Approximation of change of y as a result of change of x

At B = (2, 14)

- When $x_1 = 2, y_1 = 14$. If $\Delta x = 0.1$, we can approximate

speed at that time → Δ time, differential

$$\Delta y \approx f'(x_1).\Delta x$$
$$= f'(2).0.1$$
$$= 2(2)\cdot 0.1 = 0.4 \text{ Km (approximation)}$$

$f'(x) = 2x$

- What is the real Δy? $x_1 = 2 + 0.1 = 2.1$

$f(x_2)$ $y = 10 + x^2$

$y_2 = f(2.1) = 10 + (2.1)^2 = 14.41$ $Y_2 = y_1 + \Delta Y$

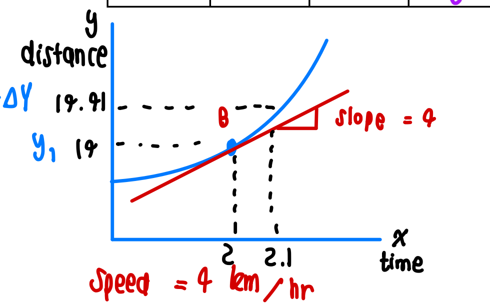
$\Delta y = y_2 - y_1 =$ 14.41 − 14 = 0.41

- We underestimate the real change of y.
- What if $\Delta x = -0.2$? Approximate the change of y.

$\Delta Y \approx f'(x_1)\,\Delta X$
$= f'(2)\,\Delta X$
$= 4\cdot(-0.2)$
$= -0.8$

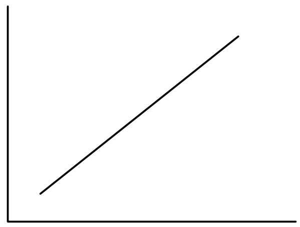
Example $y = 10 + x^2$ $f'(x) = 2x$

Point	x	y	slope
	0	10	0
A	1	11	2
B	2	14	4
C	3	19	6



Speed = 4 km / hr

$\Delta x = 0.1$ hr = 6 mins



linear line
; approximately is exact

only positive part

HW Given $y = 10 + \sqrt{x}$,

a) Find the derivative $f'(x)$.

$f'(x) = \cancel{10} + x^{\frac{1}{2}}$

$\frac{1}{2}x^{\frac{1}{2}-1} = \frac{1}{2}x^{-\frac{1}{2}}$

$= \frac{1}{2\sqrt{x}}$

b) Fill in the table

Point	X	Y	$f'(x)$ slope
	0	10	
A	1	11	0.5
B	2	11.414	0.36
C	3	11.732	0.288

$\frac{1}{2\sqrt{1}}$ $\frac{1}{2\sqrt{2}}$ $\frac{1}{2\sqrt{3}}$

1.4142 $\frac{1}{2.828} = 0.3536 \sim 0.36$

$\frac{1}{4.2426} = 0.2357 \sim 0.24$

c) Does the slope increase as x increase? no, the slope decreases as x increases

d) Approximate the change in Y when $\Delta x = 0.2$ at $x_1 = 3$. Is the approximation under- or over-estimate? over estimate

Approximately ΔY → $f'(x_1)\cdot\Delta x$
$f'(3)\cdot 0.2$
$\frac{1}{2\sqrt{x}}\cdot 0.2 = 0.288 \times 0.2$ $\frac{1}{3.464}$
$\Delta Y = 0.0576$
$= 11.732 + 0.0576$
$Y_2 = 11.7896$

Note: If the function $f(x)$ is linear, the approximation is exact.

Exactly change in Y

→ $f(x_2) = f(3+0.2) = f(3.2)$

$= y = 10 + \sqrt{3.2} = 1.788 + 10$

$Y_2 = 11.788$ $\Delta Y = 0.056$