Financial Management Time Value of Money

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Time Value of Money

- Interest rate
- Future value
- Present value
- Annuities
- Loan

The Role of Time Value in Finance

- Most financial decisions involve costs & benefits that are spread out over time.
- Question: Your father has offered to give you some money and asks that you choose one of the following two alternatives:
  - $1,000 today, or
  - $1,000 one year from now.
- What do you do?

The Role of Time Value in Finance (cont.)

- The answer depends on what ***rate of interest*** you could earn on any money you receive today.
- For example;
  - if you could deposit the $1,000 today at 12% per year, you would prefer to be paid today.
- Alternatively, if you could only earn 5% on deposited funds, you would be better off if you chose the $1,100 in one year.

Basic Concepts

- **Future Value**: compounding or growth over time
- **Present Value**: discounting to today's value
- **Single** cash flows & **series** of cash flows can be considered
- **Time lines** are used to illustrate these relationships

Simple Interest

- Simple interest is interest that is earned on only the principal.
- The dollar amount of simple interest is a function of three variables:
  - The original principal
  - The interest rate per time period
  - The number of time periods for which the principal is borrowed

$$SI = P_0(i)(n)$$

SI = simple interest in dollars

P_0 = principal at time period 0

i = interest rate per time period

n = number of time periods

Simple Interest

- Assume that you deposit \$100 in a savings account paying 8 percent **simple interest** for 10 years.
- At the end of 10 years, the amount of interest accumulated is determined as follows:

$$\$80=\$100*0.08*10$$

- The **future value** or **terminal value** of the account at the end of 10 years is;

$$FV10 = \$100 + [\$100(0.08)(10)] = \mathbf{\$180}$$

- For any simple interest rate, the future value of an account at the end of n periods is

$$FVn = P0 + SI = P0 + P0(i)(n)$$

- Equivalently,

$$FVn = P0[1 + (i)(n)]$$

Compound Interest

- The notion of **compound interest** is crucial to understanding the mathematics of finance.
- The term itself merely implies that interest paid on a loan is periodically added to the principal.
- As a result, **interest is earned on interest** as well as the initial principal.
- It is this interest-on-interest, or *compounding*, effect that accounts for the dramatic difference between simple and compound interest.

$$FV_n = P_0(1 + i)^n$$

Compound Interest

- Consider a person who deposits $100 into a savings account. If the interest rate is 8 percent, compounded annually, how much will the $100 be worth at the end of a year?

Compound Interest

- Consider a person who deposits $100 into a savings account. If the interest rate is 8 percent, compounded annually, how much will the $100 be worth at the end of two year and at the end of three year?

Concept Check

- Consider a person who deposits $100 into a savings account.
  - If the simple interest rate is 5 percent, how much will the $100 be worth at the end of 1 year to 5 years?
  - If the interest rate is 5 percent, compounded annually, how much will the $100 be worth at the end of 1 year to 5 years?

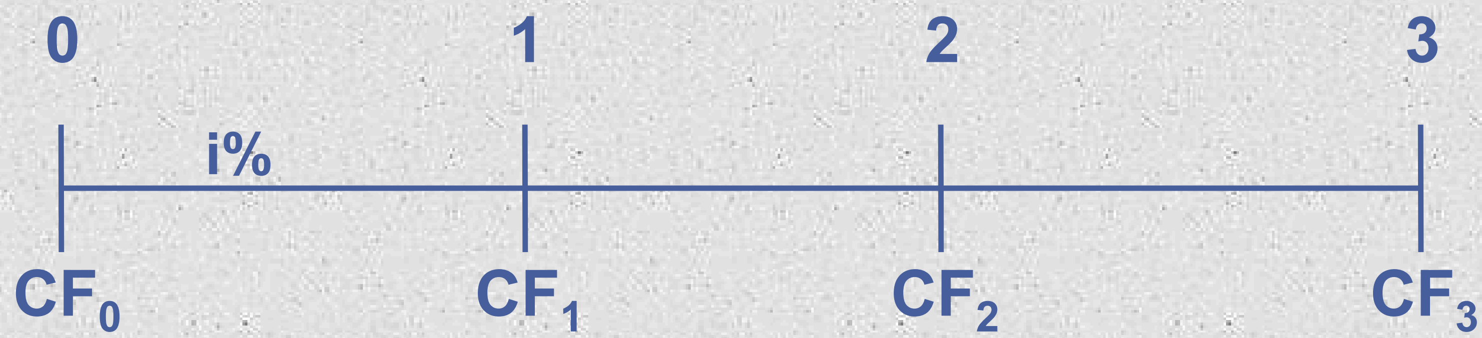
Compute the future value of $1,900 continuously compounded for

1. 5 years at a stated annual interest rate of 12 percent.
2. 3 years at a stated annual interest rate of 10 percent.
3. 10 years at a stated annual interest rate of 5 percent.
4. 8 years at a stated annual interest rate of 7 percent.

Computational Aids

- Use the Equations
- Use the Financial Tables
- Use Financial Calculators/ Spreadsheets

Time lines

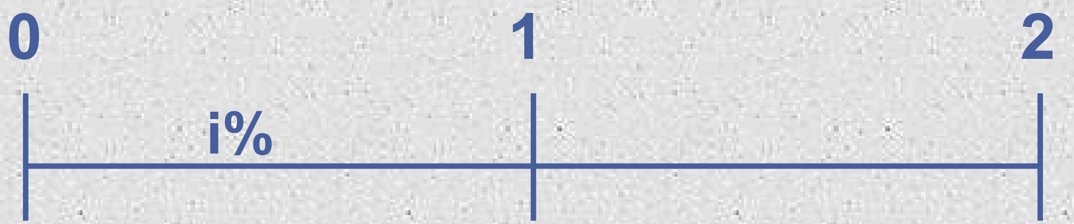


- Show the timing of cash flows.
- Tick marks occur at the end of periods
  - Time 0 is today;
  - Time 1 is the end of the first period (year, month, etc.) or the beginning of the second period.

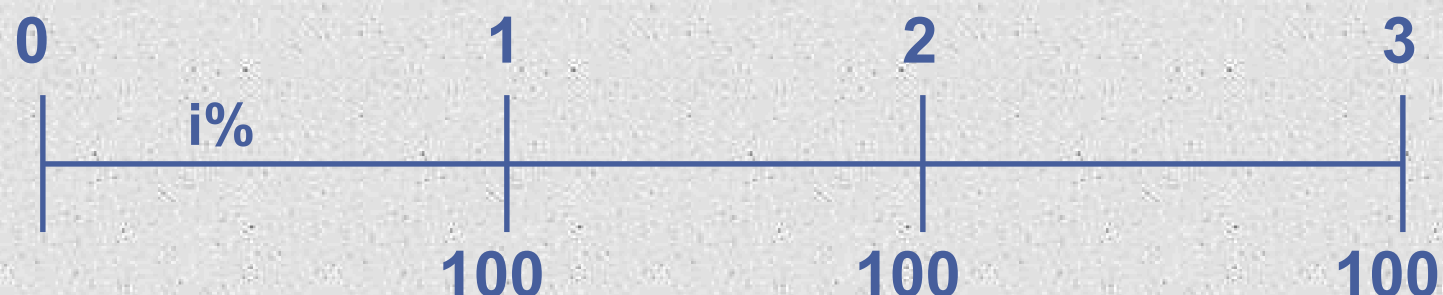
Time lines

Draw time lines: $100 lump sum due in 2 years, and 3-year $100 ordinary annuity

$100 lump sum due in 2 years



3 year $100 ordinary annuity



Time lines

Draw time lines:
Uneven cash flow stream;
CF_0 = -\$50, CF_1 = \$100, CF_2 = \$75, and CF_3 = \$50

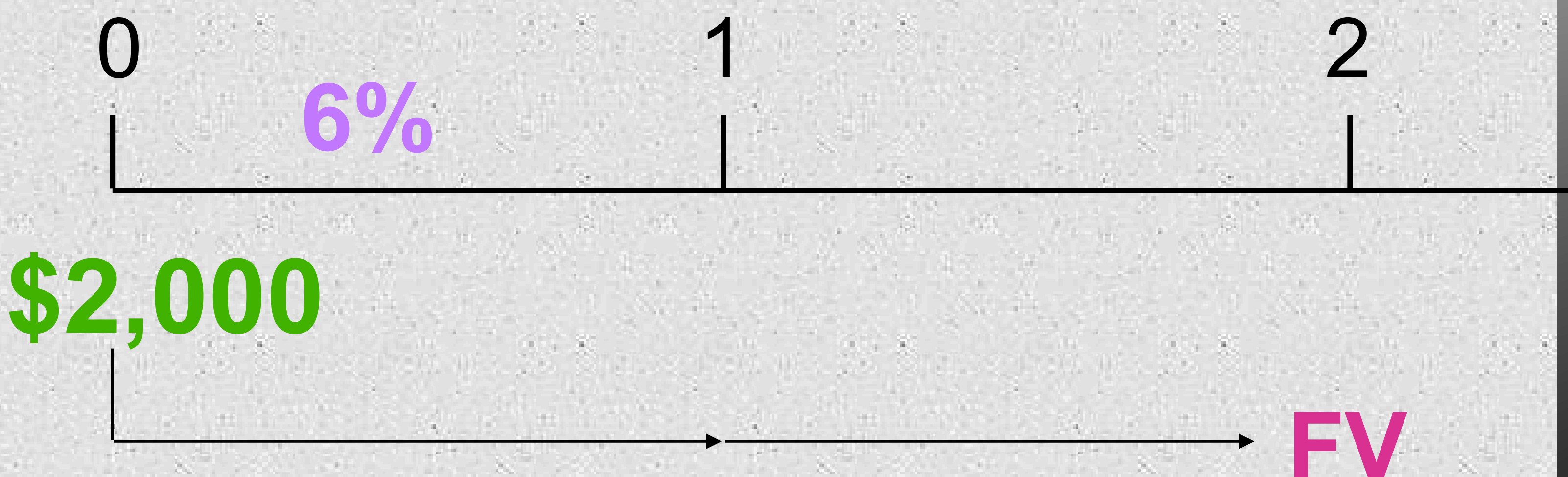
Uneven cash flow stream

Future Value and Compounding

- Future value refers to the amount of money that an investment will grow to over ***some length of time*** at ***some given interest rate***
- To determine the future value of a single cash flows, we need:
    - Present value of the cash flow (PV)
    - Interest rate (r), and
    - Time period (n)
- $FV_n = PV_0 \times (1 + r)^n$
- Future Value Interest Factor at ‘r’ rate of interest for ‘n’ time periods
- Examples on computation of future value of a single cash flow

Future Value: Example1

If you invested $2,000 today in an account that pays 6% interest, with interest compounded annually, how much will be in the account at the end of two years if there are no withdrawals?



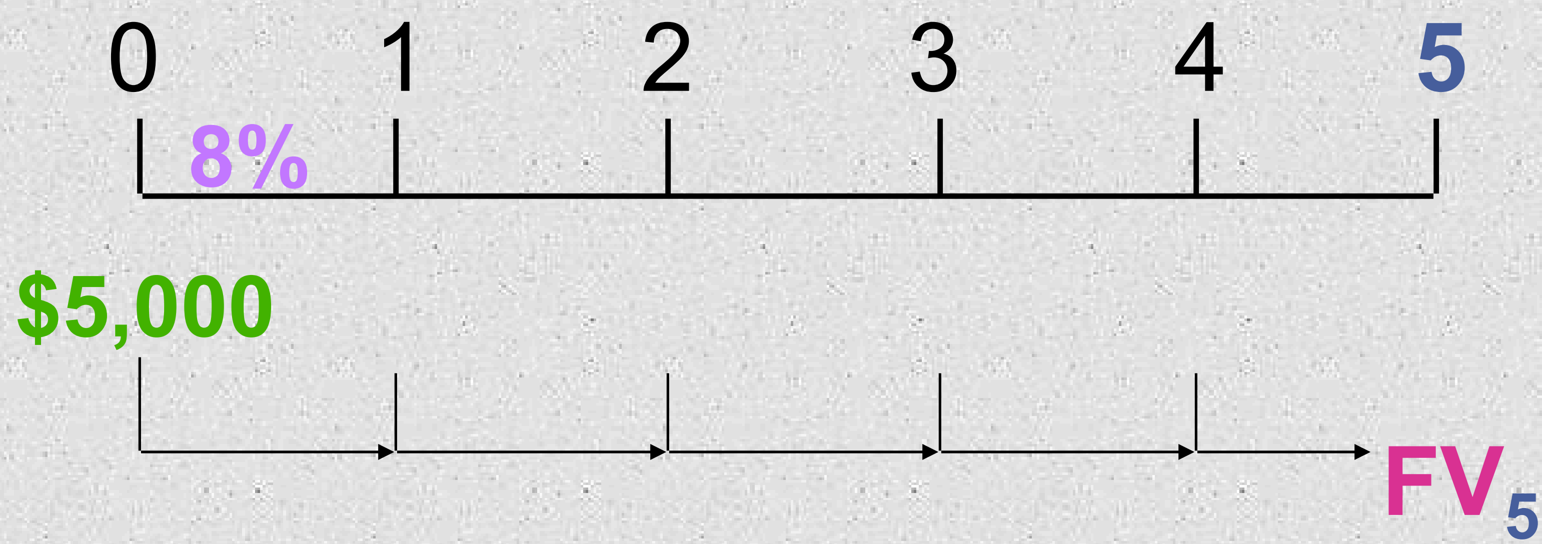
Future Value: Example1

$$FV_1 = PV\,(1+r)^n$$
$$= \$2{,}000\,(1.06)^2$$
$$= \$2{,}247.20$$

FV = future value, a value at some future point in time
PV = present value, a value today which is usually designated as time 0
r = rate of interest per compounding period
n = number of compounding periods

Future Value: Example2

- John wants to know how large his $5,000 deposit will become at an annual compound interest rate of 8% at the end of 5 years.



Use the Financial Tables

Period	Interest Rate							
	1%	2%	···	10%	···	20%	···	50%
1			···		···	⋮	···	
2			···		···	⋮	···	
3			···		···	⋮	···	
⋮	⋮	⋮	···	⋮	···	⋮	···	⋮
→ **10**	···	···	···	···	···	**X.XXX**	···	···
⋮	⋮	⋮	···	⋮	···	⋮	···	⋮
20			···		···		···	
⋮	⋮	⋮	···	⋮	···	⋮	···	⋮
50			···		···		···	

Use the Financial Tables

Table A.3 Future Value of $1 at the End of T Periods = $(1 + r)^T$

	Interest Rate								
Period	1%	2%	3%	4%	5%	6%	7%	8%	9%
1	1.0100	1.0200	1.0300	1.0400	1.0500	1.0600	1.0700	1.0800	1.0900
2	1.0201	1.0404	1.0609	1.0816	1.1025	1.1236	1.1449	1.1664	1.1881
3	1.0303	1.0612	1.0927	1.1249	1.1576	1.1910	1.2250	1.2597	1.2950
4	1.0406	1.0824	1.1255	1.1699	1.2155	1.2625	1.3108	1.3605	1.4116
5	1.0510	1.1041	1.1593	1.2167	1.2763	1.3382	1.4026	1.4693	1.5386
6	1.0615	1.1262	1.1941	1.2653	1.3401	1.4185	1.5007	1.5869	1.6771
7	1.0721	1.1487	1.2299	1.3159	1.4071	1.5036	1.6058	1.7138	1.8280
8	1.0829	1.1717	1.2668	1.3686	1.4775	1.5938	1.7182	1.8509	1.9926
9	1.0937	1.1951	1.3048	1.4233	1.5513	1.6895	1.8385	1.9990	2.1719
10	1.1046	1.2190	1.3439	1.4802	1.6289	1.7908	1.9672	2.1589	2.3674
11	1.1157	1.2434	1.3842	1.5395	1.7103	1.8983	2.1049	2.3316	2.5804
12	1.1268	1.2682	1.4258	1.6010	1.7959	2.0122	2.2522	2.5182	2.8127
13	1.1381	1.2936	1.4685	1.6651	1.8856	2.1329	2.4098	2.7196	3.0658
14	1.1495	1.3195	1.5126	1.7317	1.9799	2.2609	2.5785	2.9372	3.3417
15	1.1610	1.3459	1.5580	1.8009	2.0789	2.3966	2.7590	3.1722	3.6425
16	1.1726	1.3728	1.6047	1.8730	2.1829	2.5404	2.9522	3.4259	3.9703
17	1.1843	1.4002	1.6528	1.9479	2.2920	2.6928	3.1588	3.7000	4.3276
18	1.1961	1.4282	1.7024	2.0258	2.4066	2.8543	3.3799	3.9960	4.7171
19	1.2081	1.4568	1.7535	2.1068	2.5270	3.0256	3.6165	4.3157	5.1417
20	1.2202	1.4859	1.8061	2.1911	2.6533	3.2071	3.8697	4.6610	5.6044
21	1.2324	1.5157	1.8603	2.2788	2.7860	3.3996	4.1406	5.0338	6.1088
22	1.2447	1.5460	1.9161	2.3699	2.9253	3.6035	4.4304	5.4365	6.6586
23	1.2572	1.5769	1.9736	2.4647	3.0715	3.8197	4.7405	5.8715	7.2579
24	1.2697	1.6084	2.0328	2.5633	3.2251	4.0489	5.0724	6.3412	7.9111
25	1.2824	1.6406	2.0938	2.6658	3.3864	4.2919	5.4274	6.8485	8.6231
30	1.3478	1.8114	2.4273	3.2434	4.3219	5.7435	7.6123	10.063	13.268
40	1.4889	2.2080	3.2620	4.8010	7.0400	10.286	14.974	21.725	31.409
50	1.6446	2.6916	4.3839	7.1067	11.467	18.420	29.457	46.902	74.358
60	1.8167	3.2810	5.8916	10.520	18.679	32.988	57.946	101.26	176.03

Table A3. P.976 Jordan B., Westerfield R., Ross S., Corporate Finance 10th edition

Use the Financial Tables

Period	10%	12%	14%	15%	16%	18%	20%	24%	28%	32%	36%
1	1.1000	1.1200	1.1400	1.1500	1.1600	1.1800	1.2000	1.2400	1.2800	1.3200	1.3600
2	1.2100	1.2544	1.2996	1.3225	1.3456	1.3924	1.4400	1.5376	1.6384	1.7424	1.8496
3	1.3310	1.4049	1.4815	1.5209	1.5609	1.6430	1.7280	1.9066	2.0972	2.3000	2.5155
4	1.4641	1.5735	1.6890	1.7490	1.8106	1.9388	2.0736	2.3642	2.6844	3.0360	3.4210
5	1.6105	1.7623	1.9254	2.0114	2.1003	2.2878	2.4883	2.9316	3.4360	4.0075	4.6526
6	1.7716	1.9738	2.1950	2.3131	2.4364	2.6996	2.9860	3.6352	4.3980	5.2899	6.3275
7	1.9487	2.2107	2.5023	2.6600	2.8262	3.1855	3.5832	4.5077	5.6295	6.9826	8.6054
8	2.1436	2.4760	2.8526	3.0590	3.2784	3.7589	4.2998	5.5895	7.2058	9.2170	11.703
9	2.3579	2.7731	3.2519	3.5179	3.8030	4.4355	5.1598	6.9310	9.2234	12.166	15.917
10	2.5937	3.1058	3.7072	4.0456	4.4114	5.2338	6.1917	8.5944	11.806	16.060	21.647
11	2.8531	3.4785	4.2262	4.6524	5.1173	6.1759	7.4301	10.657	15.112	21.199	29.439
12	3.1384	3.8960	4.8179	5.3503	5.9360	7.2876	8.9161	13.215	19.343	27.983	40.037
13	3.4523	4.3635	5.4924	6.1528	6.8858	8.5994	10.699	16.386	24.759	36.937	54.451
14	3.7975	4.8871	6.2613	7.0757	7.9875	10.147	12.839	20.319	31.691	48.757	74.053
15	4.1772	5.4736	7.1379	8.1371	9.2655	11.974	15.407	25.196	40.565	64.359	100.71
16	4.5950	6.1304	8.1372	9.3576	10.748	14.129	18.488	31.243	51.923	84.954	136.97
17	5.0545	6.8660	9.2765	10.761	12.468	16.672	22.186	38.741	66.461	112.14	186.28
18	5.5599	7.6900	10.575	12.375	14.463	19.673	26.623	48.039	86.071	148.02	253.34
19	6.1159	8.6128	12.056	14.232	16.777	23.214	31.948	59.568	108.89	195.39	344.54
20	6.7275	9.6463	13.743	16.367	19.461	27.393	38.338	73.864	139.38	257.92	468.57
21	7.4002	10.804	15.668	18.822	22.574	32.324	46.005	91.592	178.41	340.45	637.26
22	8.1403	12.100	17.861	21.645	26.186	38.142	55.206	113.57	228.36	449.39	866.67
23	8.9543	13.552	20.362	24.891	30.376	45.008	66.247	140.83	292.30	593.20	1178.7
24	9.8497	15.179	23.212	28.625	35.236	53.109	79.497	174.63	374.14	783.02	1603.0
25	10.835	17.000	26.462	32.919	40.874	62.669	95.396	216.54	478.90	1033.6	2180.1
30	17.449	29.960	50.950	66.212	85.850	143.37	237.38	634.82	1645.5	4142.1	10143.
40	45.259	93.051	188.88	267.86	378.72	750.38	1469.8	5455.9	19427.	66521.	*
50	117.39	289.00	700.23	1083.7	1670.7	3927.4	9100.4	46890.	*	*	*
60	304.48	897.60	2595.9	4384.0	7370.2	20555.	56348.	*	*	*	*

Table A3. P.976 Jordan B., Westerfield R., Ross S., Corporate Finance 10th edition

Future Value of a Single Amount: Using FVIF Tables

- If Fred Moreno places $100 in a savings account paying 12% interest compounded annually, how much will he have in the account at the end of one year?

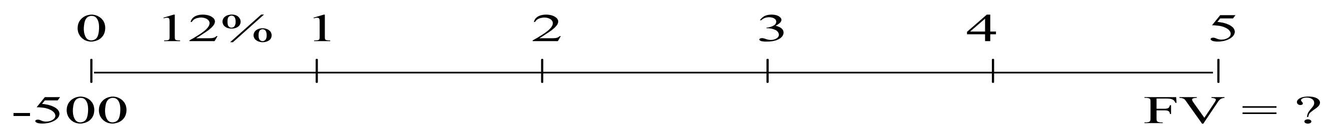
$\$100 \times (1.12)^1$	$= \$100 \times FVIF_{12\%,1}$
$\$100 \times 1.12$	$= \$112$

Future Value of a Single Amount: The Equation for Future Value

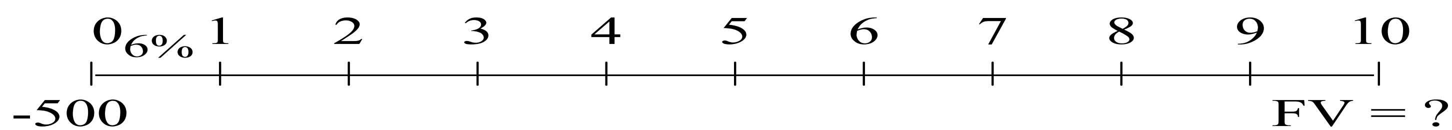
- Jane Farber places $800 in a savings account paying 6% interest compounded annually.
- She wants to know how much money will be in the account at the end of five years.

Practice Question

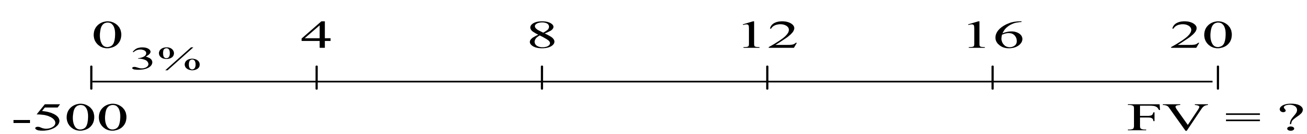
a.



b.



c.



d.

Practice Question

- The following cash-flow streams need to be analyzed:

CASH-FLOW STREAM	END OF YEAR				
	1	2	3	4	5
W	$100	$200	$200	$300	$300
X	600	–	–	–	–
Y	–	–	–	–	1,200
Z	200	–	500	–	300

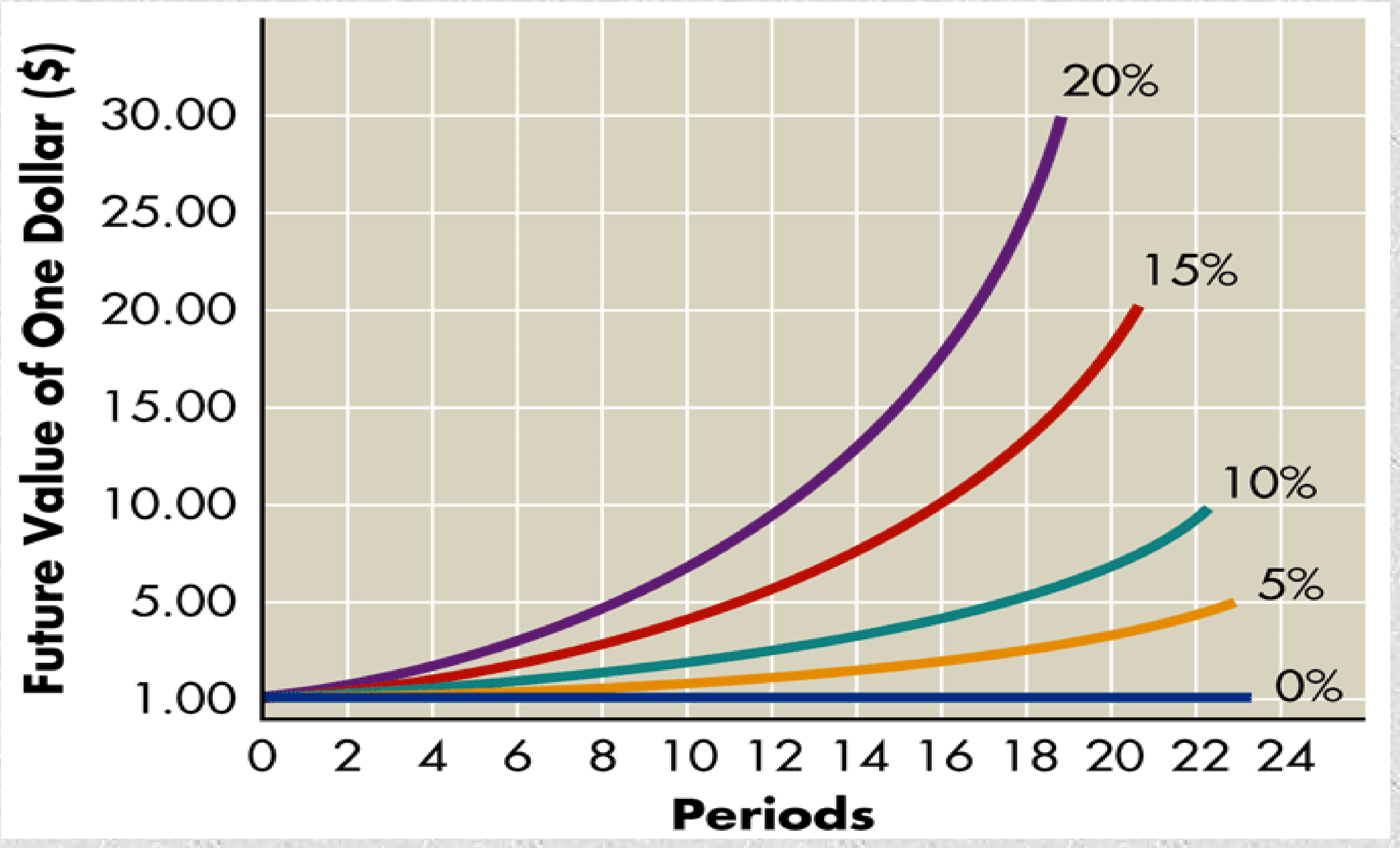
- Calculate the future (terminal) value of each stream at the end of year 5 with a compound annual interest rate of 10 percent.

Practice Question

The following are exercises in future (terminal) values:

- **a.** At the end of three years, how much is an initial deposit of $100 worth, assuming a compound annual interest rate of (i) 100 percent? (ii) 10 percent? (iii) 0 percent?
- **b**. At the end of 10 years, how much is a $100 initial deposit worth, assuming an annual interest rate of 10 percent compounded (i) annually? (ii) semiannually? (iii) quarterly? (iv) continuously?

Future Value of a Single Amount: A Graphical View of Future Value

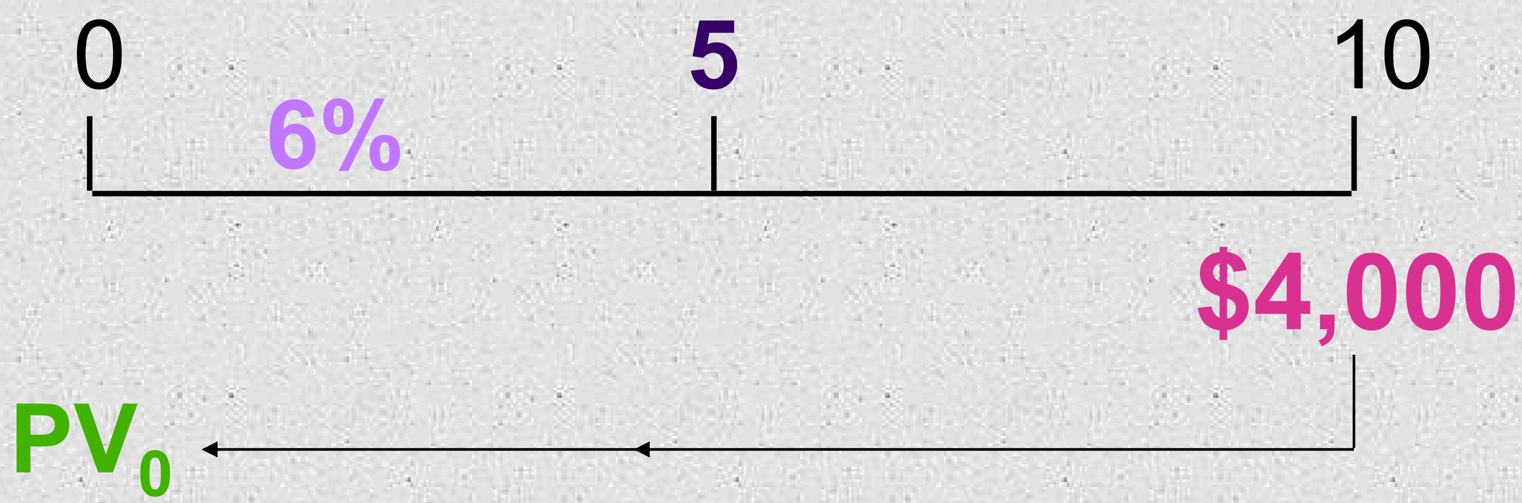


Present Value and Discounting

- The current value of future cash flows discounted at the appropriate discount rate over some length of time period
- Discounting is the process of translating a future value or a set of future cash flows into a present value.
- To compute present value of a single cash flow, we need:
  - Future value of the cash flow (FV)
  - Interest rate (r) and
  - Time Period (n)
- $PV_0 = FV_n / (1 + r)^n$
- PVIF (r,n)

Present Value and Discounting

Assume that you need to have exactly $4,000 saved 10 years from now. How much must you deposit today in an account that pays 6% interest, compounded annually, so that you reach your goal of $4,000?

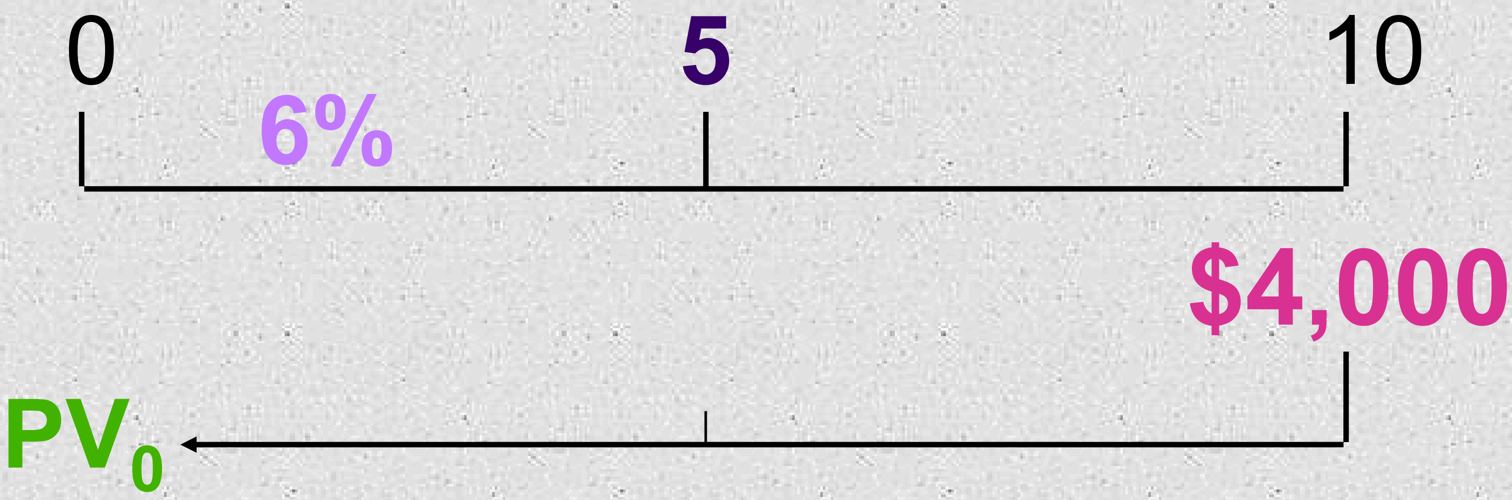


Present Value (Formula)

$$PV_0 = FV / (1+r)^{10}$$

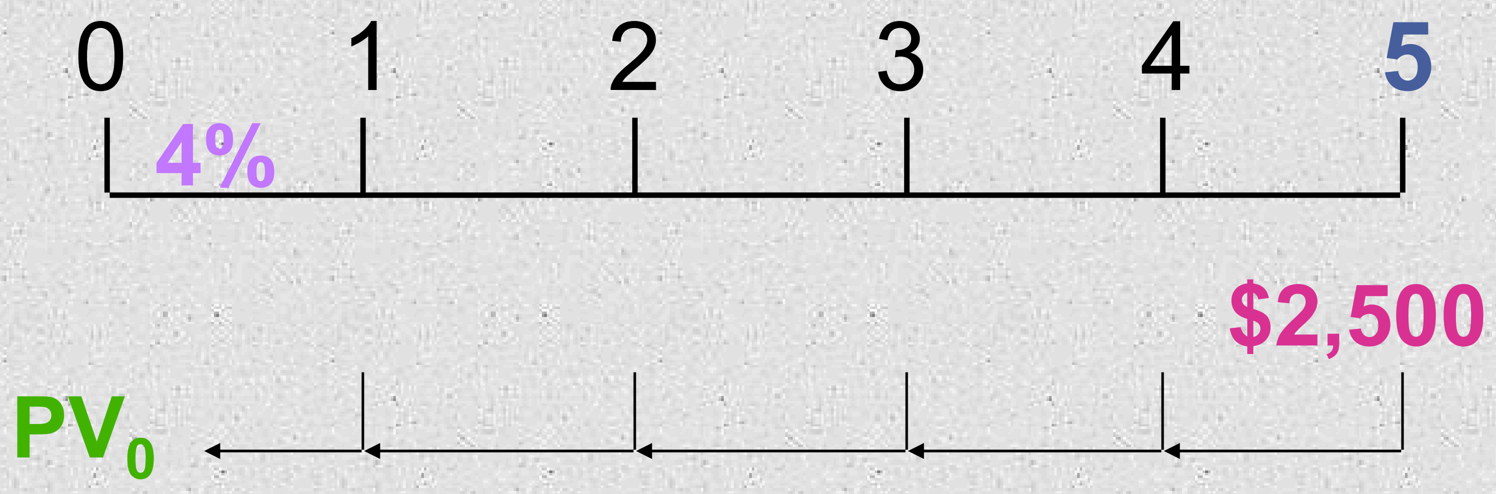
$$= \$4{,}000 / (1.06)^{10}$$

$$= \$2{,}233.58$$



Present Value Example

Joann needs to know how large of a deposit to make today so that the money will grow to $2,500 in 5 years. Assume today's deposit will grow at a compound rate of 4% annually.



Present Value Solution

- Calculation based on general formula:

$$PV_0 = FV_n / (1+r)^n$$

$$PV_0 = \$2{,}500/(1.04)^5$$

$$= \$2{,}054.81$$

Use the Financial Tables

Table A.1 Present Value of \$1 to Be Received after T Periods $= 1/(1+r)^T$

	Interest Rate								
Period	1%	2%	3%	4%	5%	6%	7%	8%	9%
1	.9901	.9804	.9709	.9615	.9524	.9434	.9346	.9259	.9174
2	.9803	.9612	.9426	.9246	.9070	.8900	.8734	.8573	.8417
3	.9706	.9423	.9151	.8890	.8638	.8396	.8163	.7938	.7722
4	.9610	.9238	.8885	.8548	.8227	.7921	.7629	.7350	.7084
5	.9515	.9057	.8626	.8219	.7835	.7473	.7130	.6806	.6499
6	.9420	.8880	.8375	.7903	.7462	.7050	.6663	.6302	.5963
7	.9327	.8706	.8131	.7599	.7107	.6651	.6227	.5835	.5470
8	.9235	.8535	.7894	.7307	.6768	.6274	.5820	.5403	.5019
9	.9143	.8368	.7664	.7026	.6446	.5919	.5439	.5002	.4604
10	.9053	.8203	.7441	.6756	.6139	.5584	.5083	.4632	.4224
11	.8963	.8043	.7224	.6496	.5847	.5268	.4751	.4289	.3875
12	.8874	.7885	.7014	.6246	.5568	.4970	.4440	.3971	.3555
13	.8787	.7730	.6810	.6006	.5303	.4688	.4150	.3677	.3262
14	.8700	.7579	.6611	.5775	.5051	.4423	.3878	.3405	.2992
15	.8613	.7430	.6419	.5553	.4810	.4173	.3624	.3152	.2745
16	.8528	.7284	.6232	.5339	.4581	.3936	.3387	.2919	.2519
17	.8444	.7142	.6050	.5134	.4363	.3714	.3166	.2703	.2311
18	.8360	.7002	.5874	.4936	.4155	.3503	.2959	.2502	.2120
19	.8277	.6864	.5703	.4746	.3957	.3305	.2765	.2317	.1945
20	.8195	.6730	.5537	.4564	.3769	.3118	.2584	.2145	.1784
21	.8114	.6598	.5375	.4388	.3589	.2942	.2415	.1987	.1637
22	.8034	.6468	.5219	.4220	.3418	.2775	.2257	.1839	.1502
23	.7954	.6342	.5067	.4057	.3256	.2618	.2109	.1703	.1378
24	.7876	.6217	.4919	.3901	.3101	.2470	.1971	.1577	.1264
25	.7798	.6095	.4776	.3751	.2953	.2330	.1842	.1460	.1160
30	.7419	.5521	.4120	.3083	.2314	.1741	.1314	.0994	.0754
40	.6717	.4529	.3066	.2083	.1420	.0972	.0668	.0460	.0318
50	.6080	.3715	.2281	.1407	.0872	.0543	.0339	.0213	.0134

Table A1. P.974 Jordan B., Westerfield R., Ross S., Corporate Finance 10th edition

Use the Financial Tables

Period	10%	12%	14%	15%	16%	18%	20%	24%	28%	32%	36%
1	.9091	.8929	.8772	.8696	.8621	.8475	.8333	.8065	.7813	.7576	.7353
2	.8264	.7972	.7695	.7561	.7432	.7182	.6944	.6504	.6104	.5739	.5407
3	.7513	.7118	.6750	.6575	.6407	.6086	.5787	.5245	.4768	.4348	.3975
4	.6830	.6355	.5921	.5718	.5523	.5158	.4823	.4230	.3725	.3294	.2923
5	.6209	.5674	.5194	.4972	.4761	.4371	.4019	.3411	.2910	.2495	.2149
6	.5645	.5066	.4556	.4323	.4104	.3704	.3349	.2751	.2274	.1890	.1580
7	.5132	.4523	.3996	.3759	.3538	.3139	.2791	.2218	.1776	.1432	.1162
8	.4665	.4039	.3506	.3269	.3050	.2660	.2326	.1789	.1388	.1085	.0854
9	.4241	.3606	.3075	.2843	.2630	.2255	.1938	.1443	.1084	.0822	.0628
10	.3855	.3220	.2697	.2472	.2267	.1911	.1615	.1164	.0847	.0623	.0462
11	.3505	.2875	.2366	.2149	.1954	.1619	.1346	.0938	.0662	.0472	.0340
12	.3186	.2567	.2076	.1869	.1685	.1372	.1122	.0757	.0517	.0357	.0250
13	.2897	.2292	.1821	.1625	.1452	.1163	.0935	.0610	.0404	.0271	.0184
14	.2633	.2046	.1597	.1413	.1252	.0985	.0779	.0492	.0316	.0205	.0135
15	.2394	.1827	.1401	.1229	.1079	.0835	.0649	.0397	.0247	.0155	.0099
16	.2176	.1631	.1229	.1069	.0930	.0708	.0541	.0320	.0193	.0118	.0073
17	.1978	.1456	.1078	.0929	.0802	.0600	.0451	.0258	.0150	.0089	.0054
18	.1799	.1300	.0946	.0808	.0691	.0508	.0376	.0208	.0118	.0068	.0039
19	.1635	.1161	.0829	.0703	.0596	.0431	.0313	.0168	.0092	.0051	.0029
20	.1486	.1037	.0728	.0611	.0514	.0365	.0261	.0135	.0072	.0039	.0021
21	.1351	.0926	.0638	.0531	.0443	.0309	.0217	.0109	.0056	.0029	.0016
22	.1228	.0826	.0560	.0462	.0382	.0262	.0181	.0088	.0044	.0022	.0012
23	.1117	.0738	.0491	.0402	.0329	.0222	.0151	.0071	.0034	.0017	.0008
24	.1015	.0659	.0431	.0349	.0284	.0188	.0126	.0057	.0027	.0013	.0006
25	.0923	.0588	.0378	.0304	.0245	.0160	.0105	.0046	.0021	.0010	.0005
30	.0573	.0334	.0196	.0151	.0116	.0070	.0042	.0016	.0006	.0002	.0001
40	.0221	.0107	.0053	.0037	.0026	.0013	.0007	.0002	.0001	*	*
50	.0085	.0035	.0014	.0009	.0006	.0003	.0001	*	*	*	*

Table A1. P.974 Jordan B., Westerfield R., Ross S., Corporate Finance 10th edition

Present Value versus Future Value

- Present value factors are reciprocals of future value factors
- Interest rates and future value are positively related
- Interest rates and present value are negatively related
- Time period and future value are positively related
- Time period and present value are negatively related

Practice Question2

Sales of the P.J. Cramer Company were $500,000 this year, and they are expected to grow at a compound rate of 20 percent for the next six years.

What will be the sales figure at the end of each of the next six years?

Unknown Interest

- Sometimes we are faced with a time-value-of money situation in which we know both the future, present values, and number of time periods involved.
- What is unknown is the compound interest rate (i) implicit in the situation.
- Assume that, if you invest $1,000 today, you will receive $3,000 in exactly 8 years.
- The compound interest rate implicit in this situation can be found by rearranging either a basic future value or present value equation.

$$FV_8 = P_0(FVIF_{i,8})$$

$$\$3{,}000 = \$1{,}000(FVIF_{i,8})$$

$$FVIF_{i,8} = \$3{,}000/\$1{,}000 = \mathbf{3}$$

Unknown Interest

- In connection with the United States Bicentennial, the Treasury once contemplated offering a savings bond for $1,000 that would be worth $1 million in 100 years. Approximately what compound annual interest rate is implied by these terms?

Practice Question 3

- In connection with the United States Bicentennial, the Treasury once contemplated offering a savings bond for $1,000 that would be worth $1 million in 100 years.
- Approximately what compound annual interest rate is implied by these terms?

Unknown Number of Compounding

- At times we may need to **know "how long it will take for a dollar amount invested today to grow to a certain future value"** given a particular compound rate of interest.
- For example, how long would it take for an investment of $1,000 to grow to $1,900 if we invested it at a compound annual interest rate of 10 percent?

Unknown Number of Year

Present Value	Years	Interest Rate	Future Value
$ 625		6%	$ 1,284
810		13	4,341
18,400		32	402,662
21,500		16	173,439

- Find the number of year for each CF

Compounding More Than Once a Year

- At times we may need to know how long it will take for a dollar today to grow to a certain future value given a particular compound rate of interest.
- For example, how long would it take for an investment of $1,000 to grow to $1,900 if we invested it at a compound annual interest rate of 10 percent?

$$FVn = P0(FVIF10\%,n)$$
$$\$1{,}900 = \$1{,}000(FVIF10\%,n)$$
$$FVIF10\%,n = \$1{,}900/\$1{,}000 = \mathbf{1.9}$$

Classifications of interest rates

- Nominal rate (i_{NOM}) – also called the quoted or state rate.
  - An annual rate that ignores compounding effects.
  - i_{NOM} is stated in contracts.
  - Periods must also be given, e.g. 8% Quarterly, Monthly or Daily interest.
- Periodic rate (i_{PER}) – amount of interest charged each period, e.g. monthly or quarterly.
  - $i_{PER} = i_{NOM} / m$
  - where m is the number of compounding periods per year.
  - m = 4 for quarterly and m = 12 for monthly compounding.

Classifications of interest rates

- Effective annual rate (EAR = EFF%) is the actual annual rate of interest that we earn, taking into account compounding.
  - EFF% for 10% semiannual investment

    EFF% $= (1 + i_{NOM} / m)^m – 1$

  For example, EAR of 10% semiannually is

    EFF% $= (1 + 0.10 / 2)^2 – 1 = 10.25\%$
  - An investor would be indifferent between an investment offering a 10.25% annual return and one offering a 10% annual return, compounded semiannually.

Concept Check

- As you increase the length of time involved, what happens to future values? What happens to present values assuming the interest is compounded?

Why is it important to consider effective rates of return?

- An investment with monthly payments is different from one with quarterly payments.
- Must put each return on an EFF% basis to compare rates of return. Must use EFF% for comparisons.
- See following values of EFF% rates at various compounding levels.

EAR_{ANNUAL}	10.00%
$EAR_{QUARTERLY}$	10.38%
$EAR_{MONTHLY}$	10.47%
$EAR_{DAILY\ (365)}$	10.52%

Can the effective rate ever be equal to the nominal rate?

- Yes, but only if annual compounding is used, i.e., if $m = 1$.
- If $m > 1$, EFF% will always be greater than the nominal rate.

When is each rate used?

- i_{NOM} written into contracts, quoted by banks and brokers. Not used in calculations or shown on time lines.
- i_{PER} Used in calculations and shown on time lines.
  If $m = 1$, $i_{NOM} = i_{PER} = EAR$.
- EAR Used to compare returns on investments with different payments per year. Used in calculations when annuity payments don't match compounding periods.

What is the FV of \$100 after 3 years under 10% semiannual compounding? Quarterly compounding?

$$FV_n = PV\left(1 + \frac{i_{NOM}}{m}\right)^{m \times n}$$

$$FV_{3S} = \$100\left(1 + \frac{0.10}{2}\right)^{2 \times 3}$$

$$FV_{3S} = \$100\,(1.05)^6 = \$134.01$$

$$FV_{3Q} = \$100\,(1.025)^{12} = \$134.49$$

Practice Question5

- Suppose that an investment promises to pay a nominal 9.6 percent annual rate of interest.
- What is the effective annual interest rate on this investment assuming that interest is compounded
  - (a) annually?
  - (b) semiannually?
  - (c) quarterly?
  - (d) monthly?
  - (e) daily (365 days)?
  - (f) continuously?

Time Value of Money

- Interest rate
- Future value
- Present value
- Annuities
- Perpetuity
- Growing Annuity
- Loan

Annuities and Perpetuities Defined

- Annuity – finite series of equal payments that occur at regular intervals
  - If the first payment occurs at the end of the period, it is called an ordinary annuity
  - If the first payment occurs at the beginning of the period, it is called an annuity due
- Perpetuity – infinite series of equal payments

Annuities and Perpetuities

- Perpetuity: PV = C / r
- Annuities:

$$PV = C\left[\frac{1 - \frac{1}{(1+r)^t}}{r}\right]$$

$$FV = C\left[\frac{(1+r)^t - 1}{r}\right]$$

Annuities

- A series of level/even/equal sized cash flows that occur at the end of each time period for a fixed time period
- Examples of Annuities:
    - Car Loans
    - House Mortgages
    - Insurance Policies
    - Some Lotteries
    - Retirement Money

Practice: Annuity

- You want to buy a new sports car from Muscle Motors for \$65,000. The contract is in the form of a 48-month annuity due at a 6.45 percent APR. What will your monthly payment be?

Practice: Annuity

- Suppose you had \$275,000 in your bank account and you invested it at 8.25% per year.
- How much could you withdraw at the <u>beginning</u> of each of the next 20 years?

Practice: Annuity

- Your grandmother just died and left you $100,000 in a trust fund that pays 6.5% interest.
- You must spend the money on your college education, and you must withdraw the money in 4 equal installments, beginning immediately.
- How much could you withdraw today and at the beginning of each of the next 3 years and end up with zero in the account?

Perpetuities

- A series of level/even/equal sized cash flows that occur at the end of each period for an infinite time period
- Examples of Perpetuities:
  - Consoles issued by British Government
  - Preferred Stock
  - Present Value of a Perpetuity

Practice Question

- An investor purchasing a British diplomat is entitled to receive annual payments from the British government forever.
- What is the price of a diplomat that pays $120 annually if **the next payment occurs one year from today**?
- The market interest rate is 5.7 percent.

Practice Question

- An investor purchasing a British diplomat is entitled to receive annual payments from the British government forever.
- What is the price of a diplomat that pays $120 annually if **the next payment occurs one year from today**?
- The market interest rate is 5.7 percent.

$$PV = C / r$$
$$PV = \$120 / .057$$
$$PV = \$2,105.26$$

Practice: Perpetuity

- What annual payment must you receive in order to earn a 6.5% rate of return on a perpetuity that has a cost of $1,250?

Growing Annuity

A growing stream of cash flows with a fixed maturity

$$PV = \frac{C}{(1+r)} + \frac{C \times (1+g)}{(1+r)^2} + \cdots + \frac{C \times (1+g)^{t-1}}{(1+r)^t}$$

$$PV = \frac{C}{r-g}\left[1 - \left(\frac{(1+g)}{(1+r)}\right)^t\right]$$

Growing Annuity: Example

- A defined-benefit retirement plan offers to pay $20,000 per year for 40 years and increase the annual payment by three-percent each year.
- What is the present value at retirement if the discount rate is 10 percent?

$$PV = \frac{\$20{,}000}{.10-.03}\left[1-\left(\frac{1.03}{1.10}\right)^{40}\right] = \$265{,}121.57$$

Practice Question

- Calculating Annuity Present Value An investment offers \$4,300 per year for 15 years, with the first payment occurring one year from now.
- If the required return is 9 percent, what is the value of the investment?
- What would the value be if the payments occurred for 40 years?
- For 75 years?
- Forever?

Practice: Growing Annuity

- Tom Adams has received a job offer from a large investment bank as a clerk to an associate banker.
- His base salary will be $45,000. He will receive his first annual salary payment one year from the day he begins to work.
- In addition, he will get an immediate $10,000 bonus for joining the company.
- His salary will grow at 3.5 percent each year.
- Each year he will receive a bonus equal to 10 percent of his salary.
- Mr. Adams is expected to work for 25 years.
- What is the present value of the offer if the discount rate is 12 percent?

Growing Perpetuity

A growing stream of cash flows that lasts forever

$$PV = \frac{C}{(1+r)} + \frac{C \times (1+g)}{(1+r)^2} + \frac{C \times (1+g)^2}{(1+r)^3} + \cdots$$

$$PV = \frac{C}{r-g}$$

Example: Growing Perpetuity

The expected dividend next year is $1.30, and dividends are expected to grow at 5% forever.

If the discount rate is 10%, what is the value of this promised dividend stream?

$$PV = \frac{\$1.30}{.10 - .05} = \$26.00$$

Example I:

- Suppose you have just won the first prize in a lottery.
- The lottery offers you two possibilities for receiving your prize.
  - The first possibility is to receive a payment of \$10,000 at the end of the year, and then, for the next 15 years this payment will be repeated, but it will grow at a rate of 5%.
  - The second possibility is to receive \$100,000 right now.
  - The interest rate is 12% during the entire period.
  - Which of the two possibilities would you take?

Practice Question

- You are planning to save for retirement over the next 30 years.
- To do this, you will invest $700 a month in a stock account and $300 a month in a bond account.
- The return of the stock account is expected to be 10 percent, and the bond account will pay 6 percent.
- When you retire, you will combine your money into an account with an 8 percent return.
- How much can you withdraw each month from your account assuming a 25-year withdrawal period?

Loan Types and Loan Amortization

- A loan might be repaid in equal installments it might be repaid in a single lump sum.
- In this section, we describe a few forms of repayment that come up quite often, and more complicated forms can usually be built up from these.
- The three basic types of loans;
  - Pure discount loans
  - Interest-only loans
  - Amortized loans.
- Working with these loans is a very EASY and straightforward application of the present value principles that we have already developed.

PURE DISCOUNT LOANS

- The *pure discount loan* is the simplest form of loan.
- With such a loan, the borrower receives money today and repays a single lump sum at some time in the future.
- A one-year, 10 percent pure discount loan would require the borrower to repay $1.10 in one year
- Suppose a borrower was able to repay $25,000 in five years. If we, acting as the lender, and we wanted a 12% interest rate on the loan, how much would we be willing to lend?

PURE DISCOUNT LOANS

- When the U.S. government borrows money on a short-term basis (a year or less), it does so by selling what are called *Treasury bills,* or *T-bills* for short.
- A T-bill is a promise by the government to repay a fixed amount at some time in the future (3,6,12 months)
- *Treasury bills are pure discount loans*.
- If a T-bill promises to repay $10,000 in 12 months, and the market interest rate is 7 percent, how much will the bill sell for in the market?

INTEREST-ONLY LOANS

- A second type of loan repayment plan calls for the borrower to pay interest each period and to repay the entire principal at some point in the future.
- Loans with such a repayment plan are called *interest-only loans*.

  "if there is just one period, a pure discount loan and an interest-only loan are the same thing"

INTEREST-ONLY LOANS

- For example, with a three-year, 10 percent, interest-only loan of $1,000
- The borrower would pay
  $1,000 x .10 = $100 in interest at the end of the first and second years.
- At the end of the third year, the borrower would return the $1,000 along with another $100 in interest for that year.
- Most corporate bonds have the general form of an interest-only loan.

AMORTIZED LOANS

- *Amortized loan* which the lender may require the borrower to repay parts of the loan amount over time.
- The process of providing for a loan to be paid off by making regular principal reductions is called *amortizing* the loan.
- A simple way of amortizing a loan is to have the borrower pay the interest each period plus some fixed amount.

AMORTIZED LOANS

- This approach is common with medium-term business loans.
- For example, suppose a business takes out a \$5,000, five-year loan at 9 percent.
- The loan agreement calls for the borrower to pay the interest on the loan balance each year and to reduce the loan balance each year by \$1,000.
- Because the loan amount declines by \$1,000 each year, it is fully paid in five years.

AMORTIZED LOANS

- In the case we are considering, notice that the total payment will decline each year.
- The reason is that the loan balance goes down, resulting in a lower interest charge each year, whereas the $1,000 principal reduction is constant.
- For example, the interest in the first year will be $5,000 x .09 = $450.
- The total payment will be $1,000 + 450 = $1,450.
- In the second year, the loan balance is $4,000,
- So the interest is $4,000 x .09 = $360,and the total payment is $1,360.
- We can calculate the total payment in each of the remaining years by preparing a simple amortization schedule as follows:

AMORTIZED LOANS

- We can calculate the total payment in each of the remaining years by preparing a simple amortization schedule as follows:

Year	Beginning Balance	Total Payment	Interest Paid	Principal Paid	Ending Balance
1	$5,000	$1,450	$ 450	$1,000	$4,000
2	4,000	1,360	360	1,000	3,000
3	3,000	1,270	270	1,000	2,000
4	2,000	1,180	180	1,000	1,000
5	1,000	1,090	90	1,000	0
Totals		$6,350	$1,350	$5,000	

Amortized Question

- Your company has just taken out a 1-year installment loan for $72,500 at a nominal rate of 11.0% but with equal end-of-month payments.
- What percentage of the <u>2nd</u> monthly payment will go toward the repayment of principal?

Amortized Question

- Suppose you borrowed $15,000 at a rate of 8.5% and must repay it in 5 equal installments at the end of each of the next 5 years. By how much would you reduce the amount you owe in the first year?

Practice Question6

- The Happy Hang Glide Company is purchasing a building and has obtained a $190,000 mortgage loan for 20 years.
- The loan bears a compound annual interest rate of 17 percent and calls for equal annual installment payments at the end of each of the 20 years.
- What is the amount of the annual payment?

Practice Question7

- You need a 30-year, fixed-rate mortgage to buy a new home for $250,000.
- Your mortgage bank will lend you the money at a 6.8 percent APR for this 360-month loan.
- However, you can only afford monthly payments of $1,200, so you offer to pay off any remaining loan balance at the end of the loan in the form of a single balloon payment.
- How large will this balloon payment have to be for you to keep your monthly payments at $1,200?

Practice Question7

- On September 1, 2007, Susan Chao bought a motorcycle for \$25,000.
- She paid \$1,000 down and financed the balance with a five-year loan at a stated annual interest rate of 8.4 percent, compounded monthly.
- She started the monthly payments exactly one month after the purchase (i.e., October 1, 2007).
- Two years later, at the end of October 2009, Susan got a new job and decided to pay off the loan.
- If the bank charges her a 1 percent **prepayment penalty** based on the loan balance, how much must she pay the bank on November 1, 2009?

End of Chapter

Chapter end practice questions

Q1

- A local finance company quotes a 15 percent interest rate on one-year loans.
- So, if you borrow $26,000, the interest for the year will be $3,900.
- Because you must repay a total of $29,900 in one year, the finance company requires you to pay $29,900/12, or $2,491.67, per month over the next 12 months.
- Is this a 15 percent loan? What rate would legally have to be quoted? What is the effective annual rate?

Q2

- An insurance company is offering a new policy to its customers.
- Typically the policy is bought by a parent or grandparent for a child at the child's birth.
- The details of the policy are as follows: The purchaser (say, the parent) makes the following six payments to the insurance company:

First birthday:	$ 800
Second birthday:	$ 800
Third birthday:	$ 900
Fourth birthday:	$ 900
Fifth birthday:	$1,000
Sixth birthday:	$1,000

- After the child's sixth birthday, no more payments are made. When the child reaches age 65, **he or she receives $350,000**. If the relevant **interest rate is 11 percent** for the first six years and 7 percent for all subsequent years, is the policy worth buying?

Q3

- A financial planning service offers a college savings program.
- The plan calls for you to make six annual payments of \$8,000 each, with the first payment occurring today, your child‘s 12th birthday.
- Beginning on your child‘s 18th birthday, the plan will provide \$20,000 per year for four years.
- What return is this investment offering?

Q4

- Your Christmas ski vacation was great, but it unfortunately ran a bit over budget.
- All is not lost: You just received an offer in the mail to transfer your $9,000 balance from your current credit card, which charges an annual rate of 18.6 percent, to a new credit card charging a rate of 8.2 percent.
- How much faster could you pay the loan off by making your planned monthly payments of $200 with the new card?
- What if there was a 2 percent fee charged on any balances transferred?

Q5

1. What is the present value of an annuity of $5,000 per year, with the first cash flow received three years from today and the last one received 25 years from today? Use a discount rate of 8 percent.
2. What is the value today of a 15-year annuity that pays $750 a year? The annuity's first payment occurs six years from

Q6

- A **15-year annuity** pays $1,500 per month, and payments are made at the end of each month.
- If the interest rate is 13 percent compounded monthly for the **first seven years**, and 9 percent compounded monthly thereafter, what is the present value of the annuity?

Q7

- You are saving for the college education of your two children.
- They are two years apart in age; one will begin college 15 years from today and the other will begin 17 years from today.
- You estimate your children's college expenses to be $35,000 per year per child, payable at the beginning of each school year.
- The annual interest rate is 8.5 percent.
- How much money must you deposit in an account **each year** to fund your children's education?
- Your deposits begin one year from today.
- You will make your last deposit when your oldest child enters college. Assume four years of college.

Q8

- You have recently won the super jackpot in the Washington State Lottery.
- You have the following two options:
  - 1. You will receive 31 annual payments of $175,000, with the first payment being delivered today.
    - The income will be taxed at a rate of 28 percent.
    - Taxes will be withheld when the checks are issued.
  - 2. You will receive $530,000 now, and you will not have to pay taxes on this amount.
    - In addition, beginning one year from today, you will receive $125,000 each year for 30 years.
    - The cash flows from this annuity will be taxed at 28 percent.
- Using a discount rate of 10 percent, which option should you select?

Q9

- You need a 30-year, fixed-rate mortgage to buy a new home for $250,000.
- Your mortgage bank will lend you the money at a 6.8 percent APR for this 360-month loan.
- However, you can only afford monthly payments of $1,200, so you offer to pay off any remaining loan balance at the end of the loan in the form of a single balloon payment.
- How large will this balloon payment have to be for you to keep your monthly payments at $1,200?