Session 2-3 Probability and Statistical Concepts – Part 1 Exercise:

1. A firm holds two $50 million bonds with call dates this week.
   The probability that Bond A will be called is 0.80.
   The probability that Bond B will be called is 0.30.
   The probability that at least one of the bonds will be called is *closest to*?

Answer:
We calculate the probability that at least one of the bonds will be called using the addition rule for probabilities:
$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, where $P(A \text{ and } B) = P(A) \times P(B)$
$P(A \text{ or } B) = 0.80 + 0.30 - (0.8 \times 0.3) = 0.86$

2. Jessica Fassler, options trader, recently wrote two put options on two different underlying stocks (AlphaDog Software and OmegaWolf Publishing), both with a strike price of $11.50. The probabilities that the prices of AlphaDog and OmegaWolf stock will decline below the strike price are 65% and 47%, respectively. The probability that at least one of the put options will fall below the strike price is approximately?

Answer:
We calculate the probability that at least one of the options will fall below the strike price using the addition rule for probabilities (A represents AlphaDog, O represents OmegaWolf):
$P(A \text{ or } O) = P(A) + P(O) - P(A \text{ and } O)$, where $P(A \text{ and } O) = P(A) \times P(O)$
$P(A \text{ or } O) = 0.65 + 0.47 - (0.65 \times 0.47) =$ approximately 0.81

3. Thomas Baynes has applied to both Harvard and Yale. Baynes has determined that the probability of getting into Harvard is 25% and the probability of getting into Yale (his father's alma mater) is 42%. Baynes has also determined that the probability of being accepted at both schools is 2.8%. What is the probability of Baynes being accepted at either Harvard or Yale?

Answer:
Using the addition rule, the probability of being accepted at Harvard or Yale is equal to:
$P(\text{Harvard}) + P(\text{Yale}) - P(\text{Harvard and Yale}) = 0.25 + 0.42 - 0.028 = 0.642$ or 64.2%.

4. There is a 30% chance that the economy will be good and a 70% chance that it will be bad. If the economy is good, your returns will be 20% and if the economy is bad, your returns will be 10%. What is your expected return?

Answer:
Expected value is the probability weighted average of the possible outcomes of the random variable. The expected return is: $((0.3) \times (0.2)) + ((0.7) \times (0.1)) = (0.06) + (0.07) = 0.13$.

5. There is a 90% chance that the economy will be good next year and a 10% chance that it will be bad. If the economy is good, there is a 60% chance that XYZ Incorporated will have EPS of $4.00 and a 40% chance that their earnings will be $3.00. If the economy is bad, there is an 80% chance that XYZ Incorporated will have EPS of $2.00 and a 20% chance that their earnings will be $1.00. What is the firm's expected EPS?

Answer:
The expected EPS is calculated by multiplying the probability of the economic environment by the probability of the particular EPS and the EPS in each case. The expected EPS in all four outcomes are then summed to arrive at the expected EPS:
$(0.90 \times 0.60 \times \$4.00) + (0.90 \times 0.40 \times \$3.00) + (0.10 \times 0.80 \times \$2.00) + (0.10 \times 0.20 \times \$1.00) = \$2.16 + \$1.08 + \$0.16 + \$0.02 = \$3.42$.

6. There is a 40% probability that the economy will be good next year and a 60% probability that it will be bad. If the economy is good, there is a 50 percent probability of a bull market, a 30% probability of a normal market, and a 20% probability of a bear market. If the economy is bad, there is a 20% probability of a bull market, a 30% probability of a normal market, and a 50% probability of a bear market. What is the probability of a bull market next year?

Answer:
Because a good economy and a bad economy are mutually exclusive, the probability of a bull market is the sum of the joint probabilities of (good economy and bull market) and (bad economy and bull market): $(0.40 \times 0.50) + (0.60 \times 0.20) = 0.32$ *or 32%.*

7. An investor is considering purchasing ACQ. There is a 30% probability that ACQ will be acquired in the next two months. If ACQ is acquired, there is a 40% probability of earning a 30% return on the investment and a 60% probability of earning 25%. If ACQ is not acquired, the expected return is 12%. What is the expected return on this investment?

Answer:
$E(r) = (0.70 \times 0.12) + (0.30 \times 0.40 \times 0.30) + (0.30 \times 0.60 \times 0.25) = 0.165$.

8. Tully Advisers, Inc., has determined four possible economic scenarios and has projected the portfolio returns for two portfolios for their client under each scenario. Tully's economist has estimated the probability of each scenario, as shown in the table below. Given this information, what is the standard deviation of returns on portfolio A?

Scenario	Probability	Return on Portfolio A	Return on Portfolio B
A	15%	18%	19%
B	20%	17%	18%
C	25%	11%	10%
D	40%	7%	9%

Answer:
$E(R) = 11.65\%$
$\sigma^2 = 0.0020506$
$= 0.15(0.18 - 0.1165)^2 + 0.2(0.17 - 0.1165)^2 + 0.25(0.11 - 0.1165)^2 + 0.4(0.07 - 0.1165)^2$
$\sigma = 0.0452836$

9. Joe Mayer, CFA, projects that XYZ Company's return on equity varies with the state of the economy in the following way:

State of Economy	Probability of Occurrence	Company Returns
Good	.20	20%
Normal	.50	15%
Poor	.30	10%

The standard deviation of XYZ's expected return on equity is *closest to:*

Answer:

In order to calculate the standard deviation of the company returns, first calculate the expected return, then the variance, and the standard deviation is the square root of the variance.

The expected value of the company return is the probability weighted average of the possible outcomes: (0.20)(0.20) + (0.50)(0.15) + (0.30)(0.10) = 0.145.

The variance is the sum of the probability of each outcome multiplied by the squared deviation of each outcome from the expected return:

$(0.2)(0.20\ 0.145)^2 + (0.5)(0.15\ 0.145)^2 + (0.3)(0.10.145)^2$

$= 0.000605 + 0.0000125 + 0.0006075 = 0.001225.$

The standard deviation is the square root of 0.001225 = 0.035 or 3.5%.

10. The covariance of the returns on investments X and Y is 18.17. The standard deviation of returns on X is 7%, and the standard deviation of returns on Y is 4%. What is the value of the correlation coefficient for returns on investments X and Y?

Answer:

The correlation coefficient = Cov (X,Y) / [(Std Dev. X)(Std. Dev. Y)] = 18.17 / 28 = 0.65

11. The returns on assets C and D are strongly correlated with a correlation coefficient of 0.80. The variance of returns on C is 0.0009, and the variance of returns on D is 0.0036. What is the covariance of returns on C and D?

Answer:

$r = Cov(C,D) / (\sigma_c \times \sigma_d)$

$\sigma_c = (0.0009)^{0.5} = 0.03$

$\sigma_d = (0.0036)^{0.5} = 0.06$

$0.8(0.03)(0.06) = 0.00144$

12. The covariance of returns on two investments over a 10year period is 0.009. If the variance of returns for investment A is 0.020 and the variance of returns for investment B is 0.033, what is the correlation coefficient for the returns?

Answer:

The correlation coefficient is:

$Cov(A,B) / [(Std\ Dev\ A)(Std\ Dev\ B)] = 0.009 / [(\sqrt{0.02})(\sqrt{0.033})] = 0.350.$

Session 2-3 Probability and Statistical Concepts – Part 2 Exercise:

1. A discount brokerage firm states that the time between a customer order for a trade and the execution of the order is uniformly distributed between three minutes and fifteen minutes. If a customer orders a trade at 11:54 A.M., what is the probability that the order is executed after noon?

Answer:
The limits of the uniform distribution are three and 15. Since the problem concerns time, it is continuous. Noon is six minutes after 11:54 A.M. The probability the order is executed after noon is (15 − 6) / (15 − 3) = 0.75.

2. Assume 30% of the CFA candidates have a degree in economics. A random sample of three CFA candidates is selected. What is the probability that none of them has a degree in economics?

Answer:
The probability of 0 successes in 3 trials is: $[3! / (0!3!)]\ (0.3)^0\ (0.7)^3 = 0.343$

3. A grant writer for a local school district is trying to justify an application for funding an afterschool program for low-income families. Census information for the school district shows an average household income of $26,200 with a standard deviation of $8,960. Assuming that the household income is normally distributed, what is the percentage of households in the school district with incomes of less than $12,000?

Answer:
Z = ($12,000 – $26,200) / $8,960 = -1.58.
From the table of areas under the standard normal curve, 5.71% of observations are more than 1.58 standard deviations below the mean.

4. The average amount of snow that falls during January in Frostbite Falls is normally distributed with a mean of 35 inches and a standard deviation of 5 inches. The probability that the snowfall amount in January of next year will be between 40 inches and 26.75 inches is *closest* to?

Answer:
To calculate this answer, we will use the properties of the standard normal distribution. First, we will calculate the Z-value for the upper and lower points and then we will determine the approximate probability covering that range. Note: This question is an example of why it is important to memorize the general properties of the normal distribution.

Z = (observation – population mean) / standard deviation
$Z_{26.75} = (26.75 - 35) / 5 = -1.65$. *(1.65 standard deviations to the left of the mean)*
$Z_{40} = (40 - 35) / 5 = 1.0$ *(1 standard deviation to the right of the mean)*

Using the general approximations of the normal distribution:

- *68% of the observations fall within ± one standard deviation of the mean. So, 34% of the area falls between 0 and +1 standard deviation from the mean.*
- *90% of the observations fall within ± 1.65 standard deviations of the mean. So, 45% of the area falls between 0 and +1.65 standard deviations from the mean.*

Here, we have 34% to the right of the mean and 45% to the left of the mean, for a total of 79%.

5. A client will move his investment account unless the portfolio manager earns at least a 10% rate of return on his account. The rate of return for the portfolio that the portfolio manager has chosen has a normal probability distribution with an expected return of 19% and a standard deviation of 4.5%. What is the probability that the portfolio manager will keep this account?

Answer:

Since we are only concerned with values that are below a 10% return this is a 1 tailed test to the left of the mean on the normal curve. With $\mu = 19$ and $\sigma = 4.5$, $P(X \geq 10) = P(X \geq \mu - 2\sigma)$ therefore looking up 2 on the cumulative Z table gives us a value of 0.0228, meaning that $(1 - 0.0228) = 97.72\%$ of the area under the normal curve is above a Z score of 2. Since the Z score of 2 corresponds with the lower level 10% rate of return of the portfolio this means that there is a 97.72% probability that the portfolio will earn at least a 10% rate of return.