EE377 HW

5604640432

10/10

1. Consider 2 firms. Suppose total cost = 20 for both firms. $MC = 0$

Firm 1's demand : $q_1 = 12 - 2P_1 + P_2$ $TC = 20$ $MC_1 = MC_2 = 0$

Firm 2's demand : $q_2 = 12 - 2P_2 + P_1$

Find 1) $BR_1(P_2)$ 3) P_1^* and P_2^* (Bertrand equilibrium in Price) 5) Find also, $q_1^*, q_2^*, \pi_1^*, \pi_2^*$

2) $BR_2(P_1)$ 4) Plot BR_1 and BR_2 and indicate an equilibrium point in your diagram

Find BR

Firm 1

$TR_1 = P_1 Q_1$

$= P_1(12 - 2P_1 + P_2)$

$= 12P_1 - 2P_1^2 + P_1P_2$

$\frac{dTR}{dP_1} = \frac{d(12P_1 - 2P_1^2 + P_1P_2)}{dP_1}$

(differentiate TR with respect to P_1 to find marginal revenue of the firm)

$MR_1 = 12 - 4P_1 + P_2$

It is given that $MC_1 = 0$. Firm maximizes its profit when $MC_1 = MR_1$, hence

$MR_1 = MC_1 \rightarrow 12 - 4P_1 + P_2 = 0$

$$\boxed{P_1 = \frac{P_2 + 12}{4}}$$ → firm 1's BR function ✓

Firm 2

$TR_2 = P_2(q_2)$

$= P_2(12 - 2P_2 + P_1)$

$= 12P_2 - 2P_2^2 + P_1P_2$

$MR_2 = \frac{dTR_2}{dP_2} = 12 - 4P_2 + P_1$

Firm sets $MC_2 = MR_2$ to maximize profit

$12 - 4P_2 + P_1 = 0$

$$\boxed{P_2 = \frac{P_1 + 12}{4}}$$ → firm 2's BR function ✓

Next, substitute BR_1 function and BR_2 function into each other to find equilibrium prices for both firms

Firm 1

$P_1 = \frac{\left(\frac{P_1 + 12}{4}\right) + 12}{4}$

$= \frac{\frac{P_1 + 60}{4}}{4}$

$P_1 = \frac{P_1 + 60}{16}$

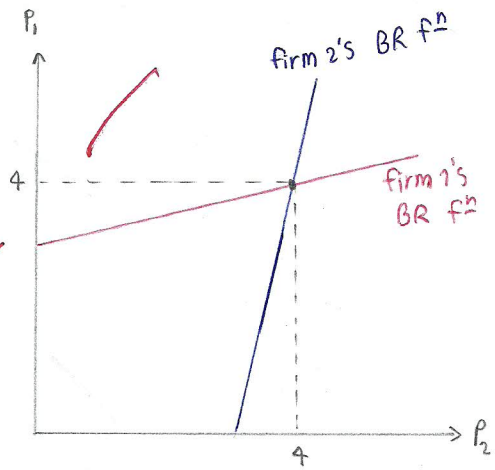
$P_1^* = 4$ Baht/unit ✓

Firm 2

$P_2 = \frac{\left(\frac{P_2 + 12}{4}\right) + 12}{4}$

$= \frac{\frac{P_2 + 60}{4}}{4}$

$P_2^* = 4$ baht/unit ✓



At equilibrium

$q_1^* = 12 - 2P_1^* + P_2^*$

$= 12 - 2(4) + 4 = 8$ units ✓

$\pi_1^* = TR_1 - TC_1$

$= P_1^* q_1^* - TC_1 = 8(4) - 20 = 12$ baht ✓

$q_2^* = 12 - 2P_2^* + P_1^*$

$= 12 - 2(4) + 4) = 8$ units ✓

$\pi_2^* = TR_2 - TC$

$= P_2^* q_2^* - TC_2 = 8(4) - 20 = 12$ baht ✓

5/5

2. Suppose firm 1 sets its price first and, after observing firm 1, firm 2 sets its price P_2.

Find P_1^*, P_2^*. Show that $\pi_1^* > \pi_2^*$ (Price leadership model)

Firm 1 sets price first, P_1^*;

$$TR_1 = (P_1)(12 - 2P_1 + P_2)$$

From last question $P_2 = \frac{P_1 + 12}{4}$

$$TR_1 = (P_1)\left(12 - 2P_1 + \frac{P_1 + 12}{4}\right)$$

$$= P_1 (12 - 2P_1 + 0.25P_1 + 3)$$

$$TR_1 = 15P_1 - 1.75P_1^2$$

Now we find the optimal P_1, first we calculate for MR_1

$$MR_1 = \frac{dTR_1}{dP_1} = 15 - 3.5P_1$$

To maximize profit, firm sets price where $MR = MC$

$$15 - 3.5P_1 = 0$$

$$P_1^* = \frac{15}{3.5} = 4.29 \text{ baht/unit}$$

hence, firm 2's optimal price is

$$P_2^* = \frac{P_1 + 12}{4} = \frac{4.29 + 12}{4}$$

$$P_2^* = 4.07 \text{ baht/unit}$$

Next, find q_1^*, q_2^*, π_1, π_2

$$q_1^* = 12 - 2P_1 + P_2$$
$$= 12 - 2(4.29) + 4.07$$
$$= 7.49 \text{ units}$$

$$q_2^* = 12 - 2P_2 + P_1$$
$$= 12 - 2(4.07) + 4.29$$
$$= 8.15 \text{ units}$$

$$\pi_1 = P_1^* \cdot q_1^* - TC_1$$
$$= 4.29(7.49) - 20$$
$$= 12.13 \text{ baht}$$

$$\pi_2 = P_2^* q_2^* - TC$$
$$= 4.07(8.15) - 20$$
$$= 13.17 \text{ baht}$$

first mover!

$\Rightarrow \pi_2 > \pi_1$

There is no first mover advantage in this case i.e. even though firm 1 can set price first, its profit is not greater than firm 2. This may be because products of the two firms are not homogeneous.

Note: • the two products are not homogeneous
• the firm who moved last charge the price slightly lower

5/5