Quiz 3

1. Use **mathematical induction** to prove that $n^2 - 7n + 12$ is nonnegative whenever n is an integer with $n \geq 3$.

Answer:
Proof by mathematical induction:

Let $P(n)$ be the statement "$n^2 - 7n + 12$ is nonnegative."
We want to prove that $P(n)$ is true for all integer $n \geq 3$.

(I) **Basis step:** Show that $P(3)$ is true.
$P(3)$: $3^2 - 7(3) + 12$ is nonnegative.
Since $3^2 - 7(3) + 12 = 9 - 21 + 12 = 0$, $3^2 - 7(3) + 12$ is nonnegative and $P(3)$ is true.

(II)**Inductive step:** Show that if $P(k)$ is true, then $P(k+1)$ is also true, for any integer $k \geq 3$.

Assume that $P(k)$: "$k^2 - 7k + 12$ is nonnegative." ———($\bigstar$) "inductive hypothesis"
We want to show that $P(k+1)$:"$(k+1)^2 - 7(k+1) + 12$ is nonnegative." Consider

$$\begin{aligned}
(k+1)^2 - 7(k+1) + 12 &= \underbrace{(k^2 + 2k + 1)}_{\geq 0} - 7(k+1) + 12 \\
&= k^2 - 7k + 12 + (2k + 1 - 7) \\
&\geq 0 + (2k + 1 - 7) \quad \text{by } (\bigstar) \text{ "inductive hypothesis : } k^2 - 7k + 12 \geq 0 \text{ "} \\
&= 2k - 6 \\
&= 2(k - 3) \\
&\geq 0 \qquad \text{since } k \geq 3, \text{ then } k - 3 \geq 0 \text{ and } 2(k-3) \geq 0 .
\end{aligned}$$

Therefore $(k+1)^2 - 7(k+1) + 12$ is nonnegative and $P(k+1)$ is true.

Hence, from (I) basis step and (II) inductive step, $P(n)$ is true for all $n \geq 3$ by mathematical induction proof. ■

Remark: It is possible to use different approaches in the inductive step (II).