

Assignment 3

1. Use the method of direct proof to show that: if y is an integer, $y^2 - y$ is an even number.
Hint: Use method of proof by cases.
2. Show that the equality $\lfloor x - y \rfloor = \lfloor x \rfloor - \lfloor y \rfloor$ is **not** valid for all real numbers x and y .
Hint: Give a counterexample.
3. Use the method of constructive proof to show that:
if r and s are two real numbers with $r < s$ then there exists a real number x such that $r < x < s$.
4. (Optional) Prove by contradiction that the difference of any rational number and any irrational number is irrational.
5. For each positive integer n let $P(n)$ be the proposition $4^n - 1$ is divisible by 3.
 - (a) Write $P(1)$: Is $P(1)$ true?
 - (b) Write $P(k)$:
 - (c) Write $P(k + 1)$:
 - (d) In a proof by mathematical induction that this divisibility property holds for all integers $n \geq 1$; what must be shown in the induction step?
6. Use mathematical induction proof to show that

$$2^n < n!$$

for all positive integer $n > 3$.

7. (Optional) A sequence a_1, a_2, \dots is defined recursively by

$$a_1 = 3, \quad a_i = 7a_{i-1} \quad \text{for } i \geq 2.$$

Show that

$$a_n = 3 \cdot 7^{n-1} \quad \text{for } n \geq 1.$$