

EE312 Macroeconomics, 2/2014 (Sec. 046402)
 Problem Sets 4 : Ch.4 Part 2 Endogenous Growth Model
 Solution

* Exam will consist of essay-type questions only.

1. In the endogenous growth model, suppose that there are three possible uses of time. Let u denote the fraction of time spent working. Let s denote the fraction of time spent neither working nor accumulating human capital (call this unemployment), and $1 - u - s$ the fraction of time spent accumulating human capital. Assume that $z = 1$ and $b = 4.2$. Also assume that the economy begins period 1 with 100 units of human capital.

- (a) Define the consumer's budget constraint, the law of motion of human capital, the firm's production function and the firm's profit function. (Fill in the blanks in the table below)

Consumers	Firm
$C = uwH^S$	$Y = zuH^d$
$H^{s'} = (1 - u - s)bH^s$	$\pi = (z - w)uH^d.$
b = efficiency of human capital accumulation technology	The demand curve is infinitely elastic at $w = z$

- (b) Define equilibrium consumption and growth of capital accumulation. (Fill in the blanks below.)

The market clears at $w = \dots z \dots$ where $uH^d = \dots uH^s \dots$

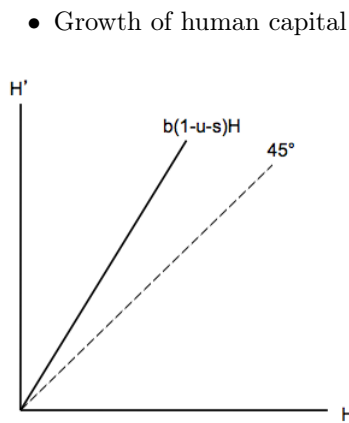
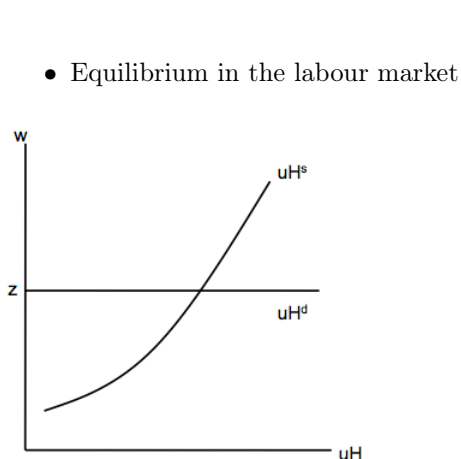
Equilibrium consumption and growth of human capital accumulation: (Define H' , $\frac{H'}{H}$ and $\frac{H'}{H} - 1$.)

$$H' = (1 - u - s)bH$$

$$\frac{H'}{H} = (1 - u - s)b$$

$$\frac{H'}{H} - 1 = (1 - u - s)b - 1$$

- (c) From question (b), draw two diagrams; one to show equilibrium in the labour market and the other to show growth of human capital.



- $H' > H$. Slope = $b(1-u-s)$ = rate of growth of human capital.

2. In the endogenous growth model, the competitive equilibrium is given by

$$Y = C = zuH$$

$$H' = b(1 - u)H$$

where Y = output; C = consumption, z = total factor productivity; H and H' = levels of current and future human capital, respectively; b = the efficiency of human capital accumulation technology; and u - time allocated to producing output.

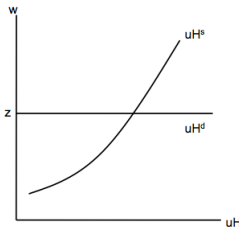
What is the most important source of economic growth in the model? What are the growth rates of consumption, output and human capital? Explain why economic growth is unbound in the model as well as compare the source of growth with Solow model. (If the space provided is not enough, please attach a separate paper.)

Solution

- Note that there are 3 sub-questions. In the exam, do not forget to answer all questions. Carefully show the steps and carefully write down all relevant details.
 - What are the growth rates of consumption, output and human capital?
 - What is the most important source of economic growth in the model?
 - Explain why economic growth is unbound in the model as well as compare the source of growth with Solow model.
- To answer the question, **define the behaviour of the agents (consumer and firm)** in the model as follows.

Consumer	Firm
$C = uwH^S$	$Y = zuH^d$
$H^{s'} = (1 - u)bH^s$	$\pi = (z - w)uH^d$
b = efficiency of human capital accumulation technology	The demand curve is infinitely elastic at $w = z$

- Then, **define competitive equilibrium** in the labour market. The market clears at $w = z$ where $uH^d = uH^s$.



- Therefore, at equilibrium, $H^s = H^d = H$.
- We can **write down equilibrium consumption and growth of human capital accumulation** as follows:

$$H' = b(1 - u)H.$$

$$\frac{H'}{H} - 1 = b(1 - u) - 1, \text{ where } b(1 - u) - 1 \text{ is constant.}$$
- Consumption grows at the same rate as human capital;**

$$\frac{C'}{C} - 1 = \frac{zuH'}{zuH} - 1 = \frac{H'}{H} - 1 = b(1 - u) - 1.$$
- Since there is no saving in the model and $w = z$, at equilibrium, consumption is always equal to output. Therefore, **output grows at the same rate as consumption (and human capital).**

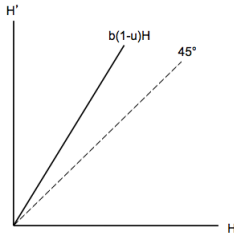
- Then, we have the **balanced growth path**.

$$\frac{H'}{H} = \frac{C'}{C} = \frac{Y'}{Y} = b(1-u) - 1$$

$$g_H = g_C = g_Y = b(1-u) - 1$$

The growth rates of consumption, output and human capital are equal to $b(1-u) - 1$. **ANS2**

- Let's assume $H' > H$, allowing human capital to grow over time.



- The slope of the graph H' is equal to $b(1-u)$ = rate of growth of human capital.
- From $H' > H$, $b(1-u) > 1$.

- Ans 1** From the balanced growth path the growth rates of consumption, output and human capital are equal to $b(1-u) - 1$. Therefore, the most important source of economic growth in the model is **accumulation of human capital, which promotes efficiency of output production in the model. The higher human capital, the more efficiency the production of human capital has.**
 - Human capital is an investment** : sacrifice of current consumption $\rightarrow$ more future production and consumption.
 - More time spent on education, increase in efficiency of human capital accumulation** promotes growth in human capital; $\frac{H'}{H} - 1 = b(1-u) - 1$, $b \uparrow$ or $u \downarrow \Rightarrow \frac{H'}{H} \uparrow$.
 - Ans 3** Economic growth is unbound in the endogenous growth model. This is different from Solow's model. The economic reasons and the comparison between the two model are as follows.
 - Production function :
Solow's model : $Y = zF(K, N)$, $y = zf(k)$.

Solow's model	An endogenous growth model (A Simplified version of Lucas's)
production function : $Y = zF(K, N)$ $y = zf(k)$ Production function is <u>subject to diminishing marginal returns</u> in capital per worker.	$Y = uzH^d$ Production function <u>does not subject to diminishing marginal returns</u> in human capital.
Source of growth is physical capital accumulation.	Source of growth is human capital accumulation
There is a limit for growth as physical capital per worker increases. Output will go to steady state output. After it reaches the steady state, it will stay there. Eventually, growth of per capita output goes down to zero.	Output continues to grow at a constant rate. as human capital increases. There is no limit for growth. Output growth is always positive. (growth rate is equal to $b(1-u) - 1$).
Economic reason for bounded growth : Physical capital is <u>rivalry</u> (once one company is using it the other cannot). production function is <u>subject to diminishing marginal returns</u> in capital per worker As physical capital increases, output increases at a decreasing rate. Therefore, we cannot always generate growth by raising physical capital.	Economic reason for unbounded growth : This is because knowledge is ' <u>non-rivalry</u> ', production function <u>does not subject to diminishing marginal returns</u> in human capital As human capital increases, output increases at a constant rate. Therefore, we can always generate growth by raising human capital.

3. From Question 2, suppose the government implement a policy to reduce u , time allocated to work, how this affects consumption path and output path. Will the economy gain from the policy? Discuss.

Figure 1: Consumption Path

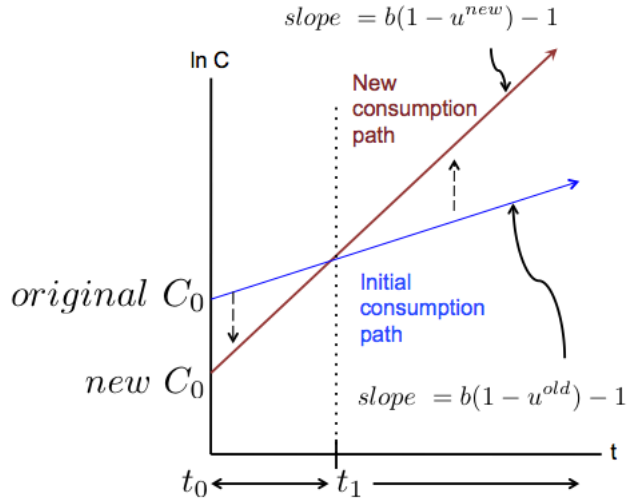


Figure 2: Labor Market

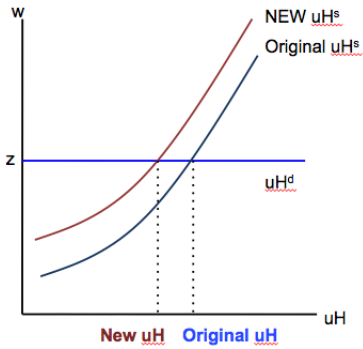
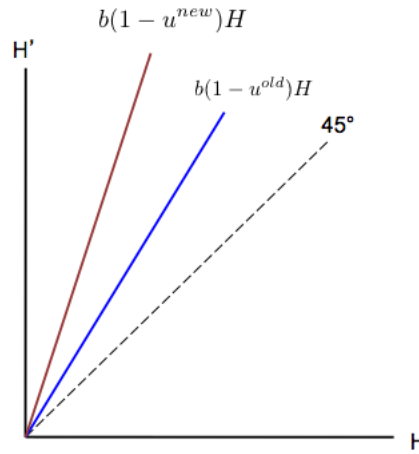


Figure 3: Dynamic of Human Capital



- A lower u results in **lower current consumption but higher consumption in the long run**. See figure 1.
 - **Lower Current Consumption.** At t_0 , the government implements a policy to reduce u .
 - * Time allocated to work (u) reduces from u^{old} to u^{new} .
 - * People work less. They supply lower labor in the labor market. [See figure 2.]
 - * Equilibrium employment decreases from *original* uH to *new* uH . The market clears at $w = z$ where $new\ uH^d = new\ uH^s$.
 - * Output decreases from $Y = z [\text{original } uH]$ to $z[\text{new } uH]$.
 - * Since there is no saving in the model, at equilibrium $w = z$, consumption is always equal to output.
 - * Consumption also decreases from *original* $C_0 = z [\text{original } uH]$ to *New* $C_0 = z[\text{new } uH]$. See figure 1 Consumption Path.

- **Higher Consumption in The Long Run** due to **higher growth rate** of output.
 - * As people spend less time to work, they devote more time to education. Time allocated to education $(1 - u)$ increases from $(1 - u^{old})$ to $(1 - u^{new})$
 - * Human capital accumulates at a faster rate [See figure 3.] .
 - * Growth rate of consumption increases from $b(1 - u^{old}) - 1$ to $b(1 - u^{new}) - 1$.
 - * Therefore, the new consumption growth path is steeper than the original consumption growth path [The slope of consumption growth path is equal to growth rate of consumption.].
- **Economy will gain a higher output and higher growth rate in The Long Run**
 - * There is a period of lower output (and consumption) [from t_0 to t_1 , see figure 1]. $C = Y$ in Equilibrium.
 - * There is a period of higher output. [from t_1 onwards, see figure 1]
 - * The long-run growth rate of the economy is higher.
- **Higher long-run growth is desirable?**
 - * There is a period of welfare loss [from t_0 to t_1 , see figure 1].
 - * There is a period of welfare gain [from t_1 onwards, see figure 1]
 - * This depends on the consumer's preference on current and future consumption. (See lecture note)
 - * The consumer may be worse off if current consumption is actually preferred.