

Assignment 7 Long-term growth model

Due on Friday 18th, 2018. In class.

Question 1

Consider a mathematical version of the Solow growth model that we discussed in class. Hopefully, this mathematical version will help you better understand all everything per the growth process in the Solow framework.

Let's focus on the case where the production function is given by the *Cobb-Douglass* function. That is we assume that

$$Y = ZF(K, N) = ZK^{\alpha}N^{1-\alpha}$$

All the notations used are defined in the same way as we discussed in class. Also, we maintain the two key exogenous assumption used in class. That is, we assume that (i) population (N) is growing at the rate “ n ” per period, and (ii) the saving rate is constant, i.e. $S = s \cdot Y$.

a) Derive the per capita output function (y : the little y). Show that the function can be expressed in terms of the capital per worker (k : capital-to-labor intensity ratio).

b) Derive the equation that describes the motion (or evaluation) of future capital per worker (k') as expressed in terms of current capital per worker (k).

c) Algebraically solve for the steady-state capital-to-labor (k^*).

- d) What is the associated level of consumption-per-capita (c^*) at the steady state?
- e) Explain what happens to the growth rate of aggregate output (Y) and aggregate consumption (C) in the steady state.
- f) What happens to the steady-state value of all the per-capita variables in the model if the economy experiences a decrease in " z " and an increase in the population growth " n ", respectively? Use the partial derivative method to show your final result. Link your result to the analytical results obtained from the diagram that we discussed in class. Provide some economic reasoning.
- g) Algebraically solve for the golden-rule allocation.

Question 2

Suppose that the economy is initially in a steady state and that some of the nation's capital stock is destroyed because of a natural disaster or a war.

- (a) Determine the long-run effects of this on the quantity of capital per worker and on output per worker.
- (b) In the short-run does aggregate capital stock grow at a rate higher or lower than growth rate of the labor force
- (c) After World War II, growth in real GDP in Germany and Japan was very high. How do your results in parts (a) and (b) shed light on this historical experience.