j[\ln(\delta) - (1-\gamma)\mu_c + \mu_d + \frac{1}{2}(1-\gamma)^2\cdot\sigma_c^2 + \underbrace{\frac{1}{2}\cdot\sigma_d^2 - (1-\gamma)\sigma_c^2\sigma_d\rho)}_{d^*}}$$

$$\frac{P_t}{d_t} = \sum_{j=1}^{\infty} e^{jd^*} = \frac{1}{1-e^{d^*}} - 1$$

$$\therefore \frac{P_t}{d_t} = \frac{1}{1-\delta e^{\infty}} - 1$$

$$P_t = d_t \frac{\delta e^{\infty}}{1-\delta e^{\infty}} \quad \#$$

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4. Consider a special case of the model of rational speculative bubbles discussed in this chapter. Assume that infinitely-lived investors are risk-neutral and that there is an asset paying a constant, one-period risk-free

return of $R_f = \delta^{-1} > 1$. There is also an infinitely-lived risky asset with price p_t at date t. The risky asset is assumed to pay a dividend of d_t which is declared at date t and paid at the end of the period, date $t+1$. Consider the price $p_t = f_t + b_t$ where

$$f_t = \sum_{i=0}^{\infty} \frac{E_t\left[d_{t+i}\right]}{R_f{}^{i+1}} \tag{1}$$

and

$$b_{t+1} = \begin{cases} \frac{R_f}{q_t} b_t + e_{t+1} & \text{with probability } q_t \\ z_{t+1} & \text{with probability } 1 - q_t \end{cases} \tag{2}$$

where $E_t\left[e_{t+1}\right] = E_t\left[z_{t+1}\right] = 0$ and where q_t is a random variable as of date $t-1$ but realized at date t and is uniformly distributed between 0 and 1.

4.a Show whether or not $p_t = f_t + b_t$ subject to the specifications in (1) and (2) is a valid solution for the price of the risky asset.

4.b Suppose that p_t is the price of a barrel of oil. If $p_t \geq p_{solar}$, then solar energy, which is in perfectly elastic supply, becomes an economically efficient perfect substitute for oil. Can a rational speculative bubble exist for the price of oil? Explain why or why not.

4.c Suppose p_t is the price of a bond that matures at date $T < \infty$. In this context, the d_t for $t \leq T$ denotes the bond's coupon and principal payments. Can a rational speculative bubble exist for the price of this bond? Explain why or why not.

4.a check 2. satisfied $E_t[b_{t+1}] = R_f b_t$ or not

$$E_t[b_{t+1}] = \frac{R_f}{q_t} b_t q_t + E_t[e_{t+1}] q_t + (1-q_t) E_t[z_{t+1}] = R_f b_t$$

4.B From $E_t[b_{t+1}] = R_f b_t$

$$\lim_{i \to \infty} E_t[b_{t+i}] = \begin{cases} +\infty & \text{if } b_t > 0 \\ -\infty & \text{if } b_t < 0 \end{cases}$$

In case of limited liability asset, we cannot have a bubble path with a price becoming negative

we consider only bubbles with $b_t > 0$. From above equation, the bubble component must be increase infinitely

But this cannot be a rational expectation if there is an upper bond on the price of oil, it would be a case if there was a perfect substitute in perfectly elastic supply. Therefore, P_t cannot rise above P_{solar}.

b_t cannot rise above $P_{solar} - P^*_t$. So, bubble path where b_t must be expected to increase to infinity cannot possibly occur.

4.c As we know, a rational speculative bubble cannot exist for the price of a bond. 6204640244_Phoom

Because at the maturity, bond price $= P_T = d_T$ and equal to 0 after date T.

So, it cannot rationally be expected to satisfy equation in 4.B and increase infinitely

moreover, rational price is $P_t = P_t^*$ and bubble path is invalid.

5. Consider an endowment economy with representative agents who maximize the following objective function:

$$\max_{C_s,\{\omega_{is}\},\forall s,i} E_t\left[\sum_{s=t}^{T} \delta^s u(C_s)\right]$$

where $T < \infty$. Explain why a rational speculative asset price bubble could not exist in such an economy.

Ans. Asset in the economy is a finite horizon, and asset price

did not have the form $P_t = f_t + b_t$ with $b_t \neq 0$ because at date T,

$P_t = f_T = d_T$ which is a final dividend payment.

So, $b_t = 0$

$E_t[b_{t+1}] = \delta^{-1} b_t$

$E_{t-1}[b_t] = E_{t-1}[0] = \delta^{-1} b_{t-1}$; $b_{t-1} = 0$

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