Student ID.....

EE431 Economics of Financial Markets and Institutions

Problem Set 1 : Measuring the interest rates

Instructions : Do all questions by using the present value tables.

1. Describe the time value of money concept and how it affects your investment programs.

   Answer. "The time value of money allows people to invest money over a period of time. At the end of the investment period, people receive not only what they have invested, but also the earnings (dividends and interest) the investment has earned. For most people, accumulation is built on the principle of investing small sums of money over a long period of time. Thus, the time value of money can really help people accumulate wealth."

2. The relationship between a bond's coupon rate, the yield required by the market and the bond's price relative to par value is as follows:

   - Coupon rate $<$ yield required by market $\rightarrow$ price_...<...par value ($>$, $<$, $\leq$, $\geq$ or $=$), the bond is sold at ..discount...
   - Coupon rate $=$ yield required by market $\rightarrow$ price ...=... par value ($>$, $<$, $\leq$, $\geq$ or $=$), the bond is sold at ..par...
   - Coupon rate $>$ yield required by market $\rightarrow$ price ...>... par value ($>$, $<$, $\leq$, $\geq$ or $=$), the bond is sold at ..premium...

3. Suppose the market interest rate is 10%, find the present value of the following securities

   (a) Security A will pay \$1,100, one year from now (at year 1).

   (b) Security B will pay \$1,210, two years from now (at year 2).

   (c) Security C will pay \$1,331, three years from now (at year 3).

   Answer

   $$\text{Present Value of Security A} = \frac{1,100}{(1+0.10)} = 1,000$$

   $$\text{Present Value of Security B} = \frac{1,210}{(1+0.10)^2} = 1,000$$

   $$\text{Present Value of Security C} = \frac{1,331}{(1+0.10)^3} = 1,000$$

4. Repeat the calculation in question 1 for the interest rate of 7%

   The question is incomplete. There is a typo. I am sorry about that.

   The correct question is "Repeat the calculation in question 3 for the interest rate of 7%.

   The solution the correct question is as follows.

$$\text{Present Value of Security A} = \frac{1,100}{(1+0.07)} = 1,028.04$$

$$\text{Present Value of Security B} = \frac{1,210}{(1+0.07)^2} = 1056.86$$

$$\text{Present Value of Security C} = \frac{1,331}{(1+0.07)} = 1,086.49$$

5. Suppose the market interest rate is 7%. Find the present value of a 20 years fixed payment loan, which pays 20,000 Baht at the end of each year.

$$\begin{aligned}
\text{Present value of the fixed payment loan} &= \frac{20,000}{(1+0.07)} + \frac{20,000}{(1+0.07)^2} + \frac{20,000}{(1+0.07)^3} + ... + + \frac{20,000}{(1+0.07)^{20}} \\
&= 20,000 \times \left(\frac{1}{(1+0.07)} + \frac{1}{(1+0.07)^2} + \frac{1}{(1+0.07)^3} + ... + + \frac{1}{(1+0.07)^{20}} \right) \\
&= 20,000 \times \text{Present Value of An Anuity of 1 Baht due at the end of 20 periods} \\
&= 20,000 \times PVIFA_{7\%,20} \\
&= 20,000 \times 10.59401 \\
&= 211,880.2
\end{aligned}$$

6. Suppose the market interest rate is 8%. A 10 years fixed payment loan with present value $1,342,016. The equal fixed payment is paid annually. Find the annual payment of this fixed payment loan.

   Let the annual payment $= PMT$

   Answer.

$$\begin{aligned}
\text{Present value of the fixed payment loans} &= PMT \times PVIFA_{8\%,10} \\
1,342,016 &= PMT \times 6.71008 \\
PMT &= \frac{1,342016}{6.71008} \\
&= 200,000
\end{aligned}$$

   The annual payment is equal to $200,000.

7. Consider a 5.5% coupon bond with the par value of 1,000 Baht, selling for 863.38505 Baht. The bond will mature in 15 years. The coupon payment is made annually.

   (a) Calculate the yield to maturity.

   Answer. Yield to maturity is defined as the discount rate that equates present value of cashflows from bonds to its market price.

$$\begin{aligned}
\text{Bond Price} &= \sum_t \frac{CF_t}{(1+k)^t}, \\
863.38505 &= \frac{55}{(1+YTM)} + \frac{55}{(1+YTM)^2} + ... + \frac{55}{(1+YTM)^{15}} + \frac{1,000}{(1+YTM)^{15}} \quad , \\
&= 55 \times PVIFA_{YTM\%,15} + 1,000 \times PVIF_{YTM\%,15}.
\end{aligned}$$

The yield to maturity usually cannot be solved directly. It is determined by using trial and error or iterative method. The method consists of a sequence of iterations, involving repeated calculations.

The bond in the question is sold at discount. Hence, the coupon rate is lower than the discount rate.

Therefore, guess the initial value of yield to maturity as 6%. Then,

$$\begin{aligned} \text{Bond Price} &= 55 \times PVIFA_{6\%,15} + 1,000 \times PVIF_{6\%,15}, \\ &= (55 \times 9.71225) + (1,000 \times 0.41727), \\ &= 534.17375 + 417.27, \\ &= 951.44375. \end{aligned}$$

951.44375$>$ 863.38505. Therefore, we increase the trial discount rate to 9% and recalculate.

$$\begin{aligned} \text{Bond Price} &= 55 \times PVIFA_{7\%,15} + 1,000 \times PVIF_{7\%,15}, \\ &= (55 \times 9.10791) + (1,000 \times 0.36245), \\ &= 500.93505+, \\ &= 863.38505 \end{aligned}$$

Therefore, yield to maturity of this bond is equal to 7%.

(b) Later in the same year, the bond interest rate is fallen to 5%. What is the price of the 5.5% coupon bond?

Answer. Bond price is equal to its present value of cashflows. Therefore, the price of the bond can be calculated as follows.

$$\begin{aligned} \text{Bond Price} &= 55 \times PVIFA_{5\%,15} + 1,000 \times PVIF_{5\%,15}, \\ &= (55 \times 10.37966) + (1,000 \times 0.48102), \\ &= 570.8813 + 481.02 \\ &= 1,051.9013 \end{aligned}$$

Therefore, the price of the 5.5% coupon bond becomes 1,051.9013, when the interest rate is 5%.

Using the answer in question 7a with that in question 7b, we can see that when the interest rate is 8%, the bond price is 863.38505 Baht.

When the interest rate is 5%, the bond price is 1,051.9013.

Thus, when interest rate increases, the bond price decreases.

When interest rate decreases, the bond price increases.

8. Suppose an investor can buy any fraction of a bond. Stefan wants to invest his money for 5 years. He requires 7% rate of return. The following options are given. How should he invest?

- Bond A is a 10% coupon bond with the par value of 1,000 Baht, 5 years to maturity, selling at 1,216.478 Baht. The coupon payment is made annually.
- Bond B is a 7% coupon bond with the par value 1,000 Baht, 5 years to maturity. The bond is selling at 900 Baht. The coupon payment is made annually.

Answer. Yield to maturity is the rate of return on a bond if held until maturity date. Bond A and Bond B have the same time to maturity. Bond B has a higher yield to maturity than Bond A. Since the investor plans to hold the bond until its maturiy, the investor should buy Bond B to get more returns.

Note:
Basically, the rate of return on a financial asset is defined as the discount rate that equates its present value of cashflows with its market price. When we calculate yield to maturity, we assume that the bond is held to its maturity date. Therefore, yield to maturity is the the rate of return on a bond if held until maturity date. The question assumes that investor can buy any fraction of a bond. This means that investors can always use up his or her 1,000 Baht to the bond regardless

of what the price of the bond.is
Bond A's price is equal to $120 \times PVIFA_{10\%.3} + 1,000 \times PVIF_{10\%,3} = 1,049.732$. With 1,049.732 Baht, you can buy 1 of Bond A. With 1,000 Baht, you can buy $\frac{1,000}{1,049.732} \approx 0.9526$ of Bond A. You buy $\frac{1,000}{1,049.732}$ of Bond A, then you would receive $\left(\frac{1,000}{1,049.732}\right) \times 120$ at the end of the year for 3 years and $\left(\frac{1,000}{1,049.732}\right) \times 1,000$ at the end of year 3. You pay 1,000 Baht for these cashflows. The rate of return on this investment is the discount rate that solves this following equation.

Price (Baht invested) = Present Value of Cashflows,

$$\begin{aligned} 1,000 &= \left\{\left[\left(\frac{1,000}{1,049.732}\right) \times 120\right] \times PVIFA_{k\%.3}\right\} \\ &\quad + \left\{\left[\left(\frac{1,000}{1,049.732}\right) \times 1,000\right] \times PVIF_{k\%,3}\right\}. \end{aligned}$$

Rearrage,

$$\begin{aligned} 1,000 &= \left(\frac{1,000}{1,049.732}\right)\{120 \times PVIFA_{k\%.3}\} \\ &\quad + \left(\frac{1,000}{1,049.732}\right)\left\{1,000 \times PVIF_{k\%,3}\right\}, \end{aligned}$$

$$\left(\frac{1,049.732}{1,000}\right) \times 1,000 = \{120 \times PVIFA_{k\%.3}\} + \left\{1,000 \times PVIF_{k\%,3}\right\},$$

$$1,049.732 = \{120 \times PVIFA_{k\%.3}\} + \left\{1,000 \times PVIF_{k\%,3}\right\}.$$

Then, the discount rate that solves the equation is equal to the yield to maturity of the bonds. In other words, *you get the same rate of return on the bond no matter how much (how many units) you buy the bond.*
The rate of return on Bond B from 1,000 Baht investment in Bond B can be calculated in the same way.
The rate of return from investing in a bond if held to maturity is equal to the bond's yield to maturity.
Therefore, for this question, the investor just simply goes for the one with the higher yield to maturtiy.

9. When interest rates fall, the prices of outstanding bonds **rises**.

10. The market price of longer maturity bonds fluctuates **more** compared with shorter maturity bonds as interest rates change.