Time Series

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Part 3: Cointegration and Error Correction Models

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Cointegration

- If $\{y_t\}$ and $\{x_t\}$ are two $I(1)$ processes. It is possible that for some $\beta \neq 0$ from $y_t - \beta x_t$ is an $I(0)$ process.
- If such a β exist, we say that y and x are cointegrated, and we call β the cointegrated parameter.
- If y and x are not cointegrated, a regression of y on x is spurious, or we say, there is no long-run relationship between y and x.

Testing for Cointegration

- If y and x are cointegrated, the OLS estimator $\hat{\beta}$ from the regression $y_t = \hat{\alpha} + \hat{\beta} x_t$ is consistent for β.
- H_0: the two series are not cointegrated $\rightarrow$ under H_0, we are running a spurious regression
- Then, apply the DF or augmented DF test to the residuals: $\hat{u}_t = y_t - \hat{\alpha} - \hat{\beta} x_t$, but using new critical values, called the Engle-Granger test.

Asymptotic Critical Values for Cointegration Test: No Time Trend

Significance level	1%	2.5%	5%	10%
Critical value	−3.90	−3.59	−3.34	−3.04

- If $t_{\hat{\theta}} < c$, we have evidence that $y_t - \beta x_t$ is $I(0)$ for some β; that is, y and x are cointegrated.

Cointegration with drift

- If y and x contain drift terms, $E(y_t)$ and $E(x_t)$ are linear (increasing) functions of time.
- The strict definition of cointegration requires $y_t - \beta x_t$ to be $I(0)$ without a trend.
- With drift we have $y_t = \delta t + g_t$ and $x_t = \lambda t + h_t$, where $\{g_t\}$ and $\{h_t\}$ are $I(1)$ processes.
- If y and x are cointegrated, there must exist β such that $g_t - \beta h_t$ is $I(0)$.
- But, $y_t - \beta x_t = (\delta - \beta\lambda)t + (g_t - \beta h_t)$ is a trend-stationary process.
- For strict form of cointegration, we need $\delta = \beta\lambda$.

Testing cointegration with drift

- We can test for cointegration between g_t and h_t without taking a stand on the trend part, by running the regression $\hat{y}_t = \hat{\alpha} + \hat{\eta} t + \hat{\beta} x_t$
- Apply the DF or augmented DF test to the residuals $\hat{u}_t$, using the following criticle values

Asymptotic Critical Values for Cointegration Test: Linear Time Trend

Significance level	1%	2.5%	5%	10%
Critical value	−4.32	−4.03	−3.78	−3.50

- This leaves the possibility that $y_t - \beta x_t$ has a linear trend, but at least it is not $I(1)$.

Error correction models

- If y_t and x_t are $I(1)$ processes and are not cointegrated, we might estimate a dynamic model in first differences.
- Consider the equation
  $\triangle y_t = \alpha_0 + \alpha_1 \triangle y_{t-1} + \gamma_0 \triangle x_t + \gamma_1 \triangle x_{t-1} + u_t$
  where $E(u_t | \triangle x_t, \triangle y_{t-1}, \triangle x_{t-1}, lags) = 0$
- Let $s_t = y_t - \beta x_t$. If y and x are cointegrated, s_t is $I(0)$
- Assume for simplicity that s_t has zero mean so that we can include lags of s_t in the equation.
- $\triangle y_t = \alpha_0 + \alpha_1 \triangle y_{t-1} + \gamma_0 \triangle x_{t-1} + \delta s_{t-1} + u_t$, then substitute $s_{t-1} = y_{t-1} - \beta x_{t-1}$.
- $\delta(y_{t-1} - \beta x_{t-1})$ is called the error correction term

Error correction models

- An error correction model explain the short-run dynamics in the relationship between y and x.
- Simple ECM (no lags of changes):
  $\triangle y_t = \alpha_0 + \gamma_0 \triangle x_t + \delta(y_{t-1} - \beta x_{t-1}) + u_t$, $\delta < 0$
- If $y_{t-1} > \beta x_{t-1}$, then y in the previous period has overshot the equilibrium. s_{t-1} will push y down.
- If $y_{t-1} < \beta x_{t-1}$, the error correction term induces a positive change in y back toward the equilibrium.
- If β is known, then simply regress $\triangle y_t$ on $\triangle x_t$ and s_{t-1}. Otherwise, estimate $\hat{s}_{t-1} = y_{t-1} - \hat{\beta} x_{t-1}$.