B.E. International Program

Faculty of Economics, Thammasat University

Semester: 1/2011
EE 425 Econometrics 1
Homework # 3

1. Given the general linear model

$$Y = X\beta + u$$

where $Y\ is\ nx1$
$X\ is\ nxk$
$\beta\ is\ kx1$
$u\ is\ nx1$

The OLS estimator and its variance are

$$\hat{\beta} = (X'X)^{-1}X'Y$$

$$\text{Var} \quad (\hat{\beta}) = \sigma_u^2(X'X)^{-1}$$

Show that in the case of $k = 2$, formulas for the estimators $\hat{\beta}_1$, $\hat{\beta}_2$ and their variances are the same as those obtained from a simple regression model.
Note: In a simple regression model, when k=2

$$Y_i = \beta_1 + \beta_2 X_i + u_i \text{ , } i = 1, 2, \dots n$$

$$\hat{\beta}_1 = \bar{Y} - \hat{\beta}\bar{X}$$

$$\hat{\beta}_2 = \frac{n\sum X_i Y_i - \sum X_i \sum Y_i}{n\sum X_i^2 - (\sum X_i)^2} = \frac{\sum x_i y_i}{\sum x_i^2} \quad \text{where } x_i = X_i - \bar{X}$$

$$y_i = Y_i - \bar{Y}$$

$$\text{var}\left(\hat{\beta}_1\right) = \sigma_u^2 \frac{\sum X_i^2}{n\sum x_i^2} = \frac{\sigma_u^2 \sum X_i^2}{n\sum X_i^2 - (\sum X_i)^2}$$

$$\text{var}\ \hat{\beta}_2 = \frac{\sigma_u^2}{\sum x_i^2} = \frac{n\sigma_u^2}{n\sum X_i^2 - (\sum X_i)^2}$$

2. Given the model $Y_i = \beta_1 + \beta_2 X_i + U_i$

where Y = consumption and X = income and you have the following data with 10 observations.

$$(X'X) = \begin{bmatrix} 10 & 1{,}700 \\ 1{,}700 & 322{,}000 \end{bmatrix}$$

$$X'Y = \begin{bmatrix} 1{,}110 \\ 205{,}500 \end{bmatrix}$$

$$\sum y_i^2 = y'y = 132{,}100$$

$$n\bar{Y}^2 = 123{,}210$$

$$\sum_{i=1}^{n} \hat{u}_i^2 = \hat{u}'\hat{u} = 337.273$$

a) Compute OLS estimators for $\hat{\beta}_1$, $\hat{\beta}_2$, $\hat{\sigma}_u^2$, $S_{\hat{\beta}_1}$, $S_{\hat{\beta}_2}$ by using

$\hat{\beta} = (X'X)^{-1}X'Y$ and var $\hat{\beta} = \sigma_u^2 (X'X)^{-1}$

b) Test the hypothesis $H_0 \colon \beta_2 = 0$, use 95% confidence level.

c) Compute R^2 and F statistic and test the overall significance of the regression. (Note that in matrix notation, it can be shown that $\sum \hat{y}_i^2 = \hat{\beta}'X'Y - n\bar{Y}^2$)

3. Given the following data

Y	X_2	X_3
1	1	2
3	2	1
8	3	-3
10	4	-5

Estimate the coefficients for the following equation

(a) $Y_i = \alpha_1 + \alpha_2 X_{2i} + u_{1i}$

(b) $Y_i = \lambda_1 + \lambda_3 X_{3i} + u_{2i}$

(c) $Y_i = \beta_1 + \beta_2 X_{2i} + \beta_3 X_{3i} + v_i$

Is $\hat{\alpha}_2 = \hat{\beta}_2$, $\hat{\lambda}_3 = \hat{\beta}_3$? Why or why not? Offer your interpretation of these coefficients.

4. The following table gives data on output and total cost of production of a commodity in the short run.

Output	Total Cost, $
1	193
2	226
3	240
4	244
5	257
6	260
7	274
8	297
9	350
10	420

To test whether the preceding data suggest the U-shaped average and marginal cost curves typically encountered in the short run, one can use the following model:

$$Y_i = \beta_1 + \beta_2 X_i + \beta_3 X_i^2 + \beta_4 X_i^3 + u_i$$

where Y=total cost and X=output. The additional explanatory variables X_i^2 and X_i^3 are derived from X.

a) Express the data in the deviation form and obtain (x′x), (x′y), and $(x'x)^{-1}$.
b) Estimate β_2, β_3, and β_4.
c) Estimate the var-cov matrix of $\hat{\beta}$i.
d) Estimate β_1. Interpret $\hat{\beta}_1$ in the context of the problem.
e) Obtain R^2 and $\bar{R}^2$.
f) A priori, what are the signs of β_2, β_3, and β_4? Why?
g) From the total cost function given previously obtain expressions for the marginal and average cost functions.
h) Fit the average and marginal cost functions to the data and comment on the fit.
i) If β_3=β_4=0, what is the nature of the marginal cost function? How would you test the hypotheses that β_3=β_4=0? Perform the test.

5. In an application of the Cobb-Douglas production function the following results were obtained:

$$\widehat{lnY_i} = 2.3542 + 0.9576\, lnX_{2i} + 0.8242\, lnX_{3i}$$
$$(0.3022) \qquad (0.3571)$$
$$R^2 = 0.8432 \qquad df = 12$$

where Y=output, X_2=labor input, and X_3=capital input, and where the figures in parentheses are the estimated standard errors of the coefficients.

a) The coefficients of the labor and capital inputs in the preceding equation give the elasticities of output with respect to labor and capital. Test the hypothesis that these elasticites are individually equal to unity.

b) Test the hypothesis that the labor and capital elasticities are equal, assuming (i) the covariance between the estimated labor and capital coefficients is zero, and (ii) it is-0.0972.

c) How would you test the overall significance of the preceding regression equation?

6. In order to study the labor force participation of urban poor families (families earning less than $3,943 in 1969), the data in Table was obtained from the 1970 Census of Population.

Table Labor Force Participation Experience of the Urban Poor: Census Tracts, New York City, 1970

Tract No.	% in Labor Force, Y*	Mean Family Income, X_2**	Mean Family Size, X_3	Unemployment Rate, X_4***
137	64.3	1,998	2.95	4.4
139	45.4	1,114	3.40	3.4
141	26.6	1,942	3.72	1.1
142	87.5	1,998	4.43	3.1
143	71.3	2,026	3.82	7.7
145	82.4	1,853	3.90	5.0
147	26.3	1,666	3.32	6.2
149	61.6	1,434	3.80	5.4
151	52.9	1,513	3.49	12.2
153	64.7	2,008	3.85	4.8
155	64.9	1,704	4.69	2.9
157	70.5	1,525	3.89	4.8
159	87.2	1,842	3.53	3.9
161	81.2	1,735	4.96	7.2
163	67.9	1,639	3.68	3.6

Y* = family heads under 65 years old.
X_2** = dollars.
X_4*** = percent of civilian labor force unemployed.
Source: Census Tracts: New York, Bureau of the Census, U.S. Department of Commerce, 1970

a) Using the regression model

(i) $Y_i = \beta_1 + \beta_2 X_{2i} + \beta_3 X_{3i} + u_i$

(ii) $Y_i = \beta_1 + \beta_2 X_{2i} + \beta_3 X_{3i} + \beta_4 X_{4i} + u_i$, obtain the estimates of the regression coefficients and interpret your results.

b) A priori, what are the expected signs of the regression coefficients in the above model and why?

c) How would you test the hypothesis that the incremental contribution of the overall unemployment rate is statistically significant? Show your ANOVA table.

d) Should any variables be dropped from the preceding model? Why?