EE 415 / EE 418 Game theory

Lecture 2 Basic theory: Normal form game and Nash Equilibrium

- Normal form game representation (how to describe a game)
- Iterated elimination of strictly dominated strategies
- Nash equilibrium

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Few terminology

- Class of static game (or simultaneous-move; each choose without knowing the other's choice)
- Complete information
- Common knowledge among all players
- Rationality (choose the best action according to your preference)=>consistent decision or choices, rational player will not play a dominated action

STATIC GAME WITH COMPLETE INFORMATION

- Normal or strategic form game representation
- Normal form game , G, specifies
- A. players in the game
  - B. strategies available to each player: S_i
  - C. payoff received by each player according to each combination of strategies that could be chosen : u_i

- $G = \{S_1, \ldots, S_n ; u_1, \ldots, u_n\}$

STATIC GAME WITH COMPLETE INFORMATION

- Begin with two players. (can extend to more)
- Timing of simultaneous game is as follows:
- 1. Player 1 chooses an strategy s_1 from his strategy set, S_1. At the same time, player 2 picks an strategy s_2 from her strategy, S_2. That is, s_1 belongs to S_1 or $s_1 \in S_1$
- 2. After both choose their strategy, they receive payoffs through payoff function:

  $u_1(s_1, s_2)$ and $u_2(s_1, s_2)$
- We call (s_1, s_2) a strategy profile (list of all the player's strategy) which belongs to $S_1 x\ S_2$

EXAMPLE 1

		Player 2		
		Left	Middle	Right
Player 1	Up	1 , 0	1 , 2	0 , 1
	Down	0 , 3	0 , 1	2 , 0

Figure 1

- N= {1, 2}
- S_1 = {Up, Down}
- S_2 = {left, middle, right}
- u_1(up, left) = 1

- Example: Cournot model, Bertrand model, first-bid auction

ITERATED ELIMINATION OF STRICTLY DOMINATED STRATEGIES

- A rational player knows not to play some choice.
- In example 1, Player 2 sees that Right is dominated by Middle.
- Following that thought, player 1 sees the game appeared as

		Player 2	
		Left	Middle
Player 1	Up	1 , 0	1 , 2
	Down	0 , 3	0 , 1

- Again, Left is dominated by Middle for player 2, leaving (up, middle) as the outcome of the game.

Iterated elimination of strictly dominated strategies

- A rational player does not play strictly dominated choices. (choice made is at least as good as every other available choice)
- Some drawbacks of this iteration: in each step of elimination, they know about each player's rationality.
- This must be assumed for players in each round, we call this common knowledge: all players know that all the players are rational and so on…..
- Sometime the process produces no prediction, see next table

EXAMPLE 2.

	L	C	R
T	0 , 4	4 , 0	5 , 3
M	4 , 0	0 , 4	5 , 3
B	3 , 5	3 , 5	6 , 6

Figure 2

- What is the prediction about the play of the game?
- All strategies are survived, not to be eliminated

Example 3. Prisoners'' dilemma

		Player 2	
		L_2	R_2
Player 1	L_1	1 , 1	5 , 0
	R_1	0 , 5	4 , 4

The Prisoners' Dilemma

- Do we see any dominated strategy?
- Think of L_1 and L_2 as Fink or Confess
- And R_1 and R_2 as Quiet or No Confess
- Game is symmetric
- Story about this game: if both are quiet, they are better off with 4. If both are fink, they are worst off with 1.
- What is the prediction of the game play?

Example 4. Battle of Sexes

		Pat: Red	Pat: White
Chris	Steak	2 , 1	0 , 0
	Chicken	0 , 0	1 , 2

The Dating Game

- If they go for different choices, each of them is equally unhappy.
- In this game, players want to cooperate, but they disagree about the best outcome.

Example 5. Matching pennies (get one or lose one)

		Player 2: Heads	Player 2: Tails
Player 1	Heads	-1 , 1	1 , -1
	Tails	1 , -1	-1 , 1

Matching Pennies

- If they show the same side, player 2 is happy and gets $1, but player 1 is not.
- Again we cannot delete any dominated strategy

Nash Equilibrium

- We need a better way to find a prediction about the strategy each player will chose
- If you can find the correct one, then it must be a best response to other player's predicted strategies.(you are willing to play the one you predict)
- Such a prediction could be called "strategically stable" or "self-enforcing, because no single player wants to deviate from his predicted strategy.
- We call such prediction 'a Nash Equilibrium".
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Nash Equilibrium

- Definition:

  In G, the strategies (s^*_1, s^*_2) are Nash equilibrium if, for each player i, s^*_i is his best response to other player strategy s^*_{-i}.

- Equivalently,

  s^*_1 must satisfy $u_1(s^*_1, s^*_2) \geq u_1(s_1, s^*_2)$ for every s_1 in S_1

  Similarly for player 2,

  s^*_2 must satisfy $u_2(s^*_1, s^*_2) \geq u_2(s^*_1, s_2)$ for every s_2 in S_2

- We can also say that s^*_1 solves max $u_1(s_1, s^*_2)$
- In NE, no player has any better strategy that yields better
- Now, let's find NE in all previous examples.

Nash Equilibrium

- From example 2,3,5, if we argue that the Nash equilibrium (or more than or mixed) exists, it must always survive the elimination of strictly dominated strategies.
- If this process kills all but (s^*_1, s^*_2), then this strategy profile must be the unique Nash Equilibrium of the game.
- However, strategies that survive the process need not to be part of any NE. (quite obvious when nothing is eliminated)
- Few observations: self-enforcing; mutual best response ; NE is not always efficient (yield highest payoff in the game); could be more than one solution and hard to say which one without conventional wisdom; or no pure strategies sometime;
- Nash (1950) showed that in any finite game (no. of players n and the strategy set Si are all finite), there exists at least one NE (may involved mixed strategies)

Let's play: classroom games

- 1. Guessing half of the average
- Need 10 students
- Write a integer number between 0 and 100
- Find the average
- The student whose choice is closest to half of the average is the winner.

LET'S PLAY: CLASSROOM GAMES

- 2. The Ultimatum Game
- Two players
- Player A offers a split of a 100 Baht bill
- If B agrees, then the game is over.
- If B refuses, it is then B's turn to offer a split, but now the bill reduced to 80 Baht
- If A agrees, then the game is over. Both get paid the agreed split.
- If A refuses, the game is over and neither get anything.