CHAPTER 4

The choice of voting rule

> Decision by majorities is as much an expedient as lighting by gas.
>
> William Gladstone

> There are two general rules. First, the more grave and important the questions discussed, the nearer should the opinion that is to prevail approach to unanimity. Second, the more the matter in hand calls for speed, the smaller the prescribed difference in the number of votes may be allowed to become: when an immediate decision has to be reached, a majority of one should suffice.
>
> Jean-Jacques Rousseau

This and the next four chapters explore the properties of various voting rules. These rules can be thought of as governing the polity itself, as when decisions are made in a town meeting or by referendum, or an assembly, or a committee of representatives of the citizenry. Following Duncan Black (1958), we shall often refer to "committee decisions" as being the outcomes of the voting process. It should be kept in mind, however, that the word "committee" is employed in this wider sense, and can imply a committee of the entire polity voting, as in a referendum. When a committee of representatives is implied, the results can be strictly related only to the preferences of the representatives themselves. The relationship between citizen and representative preferences is taken up later.

A. The unanimity rule

Since all can benefit from the provision of a public good, the obvious voting rule for providing it would seem to be unanimous consent. Knut Wicksell (1896) was the first to link the potential for all to benefit from collective action to the unanimity rule. The unanimity rule coupled with the proposal that each public good be financed by a separate tax constituted Wicksell's "new principle" of taxation. To see how the procedure might work, consider a world with two persons and one public good. Each person has a given initial income, Y_A and Y_B, and a utility function defined over the public and private goods, $U_A(X_A,G)$ and $U_B(X_B,G)$, where X is the private good, and G the public good. The public good is to be financed by a tax of t on individual A, and $(1 - t)$ on individual B. Figure 4.1 depicts individual A's indifference

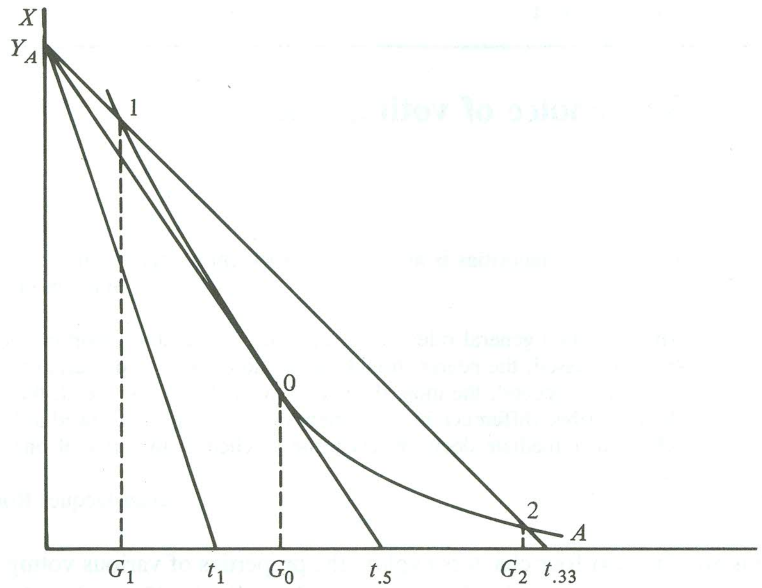

Figure 4.1. Optimal quantities for a voter at different tax prices.

curves between the private and public good. Let the prices of the private and public good be such that if A had to pay for all of the public good ($t = 1$), A's budget constraint line would be $Y_A t_1$. If A must pay only half of the cost of the public good, his budget constraint line would be $Y_A t_{.5}$, and so on. With a tax share of 0.5 A's optimal choice for a quantity of public good would be G_0. Note, however, that the tax–public good combinations $(t_{.33}, G_1)$ and $(t_{.33}, G_2)$ are on the same indifference curve as $(t_{.5}, G_0)$, and that one could calculate an infinite number of tax–public good quantity combinations from Figure 4.1 that lie upon indifference curve A. It is thus possible to map indifference curve A into a public good–tax space (Johansen, 1963).

Figure 4.2 depicts such a mapping. Points 0, 1, and 2 in Figure 4.2 correspond to points 0, 1, and 2 in Figure 4.1. Indifference curve A in Figure 4.2 is a mapping from the corresponding curve of Figure 4.1.

To map all points from Figure 4.1 into public good–tax space, we redefine each individual's utility function in terms of G and t alone. From the budget constraint, we obtain

$$\begin{aligned} X_A &= Y_A - tG \\ X_B &= Y_B - (1 - t)G. \end{aligned} \tag{4.1}$$

Substituting from (4.1) into each individual's utility function, we obtain the desired utility functions for A and B defined over G and t:

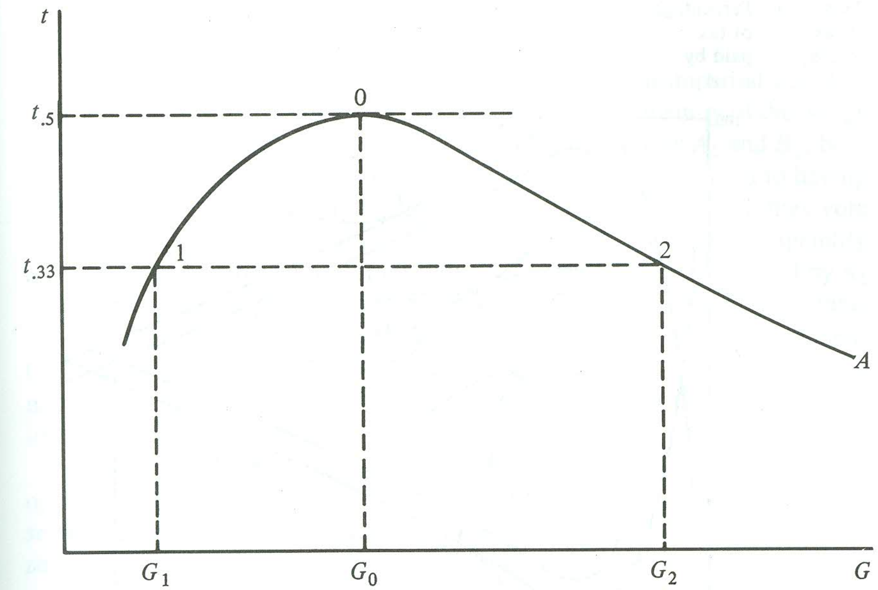

Figure 4.2. Mapping of voter preferences into tax–public good space.

$$\begin{aligned} U_A &= U_A(Y_A - tG, G) \\ U_B &= U_B(Y_B - (1 - t)G, G). \end{aligned} \tag{4.2}$$

Figure 4.3 depicts a mapping of selected indifference curves for A and B from public good–private good space into public good–tax space. A's share of the cost of the public good runs from 0, at the bottom of the vertical scale, to 1.0 at the top. B's tax share runs in the opposite direction. Thus, each point in Figure 4.3 represents a set of tax shares sufficient to cover the full cost of the quantity of public good at that point. Each point is on an indifference curve for A, and one for B. Embedded in each point is a quantity of private goods that each individual consumes as implied by his budget constraint (4.1), the quantity of the public good, and his tax share. A_1 and B_1 are the levels of utility, respectively, if each individual acted alone in purchasing the public good, and thus bore 100 percent of its cost.[1] Lower curves for A (higher for B) represent higher utilities. The set of tangency points between A and B's indifference curves, CC', represents a contract curve mapping the Pareto-possibility frontier into the public good–tax share space.

1. To simplify the discussion, we ignore spillovers from one individual's unilateral provision of the public good on the other's utility. One might think of the public good as a bridge across a stream. A_1 and B_1 represent the utilities that each individual can obtain if each builds his own bridge. Within A_1 and B_1 are points of higher utility for both that can be obtained by cooperating and building but one bridge.

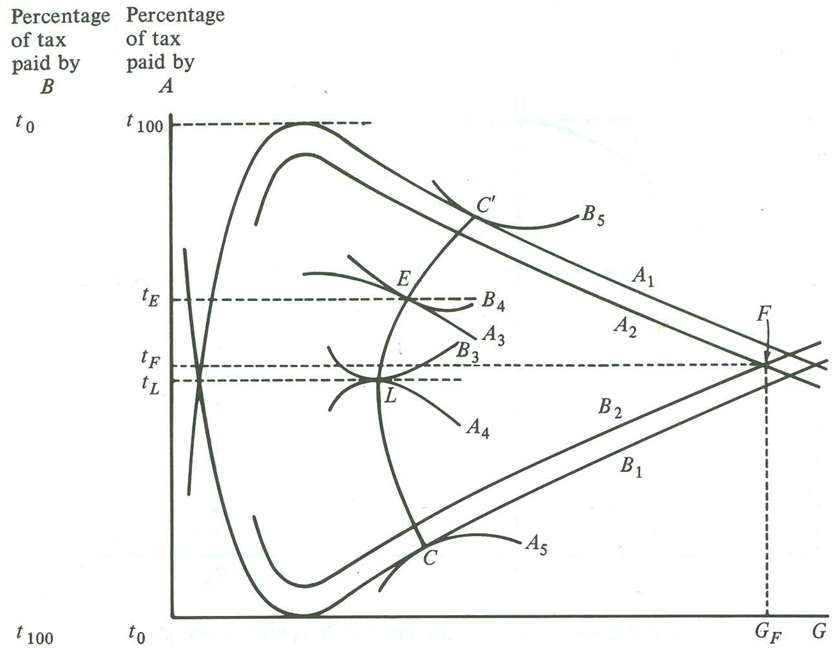

Figure 4.3. Contract curve in public good–tax space.

To see that each point on CC' is a Pareto-efficient allocation, take the total differentials of each individual's utility function with respect to t and G, holding the initial incomes (Y_A, Y_B) constant:

$$\begin{aligned} \Delta U_A &= \frac{\partial U_A}{\partial X}(-t)\mathrm{d}G + \frac{\partial U_A}{\partial G}\,\mathrm{d}G + \frac{\partial U_A}{\partial X}(-G)\mathrm{d}t \\ \Delta U_B &= \frac{\partial U_B}{\partial X}(-1+t)\mathrm{d}G + \frac{\partial U_B}{\partial G}\,\mathrm{d}G + \frac{\partial U_B}{\partial X}(G)\mathrm{d}t. \end{aligned} \tag{4.3}$$

Setting the total change in utility for each individual equal to zero, we can solve for the slope of each individual's indifference curve:

$$\begin{aligned} \left(\frac{\mathrm{d}t}{\mathrm{d}G}\right)^A &= \frac{\partial U_A/\partial G - t\partial U_A/\partial X}{\mathrm{G}(\partial U_A/\partial X)} \\ \left(\frac{\mathrm{d}t}{\mathrm{d}G}\right)^B &= -\frac{\partial U_B/\partial G - (1-t)(\partial U_B/\partial X)}{\mathrm{G}(\partial U_B/\partial X)}. \end{aligned} \tag{4.4}$$

Equating the slopes of the two indifference curves, we obtain the Samuelsonian condition for Pareto efficiency (1954):

$$\frac{\partial U_A/\partial G}{\partial U_A/\partial X} + \frac{\partial U_B/\partial G}{\partial U_B/\partial X} = 1. \tag{4.5}$$

Now consider the following public choice process. An impartial observer proposes both a pair of tax shares, t_F and $(1 - t_F)$, and a quantity of the public good, G_F. If the combination falls within the eye formed by A_1 and B_1, both individuals prefer this proposal to share the cost of the public good to having to provide all of the public good themselves. Both will vote for it, if they vote sincerely. F now becomes the status quo decision and new tax share–quantity pairs are proposed.[2] When a combination falling within the eye formed by A_2 and B_2 is hit upon, it is unanimously preferred to F. It now becomes the status quo and the process is continued until a point on CC', like E, is obtained. Once this occurs, no new proposal will be unanimously preferred, that is, can make both individuals better off, and the social choice has been, unanimously, made.

Note that for the tax shares inherent in the allocation E, each individual's optimal quantity of public good differs from the quantity of the public good selected. A prefers less of the public good, B prefers more. Given the tax shares t_E and $(1 - t_E)$, therefore, each is being "coerced" into consuming a quantity of the public good that differs from his most preferred quantity (Breton, 1974, pp. 56–66). This form of coercion can be avoided under a slightly different variant of the voting procedure (Escarraz, 1967; Slutsky, 1979). Suppose, for an initially chosen set of tax shares t and $(1 - t)$, that voters must compare all pairs of public good quantities, and a given quantity is chosen only if it is unanimously preferred to all others. This will occur only if the two individuals' indifference curves are tangent to the tax line from t at the same point. If no such quantity of public good is found for this initially chosen t, a new t is chosen, and the process repeated. This continues until a t is found at which all individuals vote for the same quantity of public good against all others. In Figure 4.3, this occurs at L for tax shares t_L and $(1 - t_L)$. L is the Lindahl equilibrium.

The outcomes of the two voting procedures just described (E and L) differ in several respects.[3] At L, the marginal rate of substitution of public for private goods for each individual is equal to his tax price:

$$\frac{\partial U_A/\partial G}{\partial U_A/\partial X} = t \qquad \frac{\partial U_B/\partial G}{\partial U_B/\partial X} = (1 - t). \tag{4.6}$$

2. Of course, the rule for selecting a new tax share or a new public good–tax share combination in the procedure described above must be carefully specified to ensure convergence to the Pareto frontier. For specifics on the characteristics of these rules, the reader is referred to the literature on Walrasian-type processes for revealing preferences on public goods as reviewed by Tulkens (1978).
3. For a detailed discussion of these differences, see Slutsky (1979).

L is an equilibrium then, in that *all* individuals prefer this quantity of public good to any other, *given each individual's assigned tax price*. E (or any other point reached via the first procedure) is an equilibrium in that at least one individual, and perhaps only one, is worse off by a movement in any direction from this point. Thus, L is preserved as the collective decision through the unanimous *agreement* of all committee members on the quantity of public good to be consumed, *at the given tax prices;* E is preserved via the *veto* power of each individual under the unanimity rule. How compelling these differences are depends upon the merits of constraining one's search for the optimum public good quantity to a given set of tax shares (search along a given horizontal line in Figure 4.3). The distribution of utilities at L arrived at under the second process depends only on the initial endowments and individual preferences, and has the (possible) advantage of being independent of the sequence of tax shares proposed, assuming L is unique. The outcome under the first procedure is dependent on the initial endowments, individual utility functions, *and* the specific set and sequence of proposed tax–public good combinations. Although this "path dependence" of the first procedure might be thought undesirable, it has the (possible) advantage of leaving the entire contract curve CC' open to selection. As demonstrated above, all points along CC' are Pareto efficient, and thus cannot be compared without additional criteria. It should be noted in this regard that if a point on CC', say E, could be selected as most preferred under some set of normative criteria, it could always be reached via the second voting procedure by first redistributing the initial endowments in such a way that L was obtained at the utility levels implied by E (McGuire and Aaron, 1969). However, the informational requirements for such a task are obviously considerable.

We have sketched here only two possible *voting* procedures for reaching the Pareto frontier. Several papers have described Walrasian/tâtonnement procedures for reaching the Pareto frontier when public goods are present. These procedures all have in common the existence of a "central planner" or "auctioneer" who gathers information of a certain type from the citizen-voter, processes the information by a given rule, and then passes a message back to the voters to begin a new round of voting. These procedures can be broadly grouped into those in which the planner calls out tax prices (the *ts* in the above example), and the citizens respond with quantity information, the process originally described by Erik Lindahl (1919) (see also Malinvaud, 1970–1, sec. 5); and those in which the planner-auctioneer calls out quantities of public goods and the citizens respond with price (marginal rate of substitution) information, as in Malinvaud (1970–1, secs. 3 and 4), and Drèze and de la Vallée Poussin (1971). A crucial part of all of these procedures is the computational rule used to aggregate the messages provided by voters and generate a new set of signals from the planner-auctioneer. It is this rule that

determines if, and when, and where on the Pareto frontier the process leads. Although there are obviously distributional implications to these rules, they are in general not designed to achieve any specific normative goal. The central planner-auctioneer's single end is to achieve a Pareto-efficient allocation of resources. These procedures are all subject to the same important distinction as to whether they allow the entire Pareto frontier to be reached or always lead to an outcome with a given set of conditions, like the Lindahl equilibrium. As such, they also share the other general properties of the unanimity rule. One particularly attractive variant on these procedures is discussed in the next section.

B.* Vernon Smith's auction mechanism

Whereas most of the tâtonnement-type procedures ask voters to state *either* a willingness-to-pay tax price or a desired quantity, Smith's (1977, 1979a,b, 1980) mechanism requires the voter to announce *both*. Each individual i announces both a bid, b_i, which is the share of the public good's cost that i is willing to cover and a proposed quantity of the public good, G_i. The tax price actually charged i is the difference between the public good's costs, c, and the aggregate bids of the other $n - 1$ voters, B_i, that is,

$$t_i G = (c - B_i)G, \tag{4.7}$$

where $B_i = \Sigma_{j \neq i} b_j$, and $G = \Sigma_{k=1}^{n} G_k/n$. The procedure selects a quantity of public good only when each voter's bid matches his tax price and each voter's proposed public good quantity equals the mean:

$$b_i = t_i \text{ and } G_i = G, \text{ for all } i. \tag{4.8}$$

After each iteration of the procedure, voters are told what their tax prices and the public good quantity would have been had (4.8) been achieved at that iteration. If a voter's bid fell short of his tax price he can adjust either his bid or proposed public good quantity to try to bring about an equilibrium. Only when all unanimously agree to both their tax prices and the public good quantity does the procedure stop.

At an equilibrium (4.8) is satisfied, and i's utility can be written as

$$V_i = U_i(G) - t_i G, \tag{4.9}$$

where the utility from consuming G is expressed in money units. Maximizing (4.9) with respect to G_i we obtain the condition for i's optimal proposed quantity for the public good

$$dV_i/dG_i = U_i'/n - t_i/n = 0$$
$$U_i' = t_i. \tag{4.10}$$