62046415IS

1) Let S_i be the price of underlying asset at date i and let D_0 be date 0 present value of dividends.

Pricing using stochastic discount factor

$$S_0 = E[m_{01} D_1] + E[m_{02} S_2] = D_0 + E[m_{02} S_2] \quad \text{— (1)}$$

let F_{02} = forward price then payoff to the long party is $f_2 = S_2 - F_{02}$

using stochastic discount factor pricing

$$E_0[m_{02} f_2] = E[m_{02}(S_2 - F_{02})] = E[m_{02} S_2] - E[m_{02} F_{02}]$$

From (1)

$$S_0 = E_0[m_{01} D_1] + E_0[m_{02} S_2] = D_0 + E_0[m_{02} S_2]$$

$$E[m_{02} F_{02}] = E[m_{02}] F_{02} = R_f^{-2} F_{02}$$

So we have

$$E[m_{02} f_2] = E[m_{02} S_2] - E[m_{02} F_{02}]$$

$$= S_0 - D_0 - R_f^{-2} F_{02}$$

From absence of arbitrage

$$F_{02} = R_f^2 (S_0 - D_0)$$

which implies that $E_0[m_{02} f_2] = 0$

62046415I5

2) From

$$P_0 = E_0\left[\sum_{t=1}^{\infty} \frac{u_c(c_t,t)}{u_c(c_0,0)} d_t\right]$$

$$u_c(c_t,t) = a\delta^{-1} e^{-ac_t} \qquad c_t = d_t$$

So

$$P_0 = E\left[\sum_{t=1}^{\infty} \delta^t e^{-a(d_t-d_0)} d_t\right]$$

---

3)

$$P_t = E_t\left[\sum_{t=1}^{\infty} \delta^j \left(\frac{c^*_{t+j}}{c^*_t}\right)^{\gamma-1} d_{t+j}\right]$$

$$\frac{P_t}{d_t} = E_t\left[\sum_{j=1}^{\infty} \delta^j e^{(\gamma-1)\ln(C_{t+j}/C_t) + \ln(d_{t+j}/d_t)}\right]$$

From $\ln(C_{t+j}/C_t) = j\mu_c + \sigma_c \sum_{j=1}^{j} n_{t+j}$

$$\ln(D_{t+j}/D_t) = j\mu_d + \sigma \sum_{i=1}^{j} \varepsilon_{t+j}$$

$$\frac{P_t}{d_t} = E_t\left[\sum_{j=1}^{\infty} \delta^j e^{(\gamma-1)(j\mu_c + \sigma_c \sum_{i=1}^{j} n_{t+j}) + j\mu_d + \sigma_d \sum_{i=1}^{j} \varepsilon_{t+j}}\right]$$

$$= E_t\left[\sum_{j=1}^{\infty} \delta^j e^{[(\gamma-1)\mu_c + \mu_c] + \sum_{j=1}^{j}[(\gamma-1)\sigma_c n_{t+j} + \sigma_d \varepsilon_{t+j}]}\right]$$

$$= \sum_{j=1}^{\infty} \delta^j e^{[(\gamma-1)\mu_c + g_d]} e^{\frac{j}{2}[(1-\gamma)^2\sigma_c^2 + \sigma_d^2 - 2(1-\gamma)\sigma_c\sigma_d\rho]}$$

$$= \sum_{j=1}^{\infty} e^{[\ln\delta - (1-\gamma)\mu_c + \mu_d + \frac{1}{2}((1-\gamma)^2\sigma_c^2 + \sigma_d^2) - (1-\gamma)\sigma_c\sigma_d\rho]}$$

$$= \frac{1}{1-\delta e^{-(1-\gamma)\mu_c + \mu_d + \frac{1}{2}((1-\gamma)^2\sigma_c^2 + \sigma_d^2) - (1-\gamma)\sigma_c\sigma_d\rho}} - 1$$

$$P = d_t \frac{\delta e^{\alpha}}{1-\delta e^{\alpha}} \quad ; \quad \alpha = \mu_d - (1-\gamma)\mu_c + \frac{1}{2}[(1-\gamma)^2\sigma_c^2 + \sigma_d^2] - (1-\gamma)\rho\sigma_c\sigma_d$$

62046415I5

4a)

check that (2) satisfy $E_t(b_{t+1}) = R_f b_t$

$$E_t[b_{t+1}] = \frac{R_f}{q_t} b_t q_t + E_t[e_{t+1}] q_t + (1-q_t) E_t[z_{t+1}] = R_f b_t$$

$\therefore$ it's a valid solution

4b)

From $E_t[b_{t+1}] = R_f b_t$

$$\lim_{i \to \infty} E_t[b_{t+i}] = \begin{cases} +\infty & \text{if } b_t > 0 \\ -\infty & \text{if } b_t < 0 \end{cases}$$

Because price cannot become negative, we consider only with $b_t > 0$.

In this case, we can see from the equation that bubble solution exist, however in reality, there will be a substitute product that will prevent the price to increase infinitely.

4c)

Because bond price must be $p_t = d_t$ and zero after date T, its price can't rationally increase forver. Thus, the bubble path is invalid.

6204641515

5) With the economy having a finite time horizon, asset prices could not have the form $p_t = f_t + b_t$ with $b_t \neq 0$ because at date T, $p_T = f_T = d_T$ which is an asset's final dividend payment. Since $b_T = 0$ with certainty, then the bubble process $E[b_{t+1}] = \delta^{-1} b_t$ implies $E_{T-1}[b_T] = E_T[0] = \delta^{-1} b_T$ or $b_{T-1} = 0$. A similar argument implies $b_t = 0$ for all previous dates, $t < T-1$.