EE320 Exercise 3

Semester 2, 2014

Chapter 5: Nonlinear Model and Differential Calculus in Economic Theory

1. Determine $\frac{dy}{dx}$ of the following function.

a) $y = 3x^2 - x^{\frac{1}{3}} + 2$

b) $y = (x^3 - 3x)^4$

c) $y = \frac{x}{x^2+1}$

d) $y = \frac{1}{(16-x^2)^{\frac{1}{2}}}$

e) $y = ln(\frac{1-4x}{1+4x})$

f) $y = x^{x^3}$

g) $y = 25^{3x^3-6}$

h) $y = x^2 lnx$

i) Are $\frac{\Delta y}{\Delta x}$ and $\frac{dy}{dx}$ similar or different ? How ? Explain.

2. Let $Q = AP^{-\alpha}$ where Q = Quantity, P= Price , A and α are positive coefficients.
a) Determine derivative of Q with respect to P.
b) Prove that price elasticity of demand is not depend on Q and P.

3. Let demand for A : $Q_A = 40\frac{P_B}{P_A}$ and demand for B : $Q_B = 10\frac{P_A}{P_B}$.
a) What is the relative price of goods A and B such that quantity demanded of A and B are equal? $[\frac{P_A}{P_B} = 2]$
b) Determine cross-price elasticity of these two products, interpret them and state how they relate. $[\varepsilon_{AB} = 1, \varepsilon_{BA} = 1]$

4. Let the demand function be $P = aQ^2 + 2abQ + ab^2$; $a>0, 0<b<1$ and $0 \leq Q< -b$. Show that $MR = P[1 + \frac{1}{|\varepsilon_D|}]$

5. Prove that price elasticity of demand of $PQ = k$ equals to -1 and interpret the meaning.

6. Let the production function be $Q = 3(7 - L)^5 + 18$, determine the level of labour such that we can observe law of diminishing return. [$L>7$]

7. A producer hire m people which can produce Q units of goods per day where $Q = \frac{10m^2}{\sqrt{m^2+19}}$. Suppose demand for this goods be $P = \frac{900}{Q+9}$, determine marginal revenue product ($MRP = \frac{dTR}{dm}$) when m = 9 and interpret the meaning. [MPR=10.71]

8. Let $C = \frac{5(2\sqrt{Y^3}+3)}{Y+10}$ where C = consumption and Y = income, determine MPC and MPS when Y =100. [MPC=0.54, MPS=0.46]

9. Let $C = 570 - \frac{160000}{Y+200}$, determine C, MPC and MPS when Y = 300, 600 and 800 million Baht respectively.

10. Let $C = a + bY$ be the consumption function.
a) Determine elasticity of consumption(C) with respect to income(Y).
b) Prove that elasticity in a) does not depend on C and Y when a = 0.