EE320 Introductory Mathematical Economics

Quiz 6

1. *(10 points)*

Suppose that a firm's production function is given by: $Q = KL$. This firm plans to produce 15 units of the good. Given that the input prices for K and L are 10 and 6, respectively, find the amount of K and L that minimize the total cost. Make sure that you show the second-order sufficient condition.

$\underline{C}$

$$\min_{K,L} \; C = 10K + 6L$$

$$\text{s.t. } \bar{Q} = KL = 15$$

$$\mathcal{L} = 10K + 6L + \lambda[15 - KL]$$

FONC.

$$\left.\begin{aligned} \mathcal{L}_K &= 10 - \lambda L = 0 \\ \mathcal{L}_L &= 6 - \lambda K = 0 \\ \mathcal{L}_\lambda &= 15 - KL = 0 \end{aligned}\right\} \; (K^*, L^*) = (3, 5)$$

SOSC.

$$|\bar{H}| = \begin{vmatrix} 0 & g_K & g_L \\ g_K & \mathcal{L}_{KK} & \mathcal{L}_{KL} \\ g_L & \mathcal{L}_{LK} & \mathcal{L}_{LL} \end{vmatrix} = \begin{vmatrix} 0 & 5 & 3 \\ 5 & 0 & -2 \\ 3 & -2 & 0 \end{vmatrix} = -60 < 0.$$

$\therefore$ SOSC are satisfied.

2. *(5 points)*

Briefly explain the economic interpretation of the Lagrange multiplier in the consumer's utility maximization problem.

Lagrange multiplier is the shadow price of resource (, in this case is the income). It tells how much the maximum utility, $u(x^*, y^*)$, if the income in the budget constraint increases by 1 unit.

3. *(5 points)*

Find the integral $\int (3x^4 + 5x^2 - 2)\,dx$.

$$\int (3x^4+5x^2-2)dx = \frac{3x^{4+1}}{4+1} + \frac{5x^{2+1}}{2+1} - 2x + C$$

$$= \frac{3x^5}{5} + \frac{5x^3}{3} - 2x + C.$$