

EE320 Exercise 4

Semester 2, 2014

Chapter 6: Optimization without constraints: One independent variable case

1. Determine the maximum or minimum value or the inflection point of the following function.

a) $y = \frac{x^3+4x^2+4x+1}{x+1}$

b) $y = 100 + 2x - \frac{1}{10}x^2$

c) $y = x^3 - 12x^2 + 36x + 8$

d) $y = 3x^2 - x + 1$

e) $y = x^3 + 3x^2$

f) $y = \frac{1}{3}x^3 - 2x^2 + 4x + 10$

g) $y = -x^3 + 4.5x^2 + 6x + 6$

h) $y = x^4 + 4x^3 - 20x^2 + 36$

i) $y = -x^2 + 4x + 7$

j) $y = 2x^2 + x$

k) $y = x^2 + 3$

2. Let $AC = \frac{300}{Q} + 50 - 10Q + 25Q^2$, determine TC, AVC, MC and Q that makes the slope of AVC and slope of MC equal to zero. Prove that MC is concave or convex function and draw the graph of AVC and MC. $[AVC = 50 - 10Q + 25Q^2, MC = 50 - 20Q + 75Q^2]$

3. Let $TR = 1400Q - 7.5Q^2$ and $TC = Q^3 - 6Q^2 + 14Q + 750$, determine profit function and illustrate by the graph.

4. Let $C = \frac{1}{3}Q^3 - 7Q^2 + 111Q + 50$ and $Q = 100 - P$, determine the following

- a) Revenue function
- b) Profit function
- c) Q which maximize profit [$Q = 11$]
- d) Maximum profit [$\Pi = 111\frac{1}{3}$]

5. Let the furniture company has the following information of the chair production,

$$TR = -Q^2 + 550Q$$

$$TC = 0.03Q^3 - Q^2 - 26Q + 12000$$

- a) Determine price and quantity of the chair such that the company will maximize profit and determine that maximum profit. [$Q=80$, $P=470$]
- b) Suppose government impose tax to each chair produced by 69.75 Baht, how could this affect price and quantity such that company will maximize profit? [$Q=75$, $P=475$]
- c) If government try to change the way of collecting taxes to be profit tax, what is the profit tax rate which would lead to the same level of revenue to government. [$t=27.95\%$]

6. Let the demand function be $Q = 60 - 2P$ and production function be $Q = 4L^{0.5}$ where Q = Quantity of goods, P = Price of goods, L = factor of production (unit:hours) cost of L is 40 Baht per hour.

- a) Write the profit function in term of Q .
- b) Find Q which maximize profit and find the maximum profit. [$Q=5$]
- c) If the producer want to use L in which $MRP = W$, how much the profit does the producer get? [$\Pi = 75$]
- d) Compare the profit from b) and c)

7. Suppose government tries to maximize tax revenues (T) from collecting tax of \$ t per unit from monopoly, given the following

$$TR = 6Q - Q^2$$

$$TC = 2Q$$

Find tax per unit(t) that maximize tax revenues(T) and determine tax revenues (T). [$t=2$, $T=1$]

8. Given that $P = 150 - 0.5Q$ and $TC = 100 + 3Q + 7Q^2$, if government subsidizes the sellers for \$3 per unit, determine the following

- a) price and quantity level where the seller can maximize profit [$Q=10$, $P=145$]
- b) if there is no subsidy to sellers, will the sellers produce and sell the products? Why?

9. Let $TFC = 4$, $AVC = 1$ and demand equation be $P = 10 - 2Q$. Determine price and quantity which the seller can maximize profit and determine that maximum profit. [$Q = 2.25$, $P = 5.5$ and $\Pi = 6.125$]

10. Let demand function be $P = 14 - 3Q$ and supply function be $P = 4 + 2Q$, if government taxes the sellers for \$ t per unit such that the government would maximize tax revenue (T), determine t , T , consumer burden and producer burden. [$t=5$, $T=5$, consumer burden = 3 and producer burden = 2]

11. Let the cost function be $C = C(Q) = Q^2 + 30Q + 200$ and the producer sell this goods for \$ 60 per unit, determine the following .

- a) Break-even quantity
- b) Quantity level which the producer would maximize profit
- c) Maximum profit

12. Let total cost function be $TC = 3Q^2 - 5Q + 75$

- a) Shut down point of the company
- b) If the products from this company are sold in perfect competitive market, determine supply function in short-run of this company.

13. a) Suppose the demand function of the monopoly is $P = 274 - Q^2$ and $MC = 4 + 3Q$, determine the consumer surplus at the production level that monopoly maximize profit. [CS= 486]

b) Suppose this monopoly can do first degree price discrimination, what is the total revenue in which monopoly can maximize profit? [TR=735]

14. Let $Y = X^3 - 18X^2 + 750X$, determine the following

- a) the value of X that cause inflection point.
- b) the value of X that cause the minimum of $\frac{dy}{dx}$

15. Let $TP = Q = 120L^2 - L^3$, determine the following

- a) value of L that maximize TP, AP, MP [$L=80$, $L=60$, $L=40$]
- b) At which level of labour(L) would be reasonable in economic sense. [$60 < L < 80$]

16. Let $TP = Q^3 - 18Q^2 + 480Q + 500$, determine the following

- a) Q which make TC, MC be extremum point and state whether it is maximum or minimum.
- b) Q at shut down point, assume the perfectly competitive market. [$Q < 9$]

17. Let $TC = 500 + 1000Q - 500Q^2 + \frac{2}{3}Q^3$, determine the following

a) AC and MC

b) Slope of MC

c) Value of Q when AC is lowest [Q=375]

18. Let demand equation be $P = \frac{80-Q}{4}; 0 \leq Q \leq 80$, determine Q that maximize TR and the maximum value of TR. [Q=40, TR=400]

19. Let $C = \frac{Q^2}{4} + 3Q + 400$ where C is total cost and Q is the quantity, determine Q which make AC be the lowest and determine the lowest value of AC. [Q=40, AC=23]