Practice Final Solutions
FN312

<u>Question 1 (30 points)</u>

Your employer offers two funds for your pension plan, a money market fund and an S&P 500 index fund. The money market fund holds 3-month Treasury bills, which currently offer a 3% safe return per year. The S&P 500 index fund offers an expected return of 10% per year with a standard deviation of 20%.

(a) (5 points) Consider portfolio A, where your level of risk aversion corresponds to a requirement of an expected return of 8% per year. What should be the composition of your portfolio? What is the standard deviation of its returns? What is your level of risk aversion?

Linearity of expected returns requires: w × 3%+(1−w) ×10% = 8%. Solving for w yields 28.6% in the risk free asset and 71.4% in the S&P. Sigma is given by 0.714 × 20% = 14.28%.

(b) (5 points) What is the Sharpe ratio of Portfolio A? Describe intuitively what the Sharpe ratio illustrates. How does it relate to the capital allocation line?

Sharpe ratio = E(rp)-rf/sigma = 8-3/14.28 which illustrates the reward to risk ratio. It is the slope of the CAL which measures the excess return being earned per unit of volatility

(c) (5 points) Illustrate Portfolio A on a graph where you also plot the risk-free asset, the S&P 500 index fund, the capital allocation line and the position of your indifference curve. Are you relatively risk averse? Explain.

(d) (5 points) Now your employer adds an emerging-market fund to the two existing funds. The emerging-market fund offers an expected return of 10% per year, the same as the S&P 500 index fund, but with a standard deviation of 30%, higher than the S&P 500 index fund. Would you consider including the emerging-market fund as part of your portfolio? Explain.

Even though the Emerging Markets fund has the same return and a higher standard deviation, its correlation with the S&P may lead to achieve lower levels of risk for the portfolio as a whole. If the correlation is sufficiently low, we may be able to diversify away some of the risk. We should certainly consider the Emerging Fund.

(e) (5 points) The correlation between the S&P 500 index fund and the emerging market fund is 0. Consider portfolio B, which consists of 80% in the S&P index fund and 20% in the emerging-market fund. Calculate portfolio B's expected return and standard deviation. If you mix portfolio B with the money-market fund to achieve an expected return of 8%, is it better than the Portfolio A?

.8*10 + 0.2*10 = 10%
sqrt(0.8*0.8*.2*.2 + .2*.2*.3*.3) = 17.09%

The weights will be the same as in part a, i.e., 28.6% in the risk free asset and 71.4% in Portfolio A. The standard deviation of the new portfolio will be $0.7143 \times 17.09\% = 12.2\%$, which is lower than for the portfolio in part a. We are clearly better off: now we can achieve the same return with a lower level of risk.

(f) (5 points) Does portfolio B correspond to the minimum global variance portfolio? If not, solve for the minimum global variance portfolio and show your working.

No, the minimum variance portfolio is when w =sig^2B/(sig^2a+sig^2b = 0.6923

Question 2 (15 points)

In 1997 the rate of return on short-term government securities (perceived to be risk-free) was about 5%. Suppose the expected rate of return required by the market for a portfolio with a beta measure of 1 is 12%. According to the capital asset pricing model:

(a) (5 points) Based on the information given, can you find that expected rate of return on the tangency portfolio? What about the expected rate of return on a stock with $\beta = 0$? If a stock has $\beta = 0$ does this mean that it has no risk?

Since the market/tangency portfolio by definition has a beta of 1, its expected rate of return is 12%.
$\beta = 0$ means no systematic risk. Hence, the portfolio's fair return is the risk-free rate, 5%.

(b) (5 points) Suppose you consider buying a share of stock at \$40. The stock is expected to pay \$3 dividends next year and you expect it to sell then for \$41. The stock risk has been evaluated by $\beta = -.5$. Is the stock overpriced or underpriced?

Using the SML, the fair rate of return of a stock with $\beta = -0.5$ is: $E(r) = 5 + (-.5)(12 - 5) = 1.5\%$.
The expected rate of return, using the expected price and dividend of next year: $E(r) = 44/40 - 1 = .10$, or 10%. Because the expected return exceeds the fair return, the stock must be underpriced.

(c) (5 points) Your friend tells you that the stock is mispriced because of the size effect. Does this imply that this stock is a large stock or a small stock? According to this explanation, why is the stock mispriced? What extensions can you make to CAPM to correct the mispricing in the stock? Write out the new model.

Beta is inversely related to size so small stocks tend to have higher expected returns than the CAPM prediction. This is because small stocks are more illiquid and may thus command a higher premium. Can add size as a factor to the factor model.

Question 3 (25 points)

The current yield curve for default-free zero coupon bonds are as follows:

Maturity (years)	YTM	Price	Forward Rate
1	10%		
2	11%		
3	12%		

(a) (6 points) Complete the third and fourth column of the above table and show your working.

1 10% $909.09 N-A (short rate = 10%)
2 11% $811.62 12.01% [(1.112 /1.10) –1]
3 12% $711.78 14.03% [(1.123 /1.112) –1]

(b) (7 points) Plot the term structure of interest rates. If the expectation hypothesis holds, what does the information in the term structure say about the course of future interest rates? What will the price and yields to maturity on the one and two-year zero coupon bonds be next year?

If the expectations hypothesis holds, then forward rates = expected future short rates.
Note that this year's upward sloping yield curve implies, according to the expectations hypothesis, a shift upward in next year's curve.

1 $892.78 [= 1000/1.1201] 12.01%
2 $782.93 [= 1000/(1.1201 x 1.1403)] 13.02%

(c) (7 points) What should be the current price of a three-year maturity bond with a 12% coupon rate paid annually? If you purchased it at that price, what would your total expected rate of return be over the next year (coupon plus price change)?

Current price = $120 x (0.90909) + $120 x (0.81162) +$1,120 x (0.71178) = $109.09 + $97.39 + $797.19 = $1,003.68
Similarly, the expected prices of zeros in 1 year can be used to calculate the expected bond value at that time: · Expected price 1 year from now = $120 x (0.89278) + $1,120 x (0.78293) = $107.1336 + $876.8816 = $984.02 · Total expected rate of return = [$120 + ($984.02 - $1,003.68)]/$1,003.68] = ($120 - $19.64)/$1,003.68 = 10%

(d) (5 points) What is the liquidity preference theory? Briefly explain

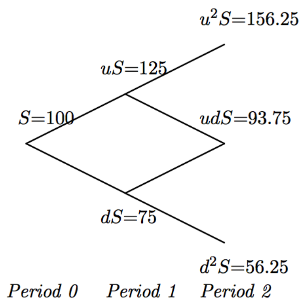
See notes.

Question 4 (25 points)

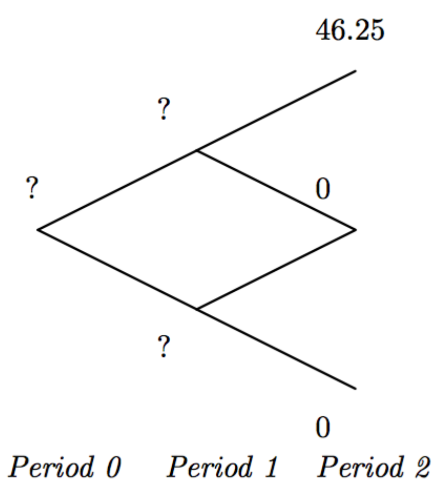
401.com's stock is trading at $100 per share. The stock price will either go up or go down by 25% in each of the next two years. The annual interest rate is 5%.

a) (5 points) Graph the tree diagram showing the current stock price of 401.com and future possible values of the stock price.

28. Parameters: $u = 1.25$, $d = 0.75$. Stock tree:



b) (10 points) Determine the price of a two-year European call option with the strike price X = $110.



First compute RNP: $q = \frac{1+0.05-0.75}{1.25-0.75} = 0.6$. Then use the risk-neutral valuation formula to compute $C_u = 26.43$, $C_d = 0$ and $C = 15.10$.

c) (5 points) If the annual interest rate increases to 7%, explain, without redoing your calculations, whether the price of the two-year European call option will increase or decrease. Explain your intuition.

If interest rates increase, the present value of the strike price would be lower and therefore the value of the call option would increase.

d) (5 points) What is the price of a two–year European put option with the strike price X = $110.

Using the put-call parity: 15.10-P = 100-110/(1+r)^2 ➔ Solve for P.

Question 5 (20 points)

Joseph Jones, a manager at Computer Science, Inc. (CSI), received 1,000 shares of company stock as part of his compensation package. The stock currently sells at $40 at share. Joseph would like to defer selling the stock until the next tax year. In January, however, he will need to sell all his holdings to provide for a down payment on his new house. Joseph is worried about the price risk involved in keeping his shares. At current prices, he would receive $40,000 for the stock. If the value of his stock holdings falls below $35,000, his ability to come up with the necessary down payment would be jeopardized. On the other hand, if the stock value rises to $45,000, he would be able to maintain a small cash reserve even after making the down payment. Joseph considers three investment strategies:

(a) Strategy A is to write January call options on the CSI shares with strike price $45. These calls are currently selling for $3 each.

(b) Strategy B is to buy January put options on CSI with strike price $35. These options also sell for $3 each.

(c) Strategy C is to establish a zero-cost collar by writing the January calls and buying the January puts.

Evaluate each of these strategies with respect to Joseph's investment goals by calculating out the profits of each strategy. What are the advantages and disadvantages of each? Which would you recommend?

(a) By writing call options (these are called covered calls given the long position in the stock), Jones takes in premium income of $3,000. If the price of the stock in January is less than or equal to $45, he will have his stock plus the premium income. But the most he can have is $45,000 + $3,000 because the stock will be called away from him if its price exceeds $45. (We are ignoring interest earned on the premium income from writing the options in this very short period of time.) The payoff as a function of the stock price in January, ST , is

Stock price Portfolio value
ST ≤ $45 (1, 000)ST + 3, 000
ST > $45 45, 000 + 3, 000 = $48, 000

This strategy offers some extra premium income (by selling the upside) but leaves substantial downside risk. At an extreme, if the stock price fell to zero, Jones would be left with only $3,000. The strategy also puts a cap on the final value at $48,000, but this is more than sufficient to purchase the house

(b) By buying put options with a \$35 strike price, Jones will be paying \$3,000 in premiums to insure a minimium level for the final value of his portfolio. That minimium value is (35)(1, 000) − 3, 000 = \$32, 000. This strategy allows for upside gain, but exposes Jones to the possibility of a moderate loss equal to the cost of the puts. The payoff structure is

Stock price	Portfolio value
$S_T \le \$35$	35, 000 − 3, 000 = \$32, 000
$S_T > \$35$	$(1,000)S_T - 3,000$

(c) The cost of the collar is zero. The value of the portfolio in January will be as follows:

Stock price	Portfolio value
$S_T \le \$35$	\$35, 000
$\$35 < S_T < \$45$	$(1,000)S_T$
$S_T \ge \$45$	\$45,000

If the stock price is less than or equal to \$35, the collar preserves the \$35,000 in principal. If the stock price exceeds \$45, the value of the portfolio can rise to a cap of \$45,000. In between, the proceeds equal 1,000 times the stock price Given the objective of Jones, the best strategy would be (c) since it satisfies the two requirements of preserving the \$35,000 in principal while offering a chance of getting \$45,000. Strategy (a) should be ruled out since it leaves Jones exposed to the risk of substantial loss of principal.

<u>Question 6 (20 points)</u>

Two futures contracts are traded on a financial asset: a three-month contract with price of \$120 and a six-month contract with price of \$122. There are no dividends on the financial asset and assume that the risk free rate is fixed for the six month period.

(a) (7 points) What is the spot price of the underlying asset at time t?

$F(t, t + 3) = \$120 = S(t) * (1 + r(3)) = S(t) * (1 + r(1))^3$, (2) Notice $(1 + r(3)) = (1 + r(1))^3$ as the one month rate will stay constant $F(t, t + 6) = \$122 = S(t) * (1 + r(6)) = S(t) * (1 + r(1))^6$. (3) Dividing equation (2) by equation (1) leads to the following expression $\frac{\$122}{\$120} = \frac{S(t) * (1 + r(1))^6}{S(t) * (1 + r(1))^3} = (1 + r(1))^3$, which can be solved to yield $r(1) = \left(\frac{\$122}{\$120}\right)^{1/3} - 1 = 0.5525\%$. We can now use either equation (1) or (2) to find S(t) $\$120 = S(t) * (1 + r(1))^3 \Rightarrow S(t) = \$118.033$

(b) (7 points) Suppose that the three-month futures contract is now trading at a price of \$119.5. Does this imply an arbitrage opportunity? How would you take advantage of this opportunity?

$S(t) = F(t, t + 1)/(1 + r(1))^3 = \frac{\$119.5}{(1 + 0.5525\%)^3} = \$117.541$. This value is smaller than \$118.033 found in part (a), thus implying the arbitrage opportunity. The following strategy exploits this arbitrage opportunity:

Transaction	Payoff at t	Payoff at t + 1
1. Long futures	0	\$S(t + 1) − 119.5 2.
2. Long asset	\$118.033	-S(t + 1)
3. Invest in Risk-free	-\$117.541	\$119.5
4. Net:	0.492	0

(c) (6 points) Suppose now the financial asset pays a dividend yield of 1%. Without redoing the calculations, do you think that six-month contract will still have a higher price than the one-month contract? Explain your reasoning.

Since d>r, the three month contract would have a higher price. See notes for explanation.

Question 7 (15 points)

Describe the differences as well as list the advantages and disadvantages of using the binomial tree versus Black Scholes approach to price options. What are some of the assumptions underlying these two pricing methods that we discussed in class that are not so realistic?

The binomial tree method for option pricing is computationally intensive while the Black Scholes is easy to compute.

However, the tree method is flexible and can be used to price not only European options but also American and exotic options while the Black Scholes method is limited to European options.

Some unrealistic assumptions include constant standard deviation, constant interest rate as these may vary over time in the real world.

The Black Scholes formula also assumes a lognormal distribution for stock returns which rules out large changes or jumps in stock prices which may happen.