Heteroskedasticity $\rightsquigarrow$ $Var(u|X) \neq$ constant

★ **Consequences**

1) $\hat{\beta}_{OLS}$ unbiased (no violation of MLR 1–4)
2) But it will require a different calculation of $Var(\hat{\beta})$ $\rightsquigarrow$ $Var(\hat{\beta}_{OLS})$ formula cannot be used.

$$Var(\hat{\beta}_j) = \frac{\sum_i (x_i - \bar{x})^2 \sigma_i^2}{SST_x^2} \quad \text{(hetero)}$$

rather than

$$Var(\hat{\beta}_{j\,OLS}) = \frac{\sigma^2}{SST_x} \quad \text{(homosk.)}$$

↳ Since we use $var(\hat{\beta}_j)$ to test hypotheses our t-test, F-tests will be invalid !?

★ **How to detect??** $\rightsquigarrow$ BP-test, White-test.

Motivation - when we have heterosk, $cor(X, \hat{u}^2) \neq 0$ ($Var(\sigma^2)$)

1) BP-test $\rightsquigarrow$ tests $cor(X, \hat{u}^2)$

stricter $\rightsquigarrow$ 2) White-test $\rightsquigarrow$ ~ $cor(X, \hat{u}^2)$ & $cor(X^2, \hat{u}^2)$ & $cor(X_i X_j, \hat{u}^2)$

See p.102 for the short form of white-test.

So, which test is more likely to find hetero? White!

- B/C we are testing more than 1 hypothesis tests for BP & White $\rightsquigarrow$ use F-tests
- Also F-distribution $\rightarrow$ χ^2-distribution as $n \to \infty$ We can use LM-test for white-test, BP-test as well.

H_0: no heteroskedasticity
H_a: heteroskedasticity

★ **Remedies** $\rightsquigarrow$ if we find heteroskedasticity, what can we do?

1) Passive way $\rightsquigarrow$ use ~~heteroskedasticity-robust~~ std. err.
   regress y x_1 x_2 .. x_k, robust
2) Active way $\rightsquigarrow$ transform the raw data so that there is no heteroskedasticity.
   weighted-least squares.

Hetero – Autocorr – Mulicollinearity are the problems that we face all the time in linear regression.

→ 1) Know what they are, their causes

2) Know the consequences

→ Bias?
→ Efficient?

} Hetero / Auto
→ $\hat{\beta}$ not biased
→ but inefficient
b/c variance not minimum

Do you also know that

$$\sqrt{var(\hat{\beta})} = \text{std. err.?}$$

→ if we use incorrect std. err., then all inferences will be invalid

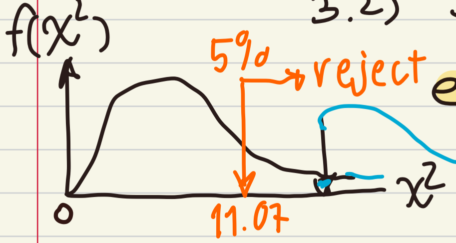
↳ t-test, F-test.

3) Know how to detect

3.1) In theory (Hetero)

Hetero → $cov(Var(u), X) = 0$
$cov(\hat{u}^2, X) = 0$
↳ BP-test, White-test

3.2) In practice (Hetero)

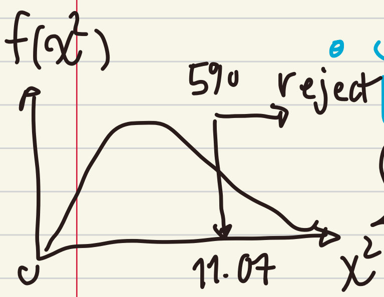


estat hettest → BP-test (P.100)
chi2(5) = 93.90

H_0: Homoske
$cov(\hat{u}^2; X) = 0$
H_a: Heterosk
$cov(\hat{u}^2; X) \neq 0$

- 5 explanatory variables in the equation. So, d.f. = 5

- Do we reject H_0?? (at 5% level)
- We check with the χ^2-table with d.f. = 5 and prob. = 0.05 ⇒ 11.07



- So, we reject H_0: homoskedasticity at 5% level and conclude that we have heterosk.

estat white → White-test
chi2(2) = 25.04

H_0: homoske
$cov(\hat{u}^2; X, X^2) = 0$
$cov(\hat{u}^2, \hat{y}, \hat{y}^2) = 0$
H_a: heteroskedastic.

no correlation between var(u) and X or interactions of X.

note: Because MLR1-4 will still be satisfied, $\hat{\beta}_{OLS}$ will be unbiased under autocorrelation.

3.3 In theory (Autocorrelation)

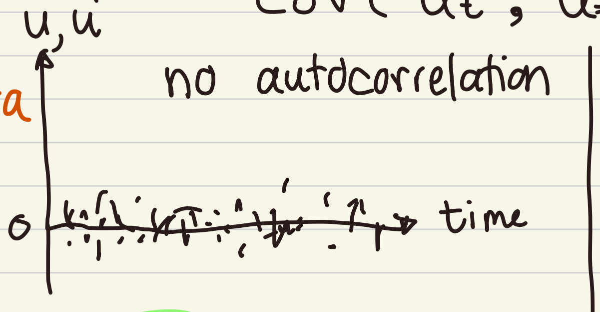
$Cov(u_i, u_{\neq i}) \neq 0$ (cross-sectional data)

$Cov(u_t, u_{\neq t}) \neq 0$ (time-series data)

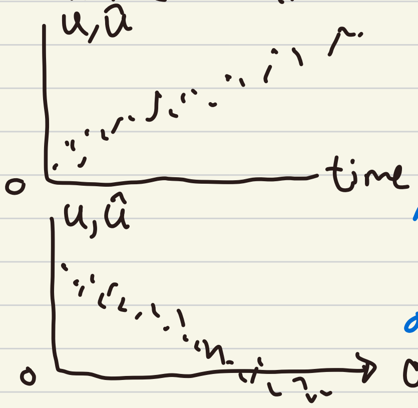
To detect

① plot the data

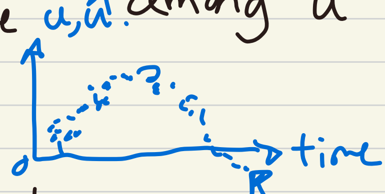
no autocorrelation



Autocorrelation



* There is/are detectable pattern(s) among "u"



something like this may be caused by specification error ⇒ e.g. add x^2 or x^3 to the model!

② → tests AR(1)

$u_t = \rho u_{t-1} + error_t$

$H_0: \rho = 0$ (no auto / serial correlation)
$H_a: \rho \neq 0$ (auto / serial correlation)

Steps: 1) estimate the main regression
$Y_t = \beta_0 + \beta_1 X_{1t} + \beta_2 X_{2t} + u_t$

2) obtain (predict $\hat{u}_t$) from step 1)

also create $\hat{u}_{t-1}$

t	y	x_1	x_2	$\hat{u}$	$\hat{u}_{t-1}$
1	100	2	5.1	0.1	–
2	102			-0.2	0.1
3				0.5	-0.2
4					0.5
5					
⋮					

3) estimate $\hat{u}_t = \rho \hat{u}_{t-1} + error_t$

regress $\hat{u}_t$ $\hat{u}_{t-1}$, no con

no intercept

4) $H_0: \hat{\rho} = 0$ (no autocorrelation)
$H_a: \hat{\rho} \neq 0$ (autocorrelation)

③ → Durbin-Watson test (DW test)

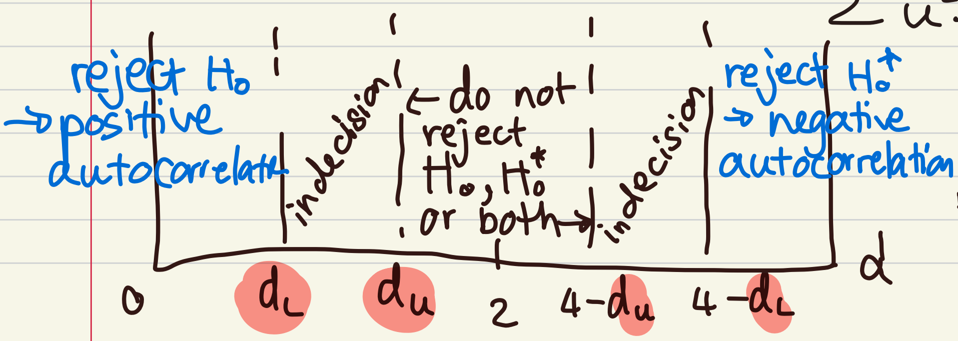
In this class, we cover 3 ways to detect.

$$d = \frac{\sum_{t=2}^{t=n} (\hat{u}_t - \hat{u}_{t-1})^2}{\sum_{t=1}^{t=n} \hat{u}_t^2}$$

$0 \leq d \leq 4$ ← since $-1 < \rho$

↳ short way $\hat{\rho} = \frac{\sum \hat{u}_t \hat{u}_{t-1}}{\sum \hat{u}_t^2}$ ⇒ $d \approx 2(1-\hat{\rho})$

same $\hat{\rho}$ as in the AR(1) test.



H_0: No positive autocorrelation
H_0^*: No negative ~