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EE325 Introductory Econometrics (Section 1 semester 1/2020)

Assignment 4

Instruction: Write your answer in either paper or digital paper. However, if you write on paper, please scan it and save as a PDF file. Submission is via BE-Moodle as a PDF file for both cases. (Please keep the file below 10MB as that is the maximum per file capacity for student.)

Due date: Friday, November 6,2020 (Before 10 P.M.)

1. From the data for 46 states in the United States for 1992, results of the regression are displayed as follows.

consumption $ income

$$\begin{aligned} \ln C_i &= 4.30 - 1.34 \ln P_i + 0.17 \ln Y_i \\ se &= (0.91)\ (0.32)\ \ \ \ (0.20) \\ \bar{R}^2 &= 0.27 \end{aligned}$$

where C_i = cigarette consumption, packs per year

P_i = real price per pack, $ per pack

Y_i = real disposable income per capita, $ per week

1.1) Do the estimation results follow the law of demand?

Yes. From the equation, the consumption of cigarette and real price are related negatively, while the consumption and real disposable income are related positively. This estimation results follow the law of demand as when price goes up, people demand less, and when people have more income, they can consume more, resulting in the increase of demand.

1.2) What is the elasticity of demand for cigarettes with respect to price? Is it statistically significant? If so, is it statistically different from 1?

1) $H_0: \beta_2 = 1$

$H_1: \beta_2 \neq 1$

2) $t_{cal}(\beta_2) = \dfrac{\hat{\beta}_2 - \beta_2}{\sigma_{\hat{\beta}_2}} \sim t_{n-3}$

$= \dfrac{-1.34 - 1}{0.32} = -7.3125$

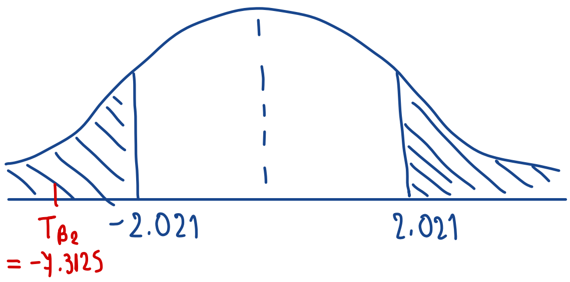
3) $\alpha = 0.05$; $t_{0.025,43} = 2.021$

$n = 46$

$n-k = 43$

$t_{upper} = 2.021$

$t_{lower} = -2.021$



$\therefore t_{\beta_2}$ lies beyond the boundaries, so we reject H_0. It is different from 1 at 95% confidence interval.

1.3) What is the income elasticity of demand for cigarettes? Is it statistically significant? If not, what might be the reasons for it?

1) $H_0 : \beta_3 = 0$

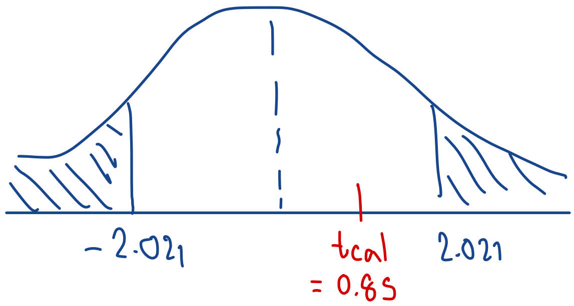
$H_1 : \beta_3 \neq 0$

2) $t_{cal}(\beta_3) = \dfrac{\hat{\beta}_3 - \beta_3}{\sigma_{\hat{\beta}_3}} = \dfrac{0.17 - 0}{0.2} = 0.85$

3) $\alpha = 0.05$; $t_{0.0025,43} = 2.021$

$n = 46$

$n - k = 43$ $\quad t_{lower} = -2.021$



$\therefore t_{\beta_3}$ lies within the boundaries, so we cannot reject H_0.

We cannot conclude that β_3 is significant at 95% confidence interval.

2. From estimating the regression equation on net financial wealth (nettfa), age of the survey respondent (age), and annual family income (inc) for people in the United States. The wealth and income variables are both recorded in thousands of dollars. The OLS estimation results for the model are given by

$$nettfa_i = \beta_1 + \beta_2 inc_i + \beta_3 age_i + u_i$$

```
reg nettfa inc age
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Source	SS	df	MS
Model	6414618.8	2	3207309.4
Residual	31528770.7	9,272	3400.42825
Total	37943389.5	9,274	4091.3726

Number of obs	=	9,275
F(2, 9272)	=	943.21
Prob > F	=	0.0000
R-squared	=	0.1691
Adj R-squared	=	0.1689
Root MSE	=	58.313

nettfa	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
inc	.9533566	.0252775	37.72	0.000	.9038072	1.002906
age	1.030777	.0591226	17.43	0.000	.9148838	1.14667
_cons	-60.69654	2.596333	-23.38	0.000	-65.78592	-55.60715

```
reg nettfa inc age agesq
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Source	SS	df	MS
Model	6567017.15	3	2189005.72
Residual	31376372.3	9,271	3384.35685
Total	37943389.5	9,274	4091.3726

Number of obs	=	9,275
F(3, 9271)	=	646.80
Prob > F	=	0.0000
R-squared	=	0.1731
Adj R-squared	=	0.1728
Root MSE	=	58.175

nettfa	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
inc	.9782522	.0254891	38.38	0.000	.928288	1.028216
age	-2.231489	.4897118	-4.56	0.000	-3.191432	-1.271547
agesq	.0377221	.0056214	6.71	0.000	.026703	.0487413
_cons	4.680388	10.08099	0.46	0.642	-15.08056	24.44134

2.1) Test the coefficient, in the first model, $\beta_3 < 1$ in the first model or not?

$nettfa_i = \beta_1 + \beta_2 inc_i + \beta_3 age_i + u_i$

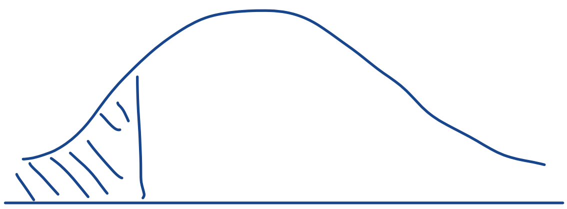
$\widehat{nettfa_i} = -60.696 + 0.9533\, inc_i + 1.3077\, age_i$

se $\quad (2.596) \quad (0.0252) \quad (0.0591)$

1) $H_0: \beta_3 \geq 1$

$H_1: \beta_3 < 1$

$\alpha = 0.05$



2) $t_{cal} = \dfrac{\hat{\beta}_3 - \beta}{se(\hat{\beta}_3)} = \dfrac{1.0307 - 1}{0.0591} = 0.520512$

$df = n - k = 9275 - 3 = 9272$

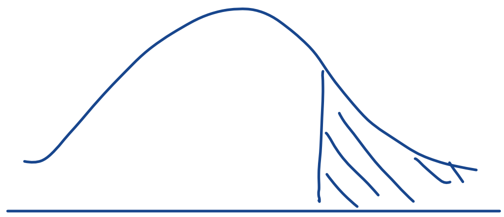
2.2) Due to estimation result by adding the age^2 variable or agesq. Perform the test whether we should include the quadratic term of the age variable or not? (Test for both t-test and F-test.) Also, interpret the meaning of this coefficient.

$$F_{cal} = \frac{RSS_R - RSS_{UR} / m}{RSS_{UR} / n-k}$$

$$= \frac{R^2_{UR} - R^2_R / m}{(1 - R^2_{UR} / n-k}$$

$$= \frac{(3.528\,770.7 - 3137\,672.3)/1}{313763.3/9,275-4}$$

$$= 45.03024$$



$f_{0.05,(1,9271)} = 3.84$

$\therefore$ reject H_0 if $f_{cal} > 3.84$

find that $45.03 > 3.82$ $\therefore$ reject H_0

$\therefore$ There is enough evidence to say that the addition of age^2 to the model has sig. at $\alpha = 0.05$

3. You are conducting an empirical investigation into the median prices of houses in 506 communities of a large metropolitan area. The sample data consist of 506 observations on the following observable variables:

P_i: the median house price in community i, in dollars;

NOX_i: the level of nitrous oxide in the air of community i, in parts per 100 million;

$DIST_i$: the weighted distance of community i from municipal area, in miles;

$ROOM_i$: the average number of rooms per house in community i;

$STRAT_i$: the average student-teacher ratio of schools in community i.

Researcher estimates the following model of median house price. The OLS estimation results for the model are given by

$$
\begin{aligned}
ln(P_i) &= 11.08 - 0.9535\,ln(NOX_i) - 0.1343\,ln(DIST_i) + 0.2545ROOM_i - 0.05245STRAT_i \\
se &= (0.3181)\ (0.1167) \qquad (0.04310) \qquad (0.01853) \qquad (0.005897) \\
RSS &= 35.1835 \quad TSS = 84.5822
\end{aligned}
$$

3.1) Interpret each of the coefficient estimates in regression equation.

$\hat{\beta}_1 = 11.06$: If $\ln(NOX) = 0$, $\ln(DIST_i) = 0$, $ROOM_i = 0$, and $STRAT_i = 0$ on average the median house price would be equal to $\$e^{11.08} = 64,860.883418$

$\hat{\beta}_2 = -0.9535$; as the level of nitrous oxide increase by 1% on average, the median house price falls by $\sim 0.9535\%$, holding another variable constant.

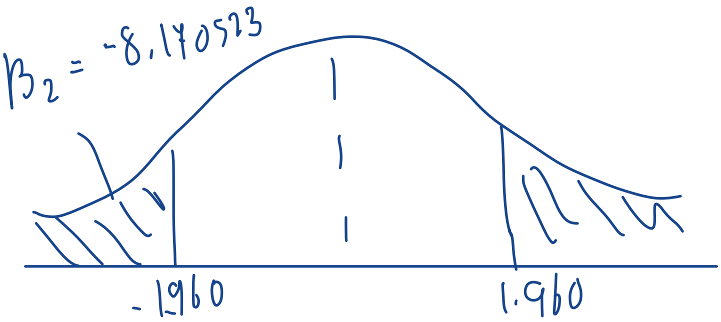
$\hat{\beta}_3 = -0.1343$, as the weighted distance from municipal area increase by 1% on average, the median house price falls by $\sim 0.1343\%$, holding other variable constant.

$\hat{\beta}_4 = 0.2545$, as the average number of rooms per house increases by 1 room per house on average, the median house price rises by $\sim 0.2545 \times 100 = 25.45\%$, holding other variable constant.

$\hat{\beta}_5 = -0.05245$, as the average student-teacher ratio of school rises by 1 student per teacher, on average the median house price decreases by $\sim 0.05245 \times 100 = 5.245\%$, holding other variable constant.

3.2) Test the individual significance of each of the slope coefficient estimates for $ln(NOX_i)$ and $ROOM_i$.

$ln(NOX_i)$



1) $H_0 : \beta_2 = 0$

$H_a : \beta_2 \neq 0$

2) $t_{cal}(\beta_2) = \dfrac{\hat{\beta}_2 - \beta_2}{\sigma_{\hat{\beta}_2}}$

$= \dfrac{-0.9535 - 0}{0.1167}$

$= -8.170523$

β_2 is significant

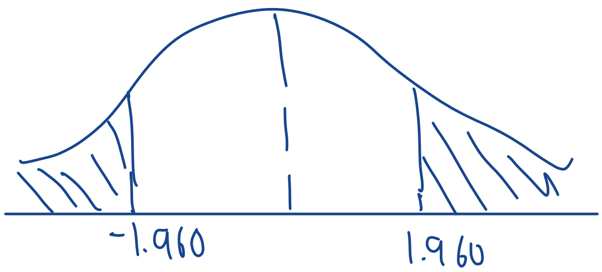
$\alpha = 0.05 \quad d.f. = 501(506-5) = \infty$

$ROOM_i$

$\alpha = 0.05 \quad d.f. = 501$

1) $H_0 : \beta_5 = 0$

$H_a : \beta_5 \neq 0$



2) $t_{cal}(\beta_5) = \dfrac{\hat{\beta}_5 - \beta_5}{\sigma_{\hat{\beta}_5}}$

$= \dfrac{0.2545 - 0}{0.01553}$

β_4 is significant

3.3) Find the R-squared, adjusted R-squared, and test the joint significance of all the slope coefficient estimates.

$$ESS = TSS - RSS = 84.5822 - 35.183 = 49.3487$$

$$R^2 = ESS/TSS = \frac{84.5822}{49.3987} = 1.7122353$$

$$\text{adjust } R^2 = \frac{1 - RSS/n-k}{TSS/n-1} = \frac{1 - 35.183/501}{84.5822/506} = \frac{-0.06822}{0.167158} = -0.408114 \ \#$$

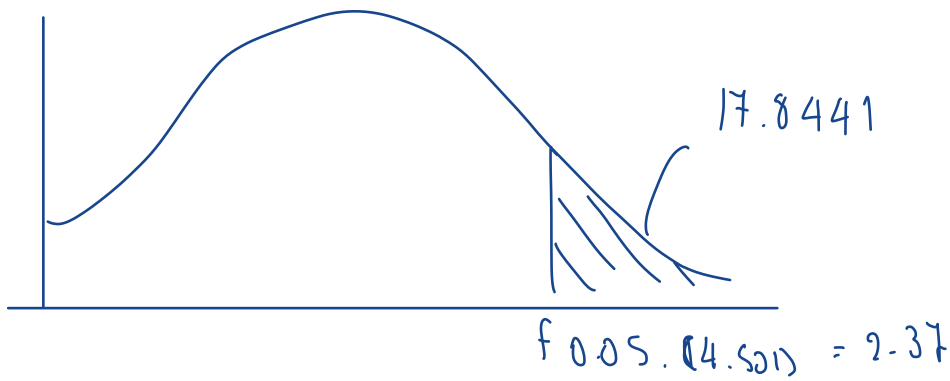
<u>Joint</u>

$H_0: \beta_2 = \beta_3 = \beta_4 = \beta_5 = 0$

H_1: Not all slope coefficient are simultaneously zero

$\alpha = 0.05$

$$F_{cal} = \frac{ESS/k-1}{RSS/n-k} = \frac{R^2/k-1}{(1-R^2)/n-k} = \frac{49.3987/4}{35.1835/501} = 175.3441$$



∴ reject H_0 → This is enough evidence to say that at least one parameter is not equal to zero, at $\alpha = 5\%$.

3.4) If researcher would like to test the proposition that the marginal effect of $ln(NOX_i)$ on $ln(P_i)$ equals the marginal effect of $ln(DIST_i)$ on $ln(P_i)$, write the restricted model and perform the test comparing restricted and unrestricted model, given that OLS estimation of this restricted regression equation yields a Residual Sum of Squares value = 41.9532.

$H_0: \beta_2 = \beta_3$ or $\beta_2 - \beta_3 = 0$

$H_1: \beta_2 \neq \beta_3$ or $\beta_2 - \beta_3 \neq 0$

$\alpha = 0.05$

$\ln(P_i) = \beta_1 + \beta_2 \ln(NOX_i) + \beta_3 \ln(DIST_i) + \beta_4 ROOM_i + \beta_5 STRAT_i + U_i$

(unrestricted)

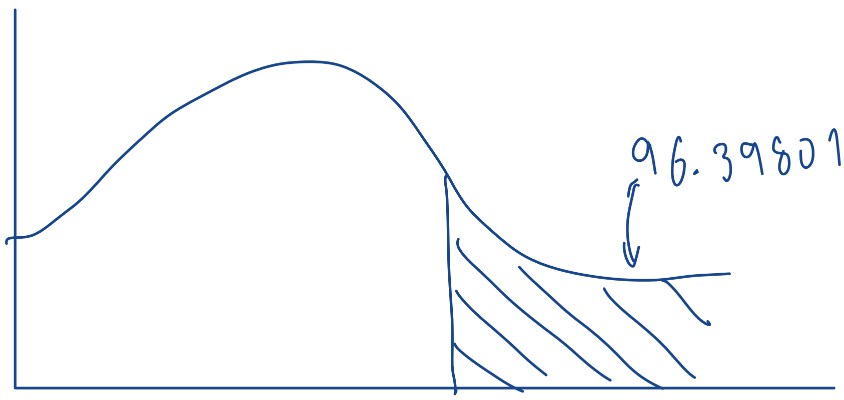
$= \beta_1 + \beta_2 \ln(NOX_i) + \beta_2 \ln(DIST_i) + \beta_4 ROOM_i + \beta_5 STRAT_i + U_i$

$= \beta_1 + \beta_2 (\ln(NOX_i)) + \ln(DIST_i) + \beta_4 ROOM_i + \beta_5 STRAT_i$

$= \beta_1 + \beta_2 \ln(NOX_i) \cdot (DIST_i) + \beta_4 ROOM_i + \beta_5 STRAT_i + U_i$

(restricted)

$$F_{cal} = \frac{RSS_R - RSS_{UR} / M}{RSS_{UR} / n-k} = \frac{41.9532 - 35.1835/1}{35.1835/506-5} = 96.39801$$



$\therefore$ reject H_0

$\therefore \beta_2 \neq \beta_3$

4. Production function (Y) of the industrial sector in Thailand. It depends on the capital factor (K) and labor factor (L) in the years 1980-2010 with the following estimation.

Model 1:

$$\ln Y_t = 18.27 + 0.536 \ln L_t + 0.024 \ln K_t$$

$$R^2 = 0.9389, RSS = 0.0124$$

Model 2:

$$\ln \left(\frac{Y}{L}\right)_t = 2.13 + 1.12 \ln \left(\frac{K}{L}\right)_t$$

$$R^2 = 0.8087, RSS = 0.0153$$

4.1) Interpret the coefficients of the independent variables in models 1 and 2.

4.2) Test the hypothesis. Is the industrial production function characterized by constant return to scale? (Hint: you can perform any type of test that you see fit.)

4.3) Can we compare the R^2 value between the two regression models? Why?

4.1) Model 1: $\ln \hat{Y}_t = 18.24 + \underbrace{0.536}_{\text{output elasticity of labor}} \ln L_t + \overbrace{0.024}^{\text{output elasticity of capital}} \ln K_t$

$\frac{\%\Delta Y}{\%\Delta K}$

$$\varepsilon^y_x = \frac{\%\Delta Y}{\%\Delta X} \qquad \frac{\%\Delta Y}{\%\Delta L}$$

$$\hat{\beta}_1 = 18.24$$

$\ln L_t = 0$, and $\ln K_t = 0$, on average output would be equal to $e^{18.24}$

$\hat{\beta}_2 = 0.536$, as labor increases by 1% on average, output increases by ~ 0.536%, holding other variables constant.

$\hat{\beta}_3 = 0.1024$, as capital increase by 1% on average, output increases by ~0.1024%, holding other variables constant.

Model 2 : $\ln\left(\frac{\hat{Y}}{L}\right)_t = 2.13 + 1.12 \ln\left(\frac{K}{L}\right)_t$

assume that $\hat{\alpha}_1 = 2.13$
$\hat{\alpha}_2 = 1.12$

$\hat{\alpha}_1 = 2.13$ if $\ln(K/L) = 0$, on average, output per labor ratio would be equal to $e^{2.13}$

$\hat{\alpha}_2 = 1.12$;
As capital per labor ratio increase by 1% on average, labor productivity rises by ~1.12%, holding other variables constant.

4.2) Model 1; $\ln Y_t = \beta_1 + \beta_2 \ln L_t + \beta_3 \ln K_t + U_t$

Unrestricted Regression

Model 2; $\ln Y_t = \beta_1 + \beta_3 \ln\left(\frac{K}{L}\right)_t + U_t$

Restricted Regression

Condition is $\beta_2 + \beta_3 = 1$

$H_0 : \beta_2 + \beta_3 = 1$

$H_1 : \beta_2 + \beta_3 \neq 1$

$\alpha = 0.05$

$$F = \frac{(R^2_{UR} - R^2_R)/m}{(1-R^2_{UR})/(n-K_{UR})}$$

$$= \frac{0.9389 - 0.8087/1}{1 - 0.9389/27}$$

$$= 54.535$$

Find critical value $f_{0.05,1,27} \approx f_{0.05,1,26} = 4.23$

Reject H_0 when $F > f_{0.05,1,27}$

because $F = 54.535 > f_{0.05,1,26} = 4.23$

so, reject H_0

To conclude, production function in Thailand isn't a constant return to scale at sig. level 0.05

4.3) We cannot compare R value between two regression models because Model 1 and Model 2 have different variables. Model 1 has $\ln Y_t$ as dependent variable, while Model 2 has $\ln\left(\frac{Y}{L}\right)$ as dependent variable. Although both models have equal number of information.