

EE 320 Introductory Mathematical Economics

Semester 2/2012

Problem Set 5 – Suggested Answers

Differential Calculus and Applications (1 independent variable)

1. Q.1 $VC = Q^3 - 5Q^2 + 12Q$; $\frac{dVC}{dQ} = MC = 3Q^2 - 10Q + 12$
- Q.2 $C = AC \cdot Q = Q^3 - 4Q^2 + 174Q$; $\frac{dC}{dQ} = MC = 3Q^2 - 8Q + 174$
- This is a long-run cost function because the fixed cost is zero.
- Q.4 (b) $= AR \cdot Q = 60Q - 3Q^2$; $MR = \frac{dR}{dQ} = 60 - 6Q$
- (c) Yes.
- (d) The MR curve is twice as steep as the AR curve.
- Q.10 (a) $MC = 6Q + 7$; $AC = 3Q + 7 + \frac{12}{Q}$
- (b) $MR = 10 - 2Q$ $AR = 10 - Q$
- (c) $MP = a + 2bL - cL^2$ $AP = a + bL - cL^2$
2. (a) $f'(x) = -4x + 8 = 0$ iff $x = 2$; the stationary value $f(2) = 15$ is a relative maximum.
- (b) $f'(x) = 10x + 1 = 0$ iff $x = -1/10$; $f(-1/10) = -1/20$ is relative minimum.
- (c) $f'(x) = 6x = 0$ iff $x = 0$; $f(0) = 3$ is a relative minimum.
3. (a) Yes.
- (b) AR function: $P = 100 - Q$. Thus, $R = PQ = (100 - Q)Q = 100Q - Q^2$.
- (c) $\pi = R - C = \frac{1}{3}Q^3 + 6Q^2 - 11Q - 50$
- (d) Set $\frac{d\pi}{dQ} = -Q^2 + 12Q - 11 = 0$. Then, only $Q^* = 11$ gives a maximum profit.
- (e) Maximum profit = 111.33
4. $Q^* = 4000$
5. (a) $\pi(Q) = Q \cdot P(Q) - C(Q) = -0.01Q^2 + 14Q - 4500$. $Q^* = 700$.

(b) This is the price elasticity with respect to demand (Q refers to quantity demanded).

$$E_d = \frac{Q}{P} \cdot \frac{dP}{dQ} = \frac{Q}{18 - 0.006Q} \cdot (-0.006) = -1 \text{ for } Q^* = 1500.$$

(c) $R(Q) = Q \cdot P(Q) = 18Q - 0.006Q^2$. $MR = R'(Q) = 18 - 0.012Q = 0$ for $Q^* = 1500$.

6. (a) $f'(x) = x^2 e^x (x + 3)$. $x^* = 0, -3$. $f(0)$ is an inflection point; $f(-3)$ is a minimum.

(b) $\frac{d(2^x)}{dx} = 2^x \ln 2$; $g'(x) = x^2 \cdot 2^x \ln 2 + 2^x (2x)$

Critical values: $x^* = 0, -2/\ln 2$;

$f''(0) > 0 \rightarrow f(0)$ is a local minimum.

$f''(-2/\ln 2) < 0 \rightarrow f(-2/\ln 2)$ is a local maximum.

7. For $x > 0$, $R = p\sqrt{x}$, $C = wx + F$, $\pi(x) = p\sqrt{x} - wx - F$.

$$\pi'(x) = 0 \text{ when } w = p/2\sqrt{x}. \rightarrow x = p^2/4w^2. \text{ Check } \pi''(x) < 0.$$

8. (a) $MC = 6Q + 7$; $AC = 3Q + 7 + \frac{12}{Q}$.

(b) $\frac{d\pi}{dQ} = 2Q - 13$; $\frac{\pi}{Q} = Q - 13 + \frac{78}{Q}$

9. a) critical values: $x = 4$ (relative maximum), $x = 8$ (relative minimum);

Inflection point: $x = 6$

b) critical values: $x = -1$ (relative minimum), $x = 5$ (relative maximum);

Inflection point: $x = 2$

c) critical values: $x = 3.5$;

Test for concavity is inconclusive.

10. $MPC = 0.8$