EE 325 HW4 Answers PART I

Gujarati, D.N. (2009) Basic Econometrics. 5^{th}ed. Singapore, McGraw-Hill. (G)

Chapter 8 Questions 8.5, 8.11, 8.14, 8.18, 8.21, 8.32

8.5

(a) Let the coefficient of log K be $\beta^* = (\beta_2 + \beta_3 - 1)$. Test the null hypothesis that $\beta^* = 0$, using the usual t test. If there are indeed constant returns to scale, the t value will be small.

(b) If we define the ratio (Y/K) as the output/capital ratio, a measure of capital productivity, and the ratio (L/K) as the labor capital ratio, then the slope coefficient in this regression gives the mean percent change in capital productivity for a percent change in the labor/capital ratio.

(c) Although the analysis is symmetrical, assuming constant returns to scale, in this case the slope coefficient gives the mean percent change in labor productivity (Y/L) for a percent change in the capital labor ratio (K/L). What distinguishes developed countries from developing countries is the generally higher capital/labor ratios in such economies.

8.11

1. Unlikely, except in the case of very high multicollinearity.
2. Likely. Such cases occur frequently in applied work.
3. Likely, actually this would be an ideal situation.
4. Likely. In this situation the regression model is useless.
5. Could occur if the significance of one coefficient is insufficient to compensate for the insignificance of the other.
6. Unlikely.

8.14

(*a*)*A priori,* salary and each of the explanatory variables are expected to be positively related, which they are. The partial coefficient of 0.280 means, *ceteris paribus,* the elasticity of CEO salary is a 0.28 percent. The coefficient 0.0174 means, *ceteris paribus*, if the rate of return on equity goes up by 1 percentage point (Note: not by 1 percent), then the CEO salary goes up by about 1.07%. Similarly, *ceteris paribus,* if return on the firm's stock goes up by 1 percentage point, the CEO salary goes up by about 0.024%.

(b) Critical t value is ± 1.96 1.96 for all individual hypothesis testing

$H_0 : \beta_1 = 0$

$H_1 : \beta_1 \neq 0$

$t = \frac{4.32}{0.32} = 13.5$

Reject null hypothesis.

It is significantly different from zero.

$H_0 : \beta_2 = 0$

$H_1 : \beta_2 \neq 0$

$t = \frac{0.280}{0.035} = 8$

Reject null hypothesis.
The elasticity is 0.280. It is significantly different from zero.

$H_0 : \beta_3 = 0$
$H_1 : \beta_3 \neq 0$
$t = \dfrac{0.0174}{0.0041} = 4.2439$
Reject null hypothesis.
It is significantly different from zero.

$H_0 : \beta_2 = 0$
$H_1 : \beta_2 \neq 0$
$t = \dfrac{0.00024}{0.00054} = 0.4444$
Fail to reject null hypothesis.
It is not significantly different from zero.

(c)

$H_0 : \beta_2 = \beta_3 = \beta_4 = 0$
$H_1 : otherwise$

To test the overall significance, that is, all the slopes are equal to zero, use the *F* test

$$F = \frac{R^2/(k-1)}{(1-R^2)/(n-k)} = \frac{0.283/3}{(0.717)/205} = 27.02$$

Under the null hypothesis, this *F* has the *F* distribution with 3 and 205 df in the numerator and denominator, respectively. Reject the null hypothesis. There is enough evidence to say that there is at least one parameter not equal to zero.

(d) Since the dependent variable is in logarithmic form and the *roe* and *ros* are in linear form, the coefficients of these variables give semi elasticities, that is, the growth rate in the dependent variable for an absolute (unit) change in the regressor.

8.18
(*a*) *Ceteris paribus,* a 1 percentage point increase in the job vacancy rate lead on average to about 5.29 pounds increase in the wages and salaries per employee; an increase of about 1 pound GDP per person lead on average to about 12 pence decline in wages and salaries per employee; an increase in import prices in the current year and the previous year lead, on average, to an increase in wages and salaries per employee of about 5 pence.

(b) Critical t value is ± 2.145 for all individual hypothesis testing
$H_0 : \beta_1 = 0$
$H_1 : \beta_1 \neq 0$

$$t = \frac{1.073}{0.797} = 1.3463$$

Fail to reject null hypothesis.
It is not significantly different from zero.

$H_0 : \beta_2 = 0$

$H_1 : \beta_2 \neq 0$

$$t = \frac{5.288}{0.812} = 6.5123$$

Reject the null hypothesis.
It is significantly different from zero.

$H_0 : \beta_3 = 0$

$H_1 : \beta_3 \neq 0$

$$t = \frac{-0.116}{0.111} = -1.0450$$

Fail to reject null hypothesis.
It is not significantly different from zero.

$H_0 : \beta_4 = 0$

$H_1 : \beta_4 \neq 0$

$$t = \frac{0.054}{0.022} = 2.4545$$

Reject the null hypothesis.
It is significantly different from zero.

$H_0 : \beta_5 = 0$

$H_1 : \beta_5 \neq 0$

$$t = \frac{0.046}{0.019} = 2.4211$$

Reject the null hypothesis.
It is significantly different from zero.

(c) A *priori*, one would expect higher per capita productivity to lead to higher wages and salaries. This is not the case in the present example, because the estimated coefficient is not statistically significantly different from zero, as the *t* value is only about –1.

(d) These are designed to capture the distributed lag effect of current and previous year import prices on wages and salaries. If import prices go up, the cost of living is expected to go up, and hence wages and salaries.

(e) The *X* variable may be dropped from the model because it has the wrong sign and because its *t* value is low, assuming of course that there is no specification error.

(f) To test the overall significance, that is, all the slopes are equal to zero, use the *F* test

$$F = \frac{R^2/(k-1)}{(1-R^2)/(n-k)} = \frac{0.934/4}{(0.66)/14} = 49.53$$

This F value is highly significant; for 4 and 14 numerator and denominator degrees of freedom, the 1% level of significance *F* value is 5.04.

8.21

(*a*) The own-price elasticity is –1.274.

(*b*) From the *t* test, we obtain:

$H_0 : \beta_2 = 0$

$H_1 : \beta_2 \neq 0$

$$t = \frac{-1.274}{0.527} = -2.4174$$

Reject the null hypothesis.

It is significantly different from zero, for the *t* value under the null hypothesis that the true price elasticity coefficient is zero.

(*c*)

$H_0 : \beta_2 = -1$

$H_1 : \beta_2 \neq -1$

$$t = \frac{-1.274 - (-1)}{0.527} = -0.5199$$

Fail to reject the null hypothesis.

Since this *t* value is not statistically significant, we do not reject the hypothesis that the true price elasticity is unity.

(*d*) Both the signs are expected to be positive, although none of these variables is statistically significant.

(*e*)Perhaps our sample size is too small to detect the statistical significance of carnation prices on the demand for roses or that of income on the demand for roses. Moreover, expenditure on roses may be such a small part of total income that one may not notice the impact of income on demand for roses.

8.32

(*a*) In Model I the slope coefficient tells us that per unit increase in the advertising expenditure, on average, retained impressions go up by 0.363 units. In Model II the (average) rate of increase in retained impressions depend on the level of advertising expenditure. Taking the derivative of Y with respect to X, you will obtain:

$$\frac{dY}{dX} = 1.0847 - 0.008X$$

This would suggest that retained impressions increase at a decreasing rate as advertising expenditure increases.

(b) &(c)We can treat Mode I 1 as the restricted version of Model II and hence can use the restricted least-squares technique to decide between the two models. Since the dependent variable in the two models is the same, we can use the R^2 version of the *F* test given in (8.7.10). The results are as follows:

$$F = \frac{(0.53 - 0.424)/1}{(1 - 0.53)/18} = \frac{0.106}{0.0261} = 4.0613$$

Under the usual assumptions of the *F* test, the preceding *F* value follows the *F* distribution with 1 df and 18 df in the numerator and denominator, respectively. For these df the critical *F* value is 4.41 (5% level) and 3.01 (10% level). It seems that we should retain the squared X variable in the model.

(*d*) As noted in (*b*), there are diminishing returns to advertising expenditure; if the coefficient of the *X*-squared term were positive, there would have been increasing returns to advertising. Equating the derivative in (*b*) to zero, we obtain: 1.0847=0.008 *X*, which gives *X* = 135.58.

At this value of X, the rate of increase of Y with respect to X is zero. Since *X* is measured in millions of dollars, we can say that at the level of expenditure of about 136 millions of dollars there is no further gain in retained impressions, which are measured in millions of impressions.