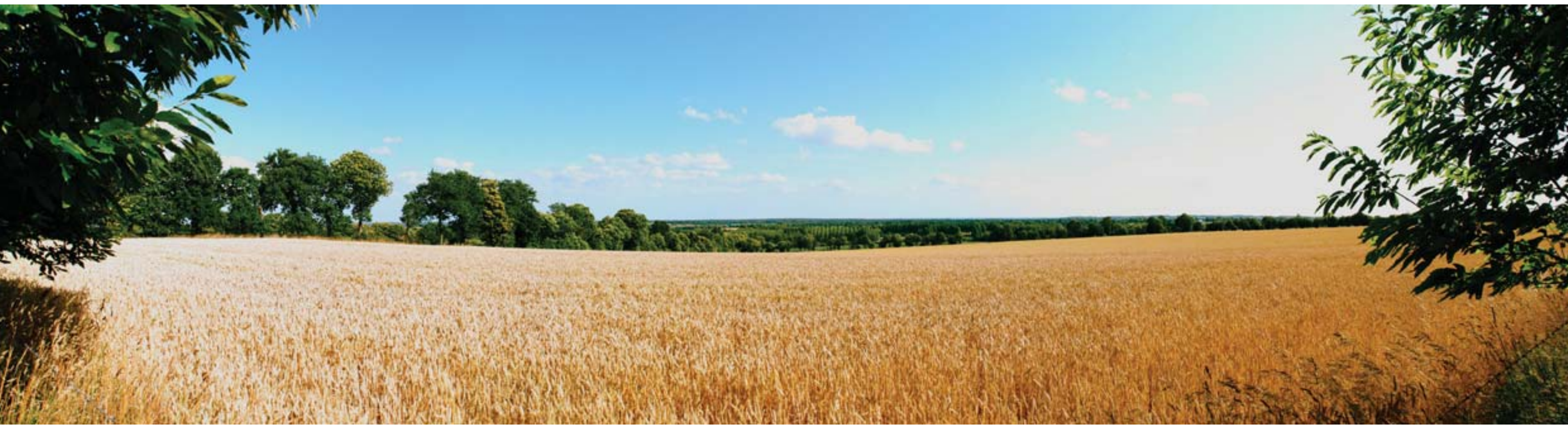

6. Extensions of The Two-Variable Linear Regression Mode

6.1 Functional Form of regression Models

We will consider the following models:

[1] The log-linear model

[2] Semilog models

[3] Reciprocal models

[4] The logarithmic reciprocal model

The Log-linear Model

The Semilog Models

The Reciprocal Models

The Logarithmic Reciprocal Models

6.2 Regression Through the Origin

In this section, we consider the case that the two-variable PRF assumes the following form:

$$Y_i = \beta_2 X_i + u_i$$

This model is called **the regression through the origin** where the intercept term $\hat{\beta}_1$ is absent from the model.

Example

Since it is the linear regression model, we can apply the Ordinary Least Square (OLS) to estimate the formula for $\hat{\beta}_2$

Let us first write the sample regression function (SRF) as:

$$Y_i = \hat{\beta}_2 X_i + \hat{u}_i$$

We would like to minimize

$$\sum \hat{u}_i{}^2 = \sum (Y_i - \hat{\beta}_2 X_i)^2$$

therefore,

$$\boxed{\hat{\beta}_2 = \frac{\sum X_i Y_i}{\sum X_i^2}}$$

Now we can find out the variance of $\hat{\beta}_2$

$$\text{var}(\hat{\beta}_2) = \frac{\sigma^2}{\sum X_i^2}$$

$$\hat{\sigma^2} = \frac{\sum \hat{u_i}^2}{n-1}$$

It should be noted that we get the condition $\sum \hat{u}_i X_i = 0$ from the normal equation. However, with the regression through the origin model, we cannot get the condition $\sum \hat{u}_i = 0$.

For the zero-intercept model, r^2 can be negative, whereas for the conventional model it cannot be negative.

Since the conventional r^2 is not appropriate for the regressions that do not contain the intercept, we therefore compute what is known as the **raw** r^2 instead:

$$\text{raw}\, r^2 = \frac{(\sum X_i Y_i)^2}{\sum X_i^2 \sum Y_i^2}$$

This raw r^2 has its value between 0 and 1, but we cannot directly compare its value to the conventional r^2 value. For this reason, some researchers do not report the r^2 value for zero intercept regression models.

6.2.1 Scaling and Units of Measurements

Consider our old example given in table 18 which refer to weekly family expenditure (Y) and Income (x), in baht.

Table 6.1: Weekly family Expenditure (Y), Baht and Income (X), (Unit:Baht)

X	Y
500	360
600	390
700	440
800	575
900	670
1000	730

By using the OLS estimation, we get the following results:

Now, we are interested in changing the units of our data. For example, we would prefer to express our sample data in the unit of 1000 baht. By using the new unit of X and Y, we can report our data in 1000 baht as in the following table.

Table 6.2: Weekly family Expenditure (Y), Baht and Income (X), (Unit: 1000 Baht)

X	Y
0.5	0.360
0.6	0.390
0.7	0.440
0.8	0.575
0.9	0.670
1	0.730

With the new unit, we would like to answer these two questions:
1. Do the units in which the regressand (Y) and regressor/s (X) are measured make any difference in the regression results?
2. If so, what is the sensible course to follow in choosing units of measurement for regression analysis?

To answer these questions, let:

$$Y_i = \hat{\beta}_1 + \hat{\beta}_2 X_i + \hat{u}_i$$

where Y is the weekly family expenditure and X is the income, in baht.

Now, let w_1 and w_2 are constants, called the **Scale factors**. For example, in our data, if we need to use the unit of 1000 baht instead, we can directly multiple the original data in table 18 with the scale factors equal to 0.001. In other words, $w_1 = w_2 = \frac{1}{1000} = 0.001$.

Define

$$Y_i^* = w_1 Y_i$$

$$X_i^* = w_1 X_i$$

Now consider the regression using Y_i^* and X_i^* variables:

$$Y_i^* = \hat{\beta}_1^* + \hat{\beta}_2^* X_i^* + \hat{u}_i{}^*$$

$\hat{u_i^*} = ?$

Our target is to find out the relationship between the following pairs:

1. $\hat{\beta}_1$ and $\hat{\beta}_1^*$

2. $\hat{\beta}_2$ and $\hat{\beta}_2^*$

3.var$(\hat{\beta}_1)$ and var$(\hat{\beta}_1^*)$

4.var$(\hat{\beta}_2)$ and var$(\hat{\beta}_2^*)$

5.$\hat{\sigma}^2$ and $\hat{\sigma}^{*2}$

6. r_{xy}^2 and $r_{x^*y^*}^2$

1. $\hat{\beta}_1$ and $\hat{\beta}_1^*$

2. $\hat{\beta}_2$ and $\hat{\beta}_2^*$

3.var($\hat{\beta}_1$) and var($\hat{\beta}_1^*$)

4.var($\hat{\beta}_2$) and var($\hat{\beta}_2^*$)

5.$\hat{\sigma}^2$ and $\hat{\sigma}^{*2}$

6. r_{xy}^2 and $r_{x^*y^*}^2$