Assignment 5
Guideline Solution

1. In the study of interest rate structure, the continuous time model can be specified as:

$$r_{t+\Delta t} - r_t = (\alpha + \beta r_t)\Delta t + \varepsilon_{t+\Delta t} \quad (1)$$

where: $E[\varepsilon_{t+\Delta t}] = 0$ and $E[\varepsilon_{t+\Delta t}^2] = \sigma^2 r_t^{2\gamma} \Delta t$

Then, the model can be transformed to be discrete time model by setting $\Delta t = 1$. The discrete time model can be stated as:

$$r_{t+1} - r_t = \Delta r_t = \alpha + \beta r_t + \varepsilon_{t+1} \quad (2)$$

where: $E[\varepsilon_{t+1}] = 0$ and $E[\varepsilon_{t+1}^2] = \sigma^2 r_t^{2\gamma}$

The above model consists of four parameters including $\alpha, \beta, \sigma^2, \gamma$. The model can be estimate using GMM. Four moment condition equations can be stated as:

Zero mean condition: $E(\varepsilon_{t+1}) = 0$

Orthogonality condition: $E(\varepsilon_{t+1} r_t) = 0$

Variance condition: $E(\varepsilon_{t+1}^2 - \sigma^2 r_t^{2\gamma}) = 0$

Zero covariance condition: $E\left(\left(\varepsilon_{t+1}^2 - \sigma^2 r_t^{2\gamma}\right) r_t\right) = 0$

The above model can be claimed as unrestricted model for other eight interest rate structure models which can be indicated as follows:

Model	α	β	σ^2	γ
(1) Unrestricted				
(2) Merton		0		0
(3) Vasicek				0
(4) CIR SR				0.5
(5) Dothan	0	0		1
(6) GBM	0			1
(7) Brennan & Schwartz				1
(8) CIR VR	0	0		1.5
(9) CEV	0			

From the given data set (`assign5-1.dta`):

1. Estimate the interest rate structure models applying all 9 models using GMM. Also perform Overidentification Test. (Hint: command for generating $\Delta r_t = r_{t+1} - r_t$ is `gen dr=f.r-r`)

```
. tsset time
        time variable:  time, 1 to 1335
                delta:  1 unit
. g dr=f.r-r
(1 missing value generated)
```

```
. *Unrestricted
. gmm (dr-{alpha}-{beta}*r) ((dr-{alpha}-{beta}*r)*r) ((dr-{alpha}-{beta}*r)^2-
{sigma2}*r^(2*{gamma})) (((dr-{alpha}-{beta}*r)^2-{sigma2}*r^(2*{gamma}))*r)
winitial(identity)
note: 1 missing value returned for equation 1 at initial values
note: 1 missing value returned for equation 2 at initial values
note: 1 missing value returned for equation 3 at initial values
note: 1 missing value returned for equation 4 at initial values

Step 1
numerical derivatives are approximate
flat or discontinuous region encountered
Iteration 0:   GMM criterion Q(b) =  .00001173
Iteration 1:   GMM criterion Q(b) =  8.321e-06  (backed up)
Iteration 2:   GMM criterion Q(b) =  6.043e-06  (not concave)
Iteration 3:   GMM criterion Q(b) =  1.604e-06
Iteration 4:   GMM criterion Q(b) =  1.662e-09
Iteration 5:   GMM criterion Q(b) =  1.344e-13

Step 2
Iteration 0:   GMM criterion Q(b) =  3.899e-10
Iteration 1:   GMM criterion Q(b) =  4.098e-18

note: model is exactly identified

GMM estimation

Number of parameters =   4
Number of moments    =   4
Initial weight matrix: Identity                 Number of obs   =      1,334
GMM weight matrix:     Robust

------------------------------------------------------------------------------
             |               Robust
             |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      /alpha |  -.0023725   .0011574    -2.05   0.040    -.0046409   -.0001041
       /beta |   .0004291   .0002873     1.49   0.135     -.000134    .0009922
     /sigma2 |   .0005043    .000324     1.56   0.120    -.0001307    .0011393
      /gamma |   .0985177   .1823933     0.54   0.589    -.2589666     .456002
------------------------------------------------------------------------------
Instruments for equation 1: _cons
Instruments for equation 2: _cons
Instruments for equation 3: _cons
Instruments for equation 4: _cons

. estat overid

  Test of overidentifying restriction:

  Hansen's J chi2(0) = 5.5e-15 (p =     .)

Note: test cannot be performed because there are no
      overidentifying restrictions.

. est store Unrestricted

. *Merton
. gmm (dr-{alpha}) ((dr-{alpha})*r) ((dr-{alpha})^2-{sigma2}) (((dr-{alpha})^2-
{sigma2})*r) winitial(identity)
note: 1 missing value returned for equation 1 at initial values
note: 1 missing value returned for equation 2 at initial values
note: 1 missing value returned for equation 3 at initial values
note: 1 missing value returned for equation 4 at initial values

Step 1
Iteration 0:   GMM criterion Q(b) =  .00001173
Iteration 1:   GMM criterion Q(b) =  4.045e-08
```

```
Iteration 2:   GMM criterion Q(b) =  4.044e-08

Step 2
Iteration 0:   GMM criterion Q(b) =  .00798141
Iteration 1:   GMM criterion Q(b) =  .00552368
Iteration 2:   GMM criterion Q(b) =  .00552368

GMM estimation

Number of parameters =   2
Number of moments    =   4
Initial weight matrix: Identity                    Number of obs   =      1,334
GMM weight matrix:     Robust

------------------------------------------------------------------------------
             |               Robust
             |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      /alpha |  -.0008137   .0006821    -1.19   0.233    -.0021507    .0005232
     /sigma2 |   .0004277   .0002902     1.47   0.141    -.0001412    .0009965
------------------------------------------------------------------------------
Instruments for equation 1: _cons
Instruments for equation 2: _cons
Instruments for equation 3: _cons
Instruments for equation 4: _cons

. estat overid

  Test of overidentifying restriction:

  Hansen's J chi2(2) = 7.36859 (p = 0.0251)

. est store Merton

. *Vasicek
. gmm (dr-{alpha}-{beta}*r) ((dr-{alpha}-{beta}*r)*r) ((dr-{alpha}-{beta}*r)^2-
{sigma2}) (((dr-{alpha}-{beta}*r)^2-{sigma2})*r) winitial(identity)
note: 1 missing value returned for equation 1 at initial values
note: 1 missing value returned for equation 2 at initial values
note: 1 missing value returned for equation 3 at initial values
note: 1 missing value returned for equation 4 at initial values

Step 1
Iteration 0:   GMM criterion Q(b) =  .00001173
Iteration 1:   GMM criterion Q(b) =  3.336e-10
Iteration 2:   GMM criterion Q(b) =  3.273e-10

Step 2
Iteration 0:   GMM criterion Q(b) =  .00063051
Iteration 1:   GMM criterion Q(b) =  .00020488
Iteration 2:   GMM criterion Q(b) =  .00020488

GMM estimation

Number of parameters =   3
Number of moments    =   4
Initial weight matrix: Identity                    Number of obs   =      1,334
GMM weight matrix:     Robust

------------------------------------------------------------------------------
             |               Robust
             |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      /alpha |  -.0026994   .0009734    -2.77   0.006    -.0046072   -.0007915
       /beta |   .0005368   .0001999     2.69   0.007      .000145    .0009286
     /sigma2 |   .0005887   .0002977     1.98   0.048     5.20e-06    .0011722
------------------------------------------------------------------------------
Instruments for equation 1: _cons
```

```
Instruments for equation 2: _cons
Instruments for equation 3: _cons
Instruments for equation 4: _cons

. estat overid

  Test of overidentifying restriction:

  Hansen's J chi2(1) = .273315 (p = 0.6011)

. est store Vasicek

. *CIR_SR
. gmm (dr-{alpha}-{beta}*r) ((dr-{alpha}-{beta}*r)*r) ((dr-{alpha}-{beta}*r)^2-
{sigma2}*r) (((dr-{alpha}-{beta}*r)^2-{sigma2}*r)*r) winitial(identity) nolog
note: 1 missing value returned for equation 1 at initial values
note: 1 missing value returned for equation 2 at initial values
note: 1 missing value returned for equation 3 at initial values
note: 1 missing value returned for equation 4 at initial values

Final GMM criterion Q(b) =   .001642

GMM estimation

Number of parameters =   3
Number of moments    =   4
Initial weight matrix: Identity                   Number of obs   =      1,334
GMM weight matrix:     Robust

------------------------------------------------------------------------------
             |               Robust
             |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      /alpha |  -.0010252   .0007112    -1.44   0.149    -.0024191    .0003686
       /beta |   .0002288   .0002532     0.90   0.366    -.0002675     .000725
     /sigma2 |   .0000917   .0000646     1.42   0.156     -.000035    .0002183
------------------------------------------------------------------------------
Instruments for equation 1: _cons
Instruments for equation 2: _cons
Instruments for equation 3: _cons
Instruments for equation 4: _cons

. estat overid

  Test of overidentifying restriction:

  Hansen's J chi2(1) = 2.19038 (p = 0.1389)

. est store CIR_SR

. *Dothan
. gmm ((dr)^2-{sigma2}*r^(2)) (((dr)^2-{sigma2}*r^(2))*r) winitial(identity)
note: 1 missing value returned for equation 1 at initial values
note: 1 missing value returned for equation 2 at initial values

Step 1
Iteration 0:   GMM criterion Q(b) =  6.056e-06
Iteration 1:   GMM criterion Q(b) =  1.070e-08
Iteration 2:   GMM criterion Q(b) =  1.070e-08

Step 2
Iteration 0:   GMM criterion Q(b) =   .0079236
Iteration 1:   GMM criterion Q(b) =  .00257705
Iteration 2:   GMM criterion Q(b) =  .00257705

GMM estimation

Number of parameters =   1
```

```
Number of moments    =   2
Initial weight matrix: Identity                   Number of obs   =      1,334
GMM weight matrix:     Robust

------------------------------------------------------------------------------
             |               Robust
             |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     /sigma2 |   8.95e-06   .0000103     0.87   0.386    -.0000113    .0000292
------------------------------------------------------------------------------
Instruments for equation 1: _cons
Instruments for equation 2: _cons

. estat overid

  Test of overidentifying restriction:

  Hansen's J chi2(1) = 3.43779 (p = 0.0637)

. est store Dothan

. *GBM
. gmm (dr-{beta}*r) ((dr-{beta}*r)*r) ((dr-{beta}*r)^2-{sigma2}*r^(2)) (((dr-
{beta}*r)^2-{sigma2}*r^(2))*r) winitial(identity)
note: 1 missing value returned for equation 1 at initial values
note: 1 missing value returned for equation 2 at initial values
note: 1 missing value returned for equation 3 at initial values
note: 1 missing value returned for equation 4 at initial values

Step 1
Iteration 0:   GMM criterion Q(b) =  .00001173
Iteration 1:   GMM criterion Q(b) =  8.547e-08
Iteration 2:   GMM criterion Q(b) =  8.547e-08

Step 2
Iteration 0:   GMM criterion Q(b) =   .0082543
Iteration 1:   GMM criterion Q(b) =  .00342544
Iteration 2:   GMM criterion Q(b) =  .00342542

GMM estimation

Number of parameters =   2
Number of moments    =   4
Initial weight matrix: Identity                   Number of obs   =      1,334
GMM weight matrix:     Robust

------------------------------------------------------------------------------
             |               Robust
             |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       /beta |   .0000248   .0001265     0.20   0.844     -.000223    .0002727
     /sigma2 |   .0000154   8.33e-06     1.84   0.065    -9.60e-07    .0000317
------------------------------------------------------------------------------
Instruments for equation 1: _cons
Instruments for equation 2: _cons
Instruments for equation 3: _cons
Instruments for equation 4: _cons

. estat overid

  Test of overidentifying restriction:

  Hansen's J chi2(2) = 4.56951 (p = 0.1018)

. est store GBM

. *B_s
```

```
. gmm (dr-{alpha}-{beta}*r) ((dr-{alpha}-{beta}*r)*r) ((dr-{alpha}-{beta}*r)^2-
{sigma2}*r^(2)) (((dr-{alpha}-{beta}*r)^2-{sigma2}*r^(2))*r) winitial(identity)
note: 1 missing value returned for equation 1 at initial values
note: 1 missing value returned for equation 2 at initial values
note: 1 missing value returned for equation 3 at initial values
note: 1 missing value returned for equation 4 at initial values

Step 1
Iteration 0:   GMM criterion Q(b) =  .00001173
Iteration 1:   GMM criterion Q(b) =  1.062e-08
Iteration 2:   GMM criterion Q(b) =  1.061e-08

Step 2
Iteration 0:   GMM criterion Q(b) =   .0202404
Iteration 1:   GMM criterion Q(b) =  .00259237
Iteration 2:   GMM criterion Q(b) =  .00259234

GMM estimation

Number of parameters =   3
Number of moments    =   4
Initial weight matrix: Identity                  Number of obs   =      1,334
GMM weight matrix:     Robust

------------------------------------------------------------------------------
             |               Robust
             |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      /alpha |  -.0008979   .0008413    -1.07   0.286    -.0025469     .000751
       /beta |   .0002882   .0002774     1.04   0.299    -.0002555    .0008319
     /sigma2 |   8.91e-06   .0000103     0.87   0.387    -.0000113    .0000291
------------------------------------------------------------------------------
Instruments for equation 1: _cons
Instruments for equation 2: _cons
Instruments for equation 3: _cons
Instruments for equation 4: _cons

. estat overid

  Test of overidentifying restriction:

  Hansen's J chi2(1) = 3.45819 (p = 0.0629)

. est store B_S

. *CIR_VR
. gmm ((dr)^2-{sigma2}*r^(3)) (((dr)^2-{sigma2}*r^(3))*r) winitial(identity)
note: 1 missing value returned for equation 1 at initial values
note: 1 missing value returned for equation 2 at initial values

Step 1
Iteration 0:   GMM criterion Q(b) =  6.056e-06
Iteration 1:   GMM criterion Q(b) =  1.720e-08
Iteration 2:   GMM criterion Q(b) =  1.720e-08

Step 2
Iteration 0:   GMM criterion Q(b) =  .01256614
Iteration 1:   GMM criterion Q(b) =  .00282087
Iteration 2:   GMM criterion Q(b) =  .00282087

GMM estimation

Number of parameters =   1
Number of moments    =   2
Initial weight matrix: Identity                  Number of obs   =      1,334
GMM weight matrix:     Robust

------------------------------------------------------------------------------
```

```
             |               Robust
             |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     /sigma2 |   1.18e-06   1.83e-06     0.65   0.517    -2.40e-06    4.77e-06
------------------------------------------------------------------------------
Instruments for equation 1: _cons
Instruments for equation 2: _cons

. estat overid

  Test of overidentifying restriction:

  Hansen's J chi2(1) = 3.76304 (p = 0.0524)

. est store CIR_VR

. *CEV
. gmm (dr-{beta}*r) ((dr-{beta}*r)*r) ((dr-{beta}*r)^2-{sigma2}*r^(2*{gamma}))
(((dr-{beta}*r)^2-{sigma2}*r^(2*{gamma}))*r) winitial(identity)
note: 1 missing value returned for equation 1 at initial values
note: 1 missing value returned for equation 2 at initial values
note: 1 missing value returned for equation 3 at initial values
note: 1 missing value returned for equation 4 at initial values

Step 1
numerical derivatives are approximate
flat or discontinuous region encountered
Iteration 0:   GMM criterion Q(b) =  .00001173  
Iteration 1:   GMM criterion Q(b) =  8.627e-06  (backed up)
Iteration 2:   GMM criterion Q(b) =  6.127e-06  (not concave)
Iteration 3:   GMM criterion Q(b) =  5.400e-06  (backed up)
Iteration 4:   GMM criterion Q(b) =  5.310e-06  

Step 2
Iteration 0:   GMM criterion Q(b) =  .01236133  
Iteration 1:   GMM criterion Q(b) =  .01075751  
Iteration 2:   GMM criterion Q(b) =  .00760436  
Iteration 3:   GMM criterion Q(b) =  .00678694  
Iteration 4:   GMM criterion Q(b) =  .00326882  
Iteration 5:   GMM criterion Q(b) =  .00316225  
Iteration 6:   GMM criterion Q(b) =  .00313882  
Iteration 7:   GMM criterion Q(b) =   .0031388  

GMM estimation 

Number of parameters =   3
Number of moments    =   4
Initial weight matrix: Identity                  Number of obs   =      1,334
GMM weight matrix:     Robust

------------------------------------------------------------------------------
             |               Robust
             |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       /beta |  -.0000453   .0001702    -0.27   0.790    -.0003789    .0002883
     /sigma2 |   .0000881   .0001436     0.61   0.539    -.0001933    .0003696
      /gamma |   .5717551   .3668112     1.56   0.119    -.1471817    1.290692
------------------------------------------------------------------------------
Instruments for equation 1: _cons
Instruments for equation 2: _cons
Instruments for equation 3: _cons
Instruments for equation 4: _cons

. estat overid

  Test of overidentifying restriction:

  Hansen's J chi2(1) = 4.18715 (p = 0.0407)
```

```
. est store CEV

. est table Unrestricted Merton Vasicek CIR_SR Dothan GBM B_S CIR_VR CEV,
star(0.1 0.05 0.01) stat(N J)

----------------------------------------------------------------------------------------------
    Variable | Unrestricted       Merton          Vasicek         CIR_SR          Dothan
-------------+--------------------------------------------------------------------------------
alpha        |
       _cons | -.00237253**    -.00081372      -.00269937***   -.00102525
-------------+--------------------------------------------------------------------------------
beta         |
       _cons |  .00042912                       .00053681***    .00022877
-------------+--------------------------------------------------------------------------------
sigma2       |
       _cons |  .00050427       .00042766       .00058872**     .00009168       8.947e-06
-------------+--------------------------------------------------------------------------------
gamma        |
       _cons |  .09851773
-------------+--------------------------------------------------------------------------------
Statistics   |
           N |       1334            1334            1334            1334            1334
           J |  5.467e-15       7.3685895       .27331476       2.1903809        3.437787
----------------------------------------------------------------------------------------------
                                                          legend: * p<.1; ** p<.05; *** p<.01

------------------------------------------------------------------------------
    Variable |      GBM              B_S           CIR_VR            CEV
-------------+----------------------------------------------------------------
alpha        |
       _cons |                  -.00089794
-------------+----------------------------------------------------------------
beta         |
       _cons |  .00002483        .0002882                       -.00004534
-------------+----------------------------------------------------------------
sigma2       |
       _cons |  .00001536*      8.909e-06       1.184e-06       .00008815
-------------+----------------------------------------------------------------
gamma        |
       _cons |                                                  .57175507
-------------+----------------------------------------------------------------
Statistics   |
           N |       1334            1334            1334            1334
           J |  4.5695129       3.4581859       3.7630421       4.1871546
------------------------------------------------------------------------------
                                         legend: * p<.1; ** p<.05; *** p<.01
```

b. Determine the most appropriated model using Wald Test.

```
. *Wald Test
. est restore Unrestricted
(results Unrestricted are active now)

. *Test Unrestricted vs Merton
. test (_b[/beta]=0) (_b[/gamma]=0)

 ( 1)  [beta]_cons = 0
 ( 2)  [gamma]_cons = 0

           chi2(  2) =    7.92
         Prob > chi2 =    0.0191

. *Test Unrestricted vs Vasicek
. test (_b[/gamma]=0)

 ( 1)  [gamma]_cons = 0

           chi2(  1) =    0.29
         Prob > chi2 =    0.5891
```

```
. *Test Unrestricted vs CIR_SR
. test (_b[/gamma]=0.5)

 ( 1)  [gamma]_cons = .5

           chi2(  1) =    4.85
         Prob > chi2 =    0.0277

. *Test Unrestricted vs Dothan
. test (_b[/alpha]=0) (_b[/beta]=0) (_b[/gamma]=1)

 ( 1)  [alpha]_cons = 0
 ( 2)  [beta]_cons = 0
 ( 3)  [gamma]_cons = 1

           chi2(  3) =   34.04
         Prob > chi2 =    0.0000

. *Test Unrestricted vs GBM
. test (_b[/alpha]=0) (_b[/gamma]=1)

 ( 1)  [alpha]_cons = 0
 ( 2)  [gamma]_cons = 1

           chi2(  2) =   27.48
         Prob > chi2 =    0.0000

. *Test Unrestricted vs B_s
. test (_b[/gamma]=1)

 ( 1)  [gamma]_cons = 1

           chi2(  1) =   24.43
         Prob > chi2 =    0.0000

. *Test Unrestricted vs CIR_VR
. test (_b[/alpha]=0) (_b[/beta]=0) (_b[/gamma]=1)

 ( 1)  [alpha]_cons = 0
 ( 2)  [beta]_cons = 0
 ( 3)  [gamma]_cons = 1

           chi2(  3) =   34.04
         Prob > chi2 =    0.0000

. *Test Unrestricted vs CEV
. test (_b[/alpha]=0)

 ( 1)  [alpha]_cons = 0

           chi2(  1) =    4.20
         Prob > chi2 =    0.0404
```

According to Wald Test, Vasicek is the most appropriated model.

2. From the model:

$$y_i = \alpha + \beta x_i + u_i \qquad (3)$$

where: y_i is dependent variable
x_i is explanatory variable
u_i is stochastic error term
$E(u_i) = 0$ but $E(x_i u_i) \neq 0$.

From the given data set (`assign5-2.dta`):

a. Estimate model (3) using OLS.

```
. reg y x

      Source |       SS           df       MS      Number of obs   =       500
-------------+----------------------------------   F(1, 498)       =    449.66
       Model |  132481.702         1  132481.702   Prob > F        =    0.0000
    Residual |  146722.774       498  294.624043   R-squared       =    0.4745
-------------+----------------------------------   Adj R-squared   =    0.4734
       Total |  279204.475       499  559.528007   Root MSE        =    17.165

------------------------------------------------------------------------------
           y |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           x |   5.431333   .2561312    21.21   0.000     4.928102    5.934564
       _cons |  -33.31333   2.673868   -12.46   0.000    -38.56678   -28.05988
------------------------------------------------------------------------------

. est store ols
```

b. Based on z_1, z_2, z_3, z_4, determine the best set of instrumental variables, then, estimate model (3) using GMM.

```
. corr x z1 z2 z3 z4
(obs=500)

             |        x       z1       z2       z3       z4
-------------+---------------------------------------------
           x |   1.0000
          z1 |   0.7215   1.0000
          z2 |   0.2352   0.1937   1.0000
          z3 |   0.5487   0.0918   0.0933   1.0000
          z4 |   0.6954   0.4738   0.1607   0.0828   1.0000

. ivregress gmm y (x=z1 z3)

Instrumental variables (GMM) regression          Number of obs   =       500
                                                 Wald chi2(1)    =    412.16
                                                 Prob > chi2     =    0.0000
                                                 R-squared       =    0.4692
GMM weight matrix: Robust                        Root MSE        =    17.217

------------------------------------------------------------------------------
             |               Robust
           y |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           x |   5.902412   .2907357    20.30   0.000     5.332581    6.472244
       _cons |  -37.03774   3.094795   -11.97   0.000    -43.10343   -30.97206
------------------------------------------------------------------------------
Instrumented:  x
Instruments:   z1 z3

. estat overid
```

```
  Test of overidentifying restriction:

  Hansen's J chi2(1) = 159.838 (p = 0.0000)

. estat endogenous x

  Test of endogeneity (orthogonality conditions)
  Ho: variables are exogenous

  GMM C statistic chi2(1) =    .0189  (p = 0.8907)

. est store gmm_z1z3

. ivregress gmm y (x=z1 z4)

Instrumental variables (GMM) regression          Number of obs   =        500
                                                 Wald chi2(1)    =      40.88
                                                 Prob > chi2     =     0.0000
                                                 R-squared       =     0.3031
GMM weight matrix: Robust                        Root MSE        =     19.727

------------------------------------------------------------------------------
             |               Robust
           y |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           x |   2.167043   .3389432     6.39   0.000     1.502726    2.831359
       _cons |  -.6716121   3.395567    -0.20   0.843    -7.326801    5.983577
------------------------------------------------------------------------------
Instrumented:  x
Instruments:   z1 z4

. estat overid

  Test of overidentifying restriction:

  Hansen's J chi2(1) = .012951 (p = 0.9094)

. estat endogenous x

  Test of endogeneity (orthogonality conditions)
  Ho: variables are exogenous

  GMM C statistic chi2(1) =  157.747  (p = 0.0000)

. est store gmm_z1z4

. ivregress gmm y (x=z3 z4)

Instrumental variables (GMM) regression          Number of obs   =        500
                                                 Wald chi2(1)    =     457.33
                                                 Prob > chi2     =     0.0000
                                                 R-squared       =     0.4615
GMM weight matrix: Robust                        Root MSE        =     17.341

------------------------------------------------------------------------------
             |               Robust
           y |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           x |   6.237532   .2916741    21.39   0.000     5.665861    6.809202
       _cons |  -40.18052   3.069851   -13.09   0.000    -46.19731   -34.16372
------------------------------------------------------------------------------
Instrumented:  x
Instruments:   z3 z4

. estat overid

  Test of overidentifying restriction:
```

```
  Hansen's J chi2(1) = 148.756 (p = 0.0000)

. estat endogenous x

  Test of endogeneity (orthogonality conditions)
  Ho: variables are exogenous

  GMM C statistic chi2(1) =  5.55593  (p = 0.0184)

. est store gmm_z3z4

. est table ols gmm_z1z3 gmm_z1z4 gmm_z3z4, star(.1 .05 .01) stat(N rss F chi2
r2 r2_a J)

------------------------------------------------------------------------------
    Variable |      ols          gmm_z1z3        gmm_z1z4        gmm_z3z4
-------------+----------------------------------------------------------------
           x |  5.4313329***    5.9024123***    2.1670429***    6.2375315***
       _cons | -33.313329***   -37.037744***   -.67161206      -40.180516***
-------------+----------------------------------------------------------------
           N |        500             500             500             500
         rss |  146722.77       148205.87       194577.03        150355.5
           F |  449.66358
        chi2 |                  412.15656       40.877192       457.33009
          r2 |  .47449706       .46918519       .30310205       .46148607
        r2_a |  .47344183        .4681193       .30170266       .46040471
           J |                  159.83754       .01295053       148.75614
------------------------------------------------------------------------------
                                           legend: * p<.1; ** p<.05; *** p<.01
```

According to GMM estimated results and Overidentification Hensen J test, the best set of instrumental variables consists of z_1 and z_4 as instrumental variables.

c. Determine whether OLS estimated results in (a) or GMM estimated results in (b) is more appropriate using the difference-in-Sargan or C statistic test.

```
. estat endogenous x

  Test of endogeneity (orthogonality conditions)
  Ho: variables are exogenous

  GMM C statistic chi2(1) =  157.747  (p = 0.0000)
```

According to the difference-in-Sargan or C statistic test, *x* is endogenous variable. Thus, GMM estimated result using z_1 and z_4 as instrumental variables is the most appropriated estimated result.