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Lecture 15

6.2 Regression Through the Origin

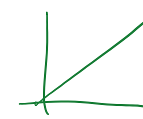
In this section, we consider the case that the two-variable PRF assumes the following form:

$$Y_i = \beta_2 X_i + u_i$$

(MODEL W/O INTERCEPT) OR INTERCEPTLESS MODEL)

This model is called **the regression through the origin** where the intercept term $\hat{\beta}_1$ is absent from the model.

RTO



$x=0,\ y\neq 0$

Example

CONSIDER CAPITAL ASSET PRICING MODEL (CAPM) FROM PORTFOLIO INVESTMENT THEORY.

$$(ER_i - r_f) = \beta_i (ER_m - r_f)$$

WHERE ER_i = EXPECTED RETURN FROM STOCK i

ER_m = EXPECTED RETURN ON THE MARKET PORTFOLIO (EX. SET 50 INDEX)

r_f = RISK-FREE RATE OF RETURN (EX: RETURN FROM GOVERNMENT BOND)

RISK

UNSYSTEMATIC RISK

SYSTEMATIC

β_i = BETA COEFFICIENT, A MEASURE OF SYSTEMATIC RISK

DEGREE THAT STOCK i's RATE OF RETURN "MOVES" WITH THE MARKET

IF $\beta_i > 1$, STOCK i IS CONSIDERED AS AN AGGRESSIVE STOCK.

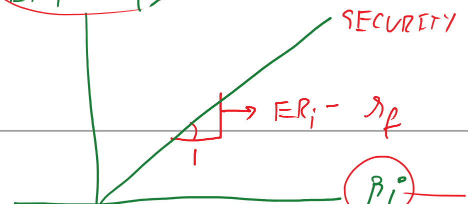
IF $\beta_i < 1$, " ——— " A DEFENSIVE STOCK.

CAPM STATES THAT "STOCK i's EXPECTED RISK PREMIUM $(ER_i - r_f)$ IS EQUAL TO $\beta_i \times$ EXPECTED MKT RISK PREMIUM $(ER_m - r_f)$"

$(ER_i - r_f)$

STOCK i's EXPECTED RISK PREMIUM

SECURITY MKT LINE



$\rightarrow ER_i - r_f$ ⟹ SO SLOPE = $\underline{ER_m - r_f}$ (MKT RISK PREMIUM)

β_i — BETA COEFFIC...

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Since it is the linear regression model, we can apply the Ordinary Least Square (OLS) to estimate the formula for $\hat{\beta}_2$

Let us first write the sample regression function (SRF) as:

$$Y_i = \hat{\beta}_2 X_i + \hat{u}_i$$

We would like to minimize

$$\sum \hat{u}_i{}^2 = \sum (Y_i - \hat{\beta}_2 X_i)^2$$

$$\frac{d \sum \hat{u}_i^2}{d \hat{\beta}_2} = 2 \sum (Y_i - \hat{\beta}_2 X_i)(-X_i) = 0$$

$$- \sum X_i Y_i + \sum \hat{\beta}_2 X_i^2 = 0$$

$$\text{SO} \quad \boxed{\hat{\beta}_2 = \frac{\sum X_i Y_i}{\sum X_i^2}}$$

therefore,

$$\boxed{\hat{\beta}_2 = \frac{\sum X_i Y_i}{\sum X_i^2}}$$

Now we can find out the variance of $\hat{\beta}_2$

$$\text{FROM} \quad \hat{\beta}_2 = \frac{\sum X_i Y_i}{\sum X_i^2} = \frac{\sum X_i (\beta_2 X_i + u_i)}{\sum X_i^2}$$

$$= \beta_2 + \frac{\sum X_i u_i}{\sum X_i^2}$$

$$\text{SO} \quad \hat{\beta}_2 - \beta_2 = \frac{\sum X_i u_i}{\sum X_i^2}$$

$$E(\hat{\beta}_2 - \beta_2)^2$$

$$= E\left(\frac{\sum X_i u_i}{\sum X_i^2}\right)^2$$

$$= \frac{1}{(\sum X_i^2)^2} E(\sum X_i u_i)^2$$

$$= \frac{\sigma^2 \sum X_i^2}{[\sum X_i^2]^2} = \frac{\sigma^2}{\sum X_i^2} = var(\hat{\beta}_2)$$

CAN BE ESTIMATED BY

$$\hat{\sigma}^2 = \frac{\sum \hat{u}_i^2}{n-1}$$

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$$\text{var}(\hat{\beta}_2) = \frac{\sigma^2}{\sum X_i^2}$$

$$\hat{\sigma^2} = \frac{\sum \hat{u}_i{}^2}{n-1}$$

OBSERVATION 1

It should be noted that we get the condition $\sum \hat{u}_i X_i = 0$ from the normal equation. However, with the regression through the origin model, we cannot get the condition $\sum \hat{u}_i = 0$.

IN SHORT: $\sum \hat{u}_i$ IS NOT NECESSARILY ZERO. (SEE APPENDIX FOR A PROOF)

MODEL WITH INTERCEPT

$$\hat{\beta}_2 = \frac{\sum x_i y_i}{\sum x_i^2}$$

$$var(\hat{\beta}_2) = \frac{\sigma^2}{\sum x_i^2}$$

WHERE $\hat{\sigma}^2 = \frac{\sum \hat{u}_i^2}{n-2}$

MODEL WITHOUT INTERCEPT

$$\hat{\beta}_2 = \frac{\sum X_i Y_i}{\sum X_i^2}$$

$$var(\hat{\beta}_2) = \frac{\sigma^2}{\sum X_i^2}$$

WHERE $\hat{\sigma}^2 = \frac{\sum \hat{u}_i^2}{n-1}$

OBSERVATION 2 r^2 MIGHT BE NEGATIVE. (HOW?)

RECALL THAT $r^2 = \underbrace{1 - \frac{RSS}{TSS}}$ $= 1 - \frac{\sum \hat{u}_i^2}{\sum y_i^2} = 1 - \frac{\sum (Y_i - \hat{Y}_i)^2}{\sum (Y_i - \bar{Y})^2}$

$$= 1 - \frac{\left(\sum Y_i^2 - \hat{\beta}_2^2 \sum X_i^2\right)}{\left(\sum Y_i^2 - n\bar{Y}^2\right)}$$

① $r^2 < 0$ IF $RSS > TSS$.

② UNDER WHICH CONDITION THAT $RSS > TSS$?

IF $\hat{\beta}_2^2 \sum X_i^2 < n\bar{Y}^2$.

THEN r^2 WILL BECOME NEGATIVE.

For the zero-intercept model, r^2 can be negative, whereas for the conventional model it cannot be negative.

Since the conventional r^2 is not appropriate for the regressions that do not contain the intercept, we therefore compute what is known as the **raw** r^2 instead:

$$\boxed{\text{raw } r^2 = \frac{(\sum X_i Y_i)^2}{\sum X_i^2 \sum Y_i^2}}$$

This raw r^2 has its value between 0 and 1, but we cannot directly compare its value to the conventional r^2 value. For this reason, some researchers do not report the r^2 value for zero intercept regression models.

intercept regression models.

6.3 Scaling and Units of Measurements

Consider our old example given in table 18 which refer to weekly family expenditure (Y) and Income (x), in baht.

Table 18. Weekly family Expenditure (Y), Baht and Income (X), (Unit:Baht)

X	Y
500	360
600	390
700	440
800	575
900	670
1000	730

By using the OLS estimation, we get the following results:

$\hat{Y}_i = 98.524 + 0.593 X_i$ (ALL MEASURED IN BAHT)

- $\hat{\beta}_1 = 98.524$
- $\hat{\beta}_2 = 0.593$
- $var(\hat{\beta}_1) = 2706.3712$ → • $se(\hat{\beta}_1) = 52.023$
- $var(\hat{\beta}_2) = 0.0046$ → • $se(\hat{\beta}_2) = 0.0678$
- $\hat{\sigma}^2 = \frac{\sum \hat{u}_i^2}{n-2} = 800.476$.

Now, we are interested in changing the units of our data. For example, we would prefer to express our sample data in the unit of 1000 baht. By using the new unit of X and Y, we can report our data in 1000 baht as in the following table.

$\boxed{X \quad Y} \rightarrow$ BAHT

$\times \frac{1}{1000}$

$\boxed{X_i^* = w_1 X_i \qquad Y_i^* = w_2 \cdot Y_i}$

MORE $\quad w_1 = w_2 = \frac{1}{1000}$

Table 19. Weekly family Expenditure (Y), Baht and Income (X), (Unit: 1000 Baht)

X	Y
0.5	0.360
0.6	0.390
0.7	0.440
0.8	0.575
0.9	0.670
1	0.730

With the new unit, we would like to answer these two questions:
1. Do the units in which the regressand (Y) and regressor/s (X) are measured make any difference in the regression results?
2. If so, what is the sensible course to follow in choosing units of measurement for regression analysis?

To answer these questions, let:

$$\boxed{Y_i = \hat{\beta}_1 + \hat{\beta}_2 X_i + \hat{u}_i}$$

ORIGINAL MODEL

$$\hat{\beta}_1 = \bar{Y} - \hat{\beta}_2 \bar{X} \qquad var(\hat{\beta}_1) = \frac{\sum X_i^2}{n \cdot \sum x_i^2} \cdot \sigma^2$$

$$\hat{\beta}_2 = \frac{\sum x_i y_i}{\sum x_i^2} \qquad var(\hat{\beta}_2) = \frac{\sigma^2}{\sum x_i^2} \qquad \hat{\sigma}^2 = \frac{\sum \hat{u}_i^2}{(n-2)}$$

where Y is the weekly family expenditure and X is the income, in baht.

Now, let w_1 and w_2 are constants, called the **Scale factors**. For example, in our data, if we need to use the unit of 1000 baht instead, we can directly multiple the original data in table 18 with the scale factors equal to 0.001. In other words, $w_1 = w_2 = \frac{1}{1000} = 0.001$.

Define

$$Y_i^* = w_1 Y_i$$

$$X_i^* = w_2 X_i$$

Now consider the regression using Y_i^* and X_i^* variables:

$$\boxed{Y_i^* = \hat{\beta}_1^* + \hat{\beta}_2^* X_i^* + \hat{u}_i^*}$$

RESCALED MODEL

$$\hat{\beta}_1^* = \bar{Y}^* - \hat{\beta}_2^* \bar{X}^* \qquad var(\hat{\beta}_1^*) = \frac{\sum X_i^{*2}}{n \sum x_i^{*2}} \cdot \sigma^{*2}$$

$$\hat{\beta}_2^* = \frac{\sum x_i^* y_i^*}{\sum x_i^{*2}} \qquad var(\hat{\beta}_2^*) = \frac{\sigma^{*2}}{\sum x_i^{*2}} \qquad \hat{\sigma}^{*2} = \frac{\sum \hat{u}_i^{*2}}{(n-2)}$$

$\hat{u}_i^* = ?$

Lecture Note: EE 325-2/2015: Introductory Econometrics—page—106

NOTE

$Y_i^* = w_1 Y_i$ OR $y_i^* = w_1 \cdot y_i$

$X_i^* = w_2 X_i$ OR $x_i^* = w_2 x_i$

$\hat{u}_i^* = w_1 \hat{u}_i$

$\bar{Y}^* = w_1 \bar{Y}$

$\bar{X}^* = w_2 \bar{X}$

SEE

$$\hat{\beta}_1^* = \bar{Y}^* - \hat{\beta}_2^* \bar{X}^*$$

$$\hat{\beta}_1^* = w_1 \bar{Y} - w_1 \hat{\beta}_2 \bar{X}$$

$$= w_1 (\bar{Y} - \hat{\beta}_2 \bar{X})$$

$$\hat{\beta}_1^* = w_1 \cdot \hat{\beta}_1$$

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ORIGINAL MODEL

RESCALED MODEL

Our target is to find out the relationship between the following pairs:

1. $\hat{\beta}_1$ and $\hat{\beta}_1^*$

2. $\hat{\beta}_2$ and $\hat{\beta}_2^*$

3. $\text{var}(\hat{\beta}_1)$ and $\text{var}(\hat{\beta}_1^*)$

4. $\text{var}(\hat{\beta}_2)$ and $\text{var}(\hat{\beta}_2^*)$

5. $\hat{\sigma}^2$ and $\hat{\sigma}^{*2}$

6. r_{xy}^2 and $r_{x^*y^*}^2$

1. $\hat{\beta}_1$ and $\hat{\beta}_1^*$

$$\hat{\beta}_1^* = w_1 \hat{\beta}_1$$

2. $\hat{\beta}_2$ and $\hat{\beta}_2^*$

$$\hat{\beta}_2^* = \left(\frac{w_1}{w_2}\right) \hat{\beta}_2$$

SO, IF $w_1 = w_2$, $\hat{\beta}_2^* = \hat{\beta}_2$.

X Y

$w_1 = \frac{1}{1000}$ BAHT → 1000 BAHT

BAHT → 1000 BAHT $w_2 = \frac{1}{1000}$

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3.var($\hat{\beta}_1$) and var($\hat{\beta}_1^*$)

$$var(\hat{\beta}_1^*) = w_1^2 \cdot var(\hat{\beta}_1)$$

4.var($\hat{\beta}_2$) and var($\hat{\beta}_2^*$)

$$var(\hat{\beta}_2^*) = \left(\frac{w_1}{w_2}\right)^2 \cdot var(\hat{\beta}_2)$$

AGAIN, IF $w_1 = w_2$, $var(\hat{\beta}_2^*) = var(\hat{\beta}_2)$.

5.$\hat{\sigma}^2$ and $\hat{\sigma}^{*2}$

$$\boxed{\hat{\sigma}^2 = \frac{\sum \hat{u}_i^2}{n-2}} \quad \text{vs.} \quad \boxed{\hat{\sigma}^{*2} = \frac{\sum \hat{u}_i^{*2}}{n-2}}$$

WHERE $\hat{u}_i^* = w_1 \cdot \hat{u}_i$.

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6. r^2_{xy} and $r^2_{x^*y^*}$

$$r^2_{xy} = r^2_{x^*y^*}$$

WHERE $x_i^* = w_2 \cdot x_i$

$y_i^* = w_1 \cdot y_i$

NOTE $x_i = X_i - \bar{X}$

$y_i = Y_i - \bar{Y}$

6.4 Regression on Standardized Variables

What will happen in the regression of Y and X if we redefine these variables as:

$$Y_i^* = \frac{Y_i - \bar{Y}}{S_Y}$$

$$X_i^* = \frac{X_i - \bar{X}}{S_X}$$

Y_i^* and X_i^* are called **Standardized Variables**

$Y_i = \beta_1 + \beta_2 X_i + u_i \rightarrow$ ORIGINAL MODEL

$Y_i^* = \beta_2^* X_i + u_i^* \rightarrow$ STANDARDIZED MODEL

EX: $\hat{CM}_i^* = \underline{-0.2026}\, PGDP_i^* \underline{- 0.7639}\, FLR_i^*$

STANDARDIZED VARIABLES

IF PGDP INCREASES BY ONE S.D., CM WILL REDUCE BY 0.2026 SD, CETERIS PARIBUS

Lecture 16
CHAPTER 7: Multiple Regression Analysis: The Problem of Analysis

Three-Variable Model: Notation and Assumptions
Let us consider the following three-variable PRF as:

$$Y_i = \beta_1 + \beta_2 X_{2i} + \beta_3 X_{3i} + u_i$$

where

Y_i is the dependent variable (regressand)

X_{2i} and X_{3i} are the regressors or the explanatory variables

u_i is the stochastic disturbance term

Remark: the subscript i is denoted the observation i from our sample data.

In case our data are time series, the subscript t will denote the t observation.

- β_1 means the average value of Y when X_2 and X_3 are set equal to zero

β_2 and β_3 are called the partial regression coefficients.

We will talk about the meaning of β_1 and β_2 shortly after knowing the assumptions of the classical linear regression model (CLRM)

Under the CLRM, we assume:

1. Zero mean value of u_i

$$\boxed{\underbrace{E(u_i \mid X_{2i}, X_{3i})}_{\text{CONDITIONAL MEAN VALUE OF DISTURBANCE TERM}} = 0 \quad \text{FOR EACH } i}$$

2. No serial correlation OR NO AUTOCORRELATION PROBLEM

$$\boxed{\operatorname{cov}(u_i, u_j) = 0 \quad \text{FOR ALL } i \neq j}$$

FAMILY i	Y_i	X_{2i}	X_{3i}
1			
2			
3			
4			
5			
6			
⋮			
n			

$$\underbrace{Y_i = \beta_1 + \beta_2 X_{2i} + \beta_3 X_{3i} + u_i}_{\text{CROSS SECTIONAL MODEL}}$$

$Y_t = \beta_1 + \beta_2 X_{2t} + \beta_3 X_{3t} + u_t$ → TIME-SERIES MODEL

t	Y_t	X_{2t}	X_{3t}
1990			
⋮			
2015			

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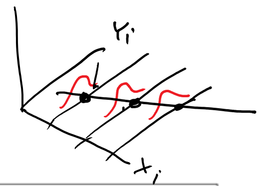
= EQUAL

= SPREAD



3. Homoscedasticity

$$\boxed{var(u_i) = \sigma^2}$$ [= VARIANCE OF u_i IS CONSTANT]

4. Zero covariance between u_i and each X variable, or

$$COV(u_i, X_{2i}) = COV(u_i, X_{3i}) = 0.$$

5. No specification bias or

The model is correctly specified.

6. No exact collinearity between the X variables ~~or~~ (NO MULTICOLLINEARITY)

EXAMPLE OF EXACT LINEAR RELATIONSHIP

$$X_{3i} = 2X_{2i}$$

OR $2X_{2i} - X_{3i} = 0$

ABSTRACT FORM: $\boxed{\lambda_2 X_{2i} + \lambda_3 X_{3i} = 0}$

SO, THAT MEANS, TO HAVE NO MULTICOLLINEARITY, THERE MUST BE NO SET OF λ_2 AND λ_3 (NOT BOTH ZERO) SUCH THAT $\lambda_2 x_{2i} + \lambda_3 x_{3i} = 0$!

X_{2i}	X_{3i}
2	4
4	8
6	12
8	16

$X_{3i} = 2X_{2i}$ OR $2X_{2i} - X_{3i} = 0$

EXACT LINEAR RELATIONSHIP

PERFECT MULTICOLLINEARITY

↓

YOU CANNOT ESTIMATE β_2 AND β_3!

Lecture Note: EE 325-2/2015: Introductory Econometrics—page—111

X_{2i}	X_{3i}	v_i
2	4	0
4	10	+2
6	12	0
8	19	+3

$X_{3i} = 2X_{2i} + v_i$

INEXACT LINEAR RELATIONSHIP

IMPERFECT MULTICOLLINEARITY

↓

YOU STILL CAN GET $\hat{\beta}_2$ AND $\hat{\beta}_3$

$$Y_i = \beta_1 + \beta_2 X_{2i} + \beta_3 X_{3i} + u_i$$
$$= \beta_1 + \beta_2 X_{2i} + \beta_3 (2X_{2i}) + u_i$$
$$= \beta_1 + (\beta_2 + 2\beta_3) X_{2i} + u_i$$

$$= \beta_1 + (\beta_2 + 2\beta_3) X_{2i} + u_i$$

$$= \beta_1 + \alpha X_{2i} + u_i$$

SEE THAT WE CANNOT SEPARATE MARGINAL IMPACT OF X_2 ON Y AND MARGINAL IMPACT OF X_3 ON Y!

By the above assumptions, we can find out the conditional expectation of Y_i:

FROM $Y_i = \beta_1 + \beta_2 X_{2i} + \beta_3 X_{3i} + u_i$

$\underbrace{E[Y_i \mid X_{2i}, X_{3i}]}$ $= \beta_1 + \beta_2 X_{2i} + \beta_3 X_{3i}$. SINCE $E[u_i \mid X_{2i}, X_{3i}] = 0$.

AVERAGE RESPONSE OF Y FOR THE GIVEN VALUES OF REGRESSORS (X_{2i} AND X_{3i})

The meaning of partial coefficients:

β_2

β_2 MEASURES "THE NET" OR "THE DIRECT IMPACT" OF A UNIT CHANGE IN X_2 ON THE MEAN VALUE OF Y, HOLDING THE VALUE OF X_3 CONSTANT

β_3

β_3 MEASURES "THE NET" OR "THE DIRECT IMPACT" OF A UNIT CHANGE IN X_3 ON THE MEAN VALUE OF Y, HOLDING THE VALUE OF X_2 CONSTANT

≡ HOLDING THE INFLUENCE OF X_2 THAT MIGHT AFFECT Y

(READ PAGE 191–192) ON EXAMPLE OF CM MODEL.

7.1 OLS Estimation of the Partial Regression Coefficients

In order to find the OLS estimators, we need to write down the sample regression function (SRF) corresponding to the PRF:

$$Y_i = \hat{\beta}_1 + \hat{\beta}_2 X_{2i} + \hat{\beta}_3 X_{3i} + \hat{u}_i$$

JOB: $\min_{\hat\beta_1, \hat\beta_2, \hat\beta_3} \sum \hat{u}_i^2 = \sum (\underbrace{Y_i - \hat{Y}_i}_{\hat{u}_i})^2 = \sum (Y_i - \hat\beta_1 - \hat\beta_2 X_{2i} - \hat\beta_3 X_{3i})^2$

F.O.C.s

$$\frac{\partial \sum \hat{u}_i^2}{\partial \hat\beta_1} = 2 \sum (\underbrace{Y_i - \hat\beta_1 - \hat\beta_2 X_{2i} - \hat\beta_3 X_{3i}}_{= \hat{u}_i})(-1) = 0 \equiv \sum \hat{u}_i = 0 \quad (1)$$

$$\frac{\partial \sum \hat{u}_i^2}{\partial \hat\beta_2} = 2 \sum (\underbrace{Y_i - \hat\beta_1 - \hat\beta_2 X_{2i} - \hat\beta_3 X_{3i}}_{= \hat{u}_i})(-X_{2i}) = 0 \equiv \sum \hat{u}_i X_{2i} = 0 \quad (2)$$

$$\frac{\partial \sum \hat{u}_i^2}{\partial \hat\beta_3} = 2 \sum (\underbrace{Y_i - \hat\beta_1 - \hat\beta_2 X_{2i} - \hat\beta_3 X_{3i}}_{})(-X_{3i}) = 0 \equiv \sum \hat{u}_i X_{3i} = 0 \quad (3)$$

From the FOC, we then get the normal equations:

$$\bar{Y} = \hat{\beta}_1 + \hat{\beta}_2\bar{X}_2 + \hat{\beta}_3\bar{X}_3$$
$$\sum Y_i X_{2i} = \hat{\beta}_1 \sum X_{2i} + \hat{\beta}_2 \sum X_{2i}^2 + \hat{\beta}_3 \sum X_{2i}X_{3i}$$
$$\sum Y_i X_{3i} = \hat{\beta}_1 \sum X_{3i} + \hat{\beta}_2 \sum X_{2i}X_{3i} + \hat{\beta}_3 \sum X_{3i}^2$$

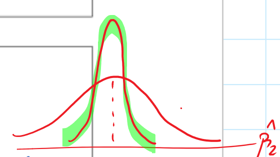
We therefore get:

$$\hat{\beta}_1 = \bar{Y} - \hat{\beta}_2\bar{X}_2 - \hat{\beta}_3\bar{X}_3$$
$$\hat{\beta}_2 = \frac{(\sum y_i x_{2i})(\sum x_{3i}^2) - (\sum y_i x_{3i})(\sum x_{2i}x_{3i})}{(\sum x_{2i}^2)(\sum x_{3i}^2) - (\sum x_{2i}x_{3i})^2}$$
$$\hat{\beta}_3 = \frac{(\sum y_i x_{3i})(\sum x_{2i}^2) - (\sum y_i x_{2i})(\sum x_{2i}x_{3i})}{(\sum x_{2i}^2)(\sum x_{3i}^2) - (\sum x_{2i}x_{3i})^2}$$

Variance and Standard Errors of OLS Estimators → PURPOSES ARE
CONSTRUCT CI — ①
DO HYPOTHESIS TESTING — ②

$$var(\hat{\beta}_1) = \left[\frac{1}{n} + \frac{\bar{X}_2^2 \sum x_{3i}^2 + \bar{X}_3^2 \sum x_{2i}^2 - 2\bar{X}_2\bar{X}_3 \sum x_{2i}x_{3i}}{\sum x_{2i}^2 \sum x_{3i}^2 - (\sum x_{2i}x_{3i})^2}\right] * \sigma^2$$
$$se(\hat{\beta}_1) = +\sqrt{var(\hat{\beta}_1)}$$

$$var(\hat{\beta}_2) = \frac{\sum x_{3i}^2}{(\sum x_{2i}^2)(\sum x_{3i}^2) - (\sum x_{2i}x_{3i})^2} * \sigma^2$$
$$var(\hat{\beta}_2) = \frac{\sigma^2}{\sum x_{2i}^2(1 - r_{23}^2)}$$
$$se(\hat{\beta}_2) = +\sqrt{var(\hat{\beta}_2)}$$



GIVEN σ^2, $\sum x_{2i}^2$, THE HIGHER THE r_{23}, THE HIGHER $var(\hat{\beta}_2)$ ☹

$$var(\hat{\beta}_3) = \frac{\sum x_{2i}^2}{(\sum x_{2i}^2)(\sum x_{3i}^2) - (\sum x_{2i}x_{3i})^2} * \sigma^2$$
$$var(\hat{\beta}_3) = \frac{\sigma^2}{\sum x_{3i}^2(1 - r_{23}^2)}$$
$$se(\hat{\beta}_3) = +\sqrt{var(\hat{\beta}_3)}$$

Lecture Note: EE 325-2/2015: Introductory Econometrics—page—113

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$$cov(\hat{\beta}_2, \hat{\beta}_3) = \frac{-r_{23}\sigma^2}{(1-r_{23}^2)\sqrt{\sum x_{2i}^2}\sqrt{\sum x_{3i}^2}}$$

$$\hat{\sigma}^2 = \frac{\sum \hat{u}_i{}^2}{n-3}$$

WE ESTIMATE 3 STATISTICS $\hat{\beta}_1, \hat{\beta}_2, \hat{\beta}_3$, WE LOSE 3 d.f.

7.2 Properties of OLS Estimators

① THE THREE-VARIABLE REGRESSION SURFACE PASSES THROUGH THE MEANS: $\bar{Y}, \bar{X}_2, \bar{X}_3$

i.e., $\bar{Y} = \hat{\beta}_1 + \hat{\beta}_2 \bar{X}_2 + \hat{\beta}_3 \bar{X}_3$.

② THE MEAN VALUE OF THE ESTIMATED Y_i: $(\hat{Y}_i)$ = THE MEAN VALUE OF ACTUAL Y_i: $\bar{\hat{Y}} = \bar{Y}$

PROOF:

$$\hat{Y}_i = \hat{\beta}_1 + \hat{\beta}_2 X_{2i} + \hat{\beta}_3 X_{3i}$$

$$\hat{Y}_i = \underbrace{(\bar{Y} - \hat{\beta}_2 \bar{X}_2 - \hat{\beta}_3 \bar{X}_3)}_{\text{FROM PAGE 113}} + \hat{\beta}_2 X_{2i} + \hat{\beta}_3 X_{3i}$$

$$\hat{Y}_i = \bar{Y} + \hat{\beta}_2 (X_{2i} - \bar{X}_2) + \hat{\beta}_3 (X_{3i} - \bar{X}_3)$$

$$\hat{Y}_i = \bar{Y} + \hat{\beta}_2 x_{2i} + \hat{\beta}_3 x_{3i}$$

$$\frac{\sum \hat{Y}_i}{n} = \frac{\sum \bar{Y}}{n} + \frac{\hat{\beta}_2 \sum x_{2i}}{n} + \frac{\hat{\beta}_3 \sum x_{3i}}{n}$$

$$\bar{\hat{Y}} = \frac{n\bar{Y}}{\cancel{n}} + 0 + 0 \qquad \text{(WHY?)}$$

SO $\bar{\hat{Y}} = \bar{Y}$. #

Properties of OLS Estimators (Cont:)

(3) $\sum \hat{u}_i = 0$ AND $\frac{\sum \hat{u}_i}{n} = \overline{\hat{u}_i} = 0$

(4) $\hat{u}_i$ ARE UNCORRELATED WITH X_{2i}, X_{3i}:

$\underbrace{\sum \hat{u}_i X_{2i} = 0}_{\text{FROM FOC. (2)}}$, $\underbrace{\sum \hat{u}_i X_{3i} = 0}_{\text{FROM FOC. (3)}}$ PAGE 112

(5) $\hat{u}_i$ ARE UNCORRELATED WITH $\hat{Y}_i$:

$\sum \hat{u}_i \hat{Y}_i = 0.$

CALLED "RESIDUALS"

PROOF: $\hat{Y}_i = \hat{\beta}_1 + \hat{\beta}_2 X_{2i} + \hat{\beta}_3 X_{3i}$

$$\sum \hat{u}_i \hat{Y}_i = \sum \hat{\beta}_1 \hat{u}_i + \sum \hat{\beta}_2 \hat{u}_i X_{2i} + \sum \hat{\beta}_3 \hat{u}_i X_{3i}$$

$$= \hat{\beta}_1 \underbrace{\sum \hat{u}_i}_{=0} + \hat{\beta}_2 \underbrace{\sum \hat{u}_i X_{2i}}_{=0} + \hat{\beta}_3 \underbrace{\sum \hat{u}_i X_{3i}}_{=0}$$

$$\sum \hat{u}_i \hat{Y}_i = 0. \quad \#$$