ExerciseIII (Solution)

1 $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$; u and v are basic variables (Pivot variables); w and y are free variables.

The general solution is $x = \begin{bmatrix} 2w - y \\ -w \\ w \\ y \end{bmatrix} = w \begin{bmatrix} 2 \\ -1 \\ 1 \\ 0 \end{bmatrix} + y \begin{bmatrix} -1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$; $r = 2$.

2. $U = \begin{bmatrix} 1 & 2 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$; u is the basic (pivot) variables
v is free. The general solution to $\underline{A}\underline{x} = 0$ is
$\underline{x} = \begin{bmatrix} -2v \\ v \end{bmatrix}$; $\underline{A}\underline{x} = \underline{b}$ is consistent if $b_1 = 0$, $b_3 - 4b$
and $b_4 = 0$; $\underline{x} = v\begin{bmatrix} -2 \\ 1 \end{bmatrix} + \begin{bmatrix} b_2 \\ 0 \end{bmatrix}$; r-

3 $\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} -2v - 3 \\ v \\ 2 \end{bmatrix} = v \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} -3 \\ 0 \\ 2 \end{bmatrix}$.

4 n

5 4 , pivots = 5, 2, 6 , NO.

6. $a = 4$ leads to a row exchange; $3b + 10a = 40$ leads to a singular matrix; $c = 0$ leads to a row exchange; $c = 3$ leads to a singular matrix.

7 a) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & v_3/v_2 & 1 & 0 \\ 0 & v_4/v_2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & v_1 & 0 & 0 \\ 0 & v_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ b) $\begin{bmatrix} 1 & -v_1/v_2 & 0 & 0 \\ 0 & 1/v_2 & 0 & 0 \\ 0 & -v_3/v_2 & 0 & 0 \\ 0 & -v_4/v_2 & 0 & 0 \end{bmatrix}$

8. $A_1^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$; $A_2^{-1} = \begin{bmatrix} \frac{3}{4} & \frac{1}{2} & \frac{1}{4} \\ \frac{1}{2} & 1 & \frac{1}{2} \\ \frac{1}{4} & \frac{1}{2} & \frac{3}{4} \end{bmatrix}$; $A_3^{-1} = \begin{bmatrix} 0 & -1 & 1 \\ -1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$.