1. (50 points) Consider the following two-period game between an economy's monetary authority and its labour force. The timing of the game is:

   - Nature picks the monetary authority's type. The monetary authority is *weak* with probability 0.4 and *strong* with probability 0.6.
   - The monetary authority picks first period inflation to be either HIGH or LOW
   - The labour force forms *high* or *low* expectations of second period inflation.
   - The authority picks second period inflation: HIGH if weak; LOW if strong.

   Strong monetary authorities prefer low inflation; weak monetary authorities are less prepared to adopt policies that they believe may compromise national income in the short-run, and prefer a situation that results in higher inflation. The authority receives a payoff of 100 if it chooses its preferred first period inflation level, zero otherwise. In addition, the authority receives a bonus payoff if labour force expectations of second period inflation are low. These benefit the strong authority by keeping down inflationary pressures, gaining it a bonus payoff of 200; while the weak authority gains a bonus of 50. (The (present value) of the bonus to the weak authority is lower since it cares less about the future.) The labour force simply wants to get its expectations of second period inflation correct. It receives a payoff of zero if it is correct, -100 otherwise.

   (a) Draw this game in extensive form

   (b) Show that there is no equilibrium in which a weak monetary authority chooses LOW first period inflation

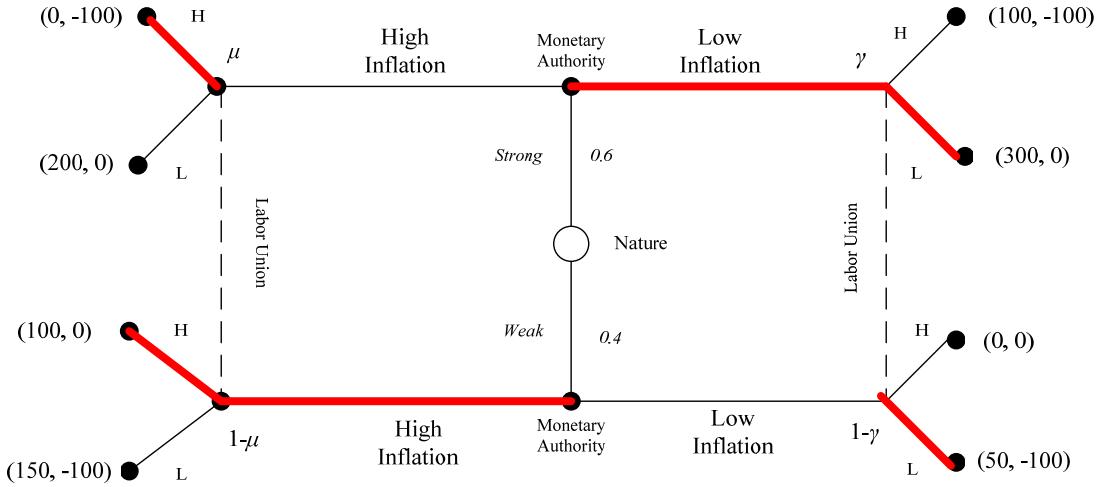
   (c) Is there a separating (perfect Bayesian) equilibrium in which a strong monetary authority chooses LOW first period inflation, and a weak monetary authority chooses HIGH first period inflation?

   (d) What out-of-equilibrium beliefs would the labour force have to hold to support a pooling (perfect Bayesian) equilibrium in which both types of monetary authority chose HIGH first period inflation?

   (e) Are these beliefs compatible with the intuitive criterion? If not, then why not?

2. (30 points) Since education may not reflect workers' true ability, firms should not use worker's education in worker's wage setting decision. Discuss.

1. Separating PBE with a Strong monetary authority sending Low inflation level, and a Weak monetary authority sending High: (Low, High) – Reasonable!



a) Labor Union's beliefs (responder beliefs) in this separating PBE

- $\mu = 0$, i.e., after observing a High inflation announcement, the Labor Union concentrates all its beliefs on the Monetary authority being Weak.
- $\gamma = 1$, i.e., after observing a Low inflation announcement, the Labor Union concentrates all its beliefs on the Monetary authority being Strong.

b) Given the labor union's beliefs, which is the labor union's optimal action, after observing every possible inflation announcement from the monetary authority?

- After observing High inflation, the Labor Union responds with H since beliefs are $\mu = 0$ and 0>-100.
- After observing Low inflation, the Labor Union responds with L since beliefs are $\gamma = 1$, and 0>-100.

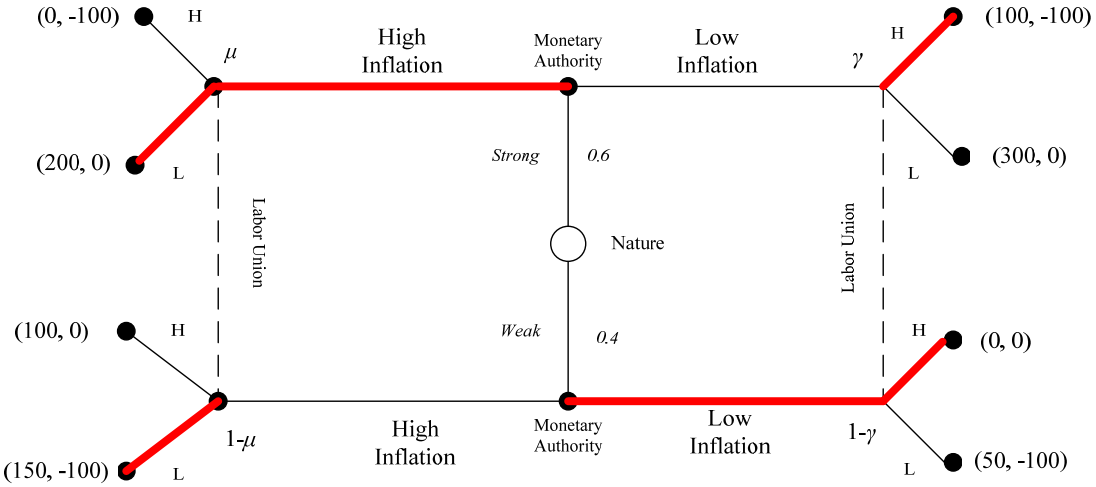
c) Given the previous points, what is the monetary authority optimal action (whether to announce High or Low inflation levels) when it is a Strongly committed central bank? What is its optimal action when it is a Weakly committed central bank?

- Strong monetary authority selects Low inflation (as prescribed in this strategy profile) since 300>0.
- Weak monetary authority selects High inflation (as prescribed in this strategy profile) since 100>50.

d) Can this separating PBE be supported from your answer in *c)*?

Yes.

2. Separating PBE with a Strong monetary authority sending High inflation level, and a Weak monetary authority sending Low: (High, Low) – Crazy!



a) Labor Union's beliefs (responder beliefs) in this separating PBE

- $\mu = 1$, i.e., after observing a High inflation announcement, the Labor Union concentrates all its beliefs on the Monetary authority being Strong.
- $\gamma = 0$, i.e., after observing a Low inflation announcement, the Labor Union concentrates all its beliefs on the Monetary authority being Weak.

b) Given the labor union's beliefs, which is the labor union's optimal action, after observing every possible inflation announcement from the monetary authority?

- After observing High inflation, the Labor Union responds with L since beliefs are $\mu = 1$ and 0>-100.
- After observing Low inflation, the Labor Union responds with H since beliefs are $\gamma = 0$, and 0>-100.

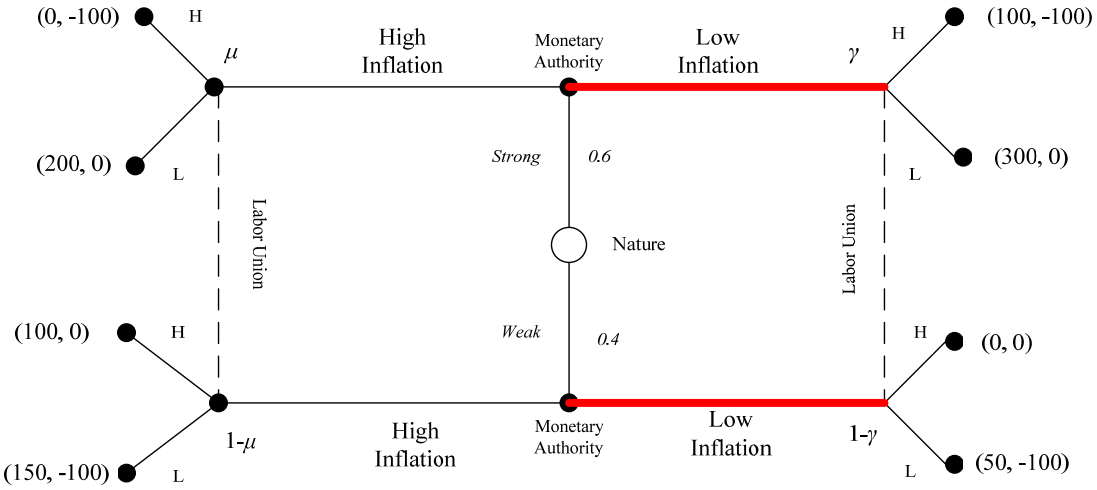
c) Given the previous points, what is the monetary authority optimal action (whether to announce High or Low inflation levels) when it is a Strongly committed central bank? What is its optimal action when it is a Weakly committed central bank?

- Strong monetary authority selects High inflation (as prescribed in this strategy profile) since 200>100.
- Weak monetary authority selects High inflation (contradicting this strategy profile) since 150>0.

d) Can this separating PBE be supported from your answer in *c)*?

No, since the Weak type of monetary authority has incentives to deviate towards High inflation.

3. *Pooling PBE with both types of Monetary authorities forecast a Low inflation : (Low, Low)*



a) Labor Union's beliefs (responder beliefs) in this pooling PBE

Using Bayes' rule, the Labor Union's beliefs after observing Low inflation (in equilibrium) are

$$\gamma = \frac{p * 1}{p * 1 + (1-p) * 1} = \frac{p}{p + 1 - p} = p = 0.6$$

And after observing High inflation (off-the-equilibrium), beliefs are

$$\mu = \frac{p * 0}{p * 0 + (1-p) * 0} = \frac{0}{0}$$

Thus, off-the-equilibrium beliefs cannot be specified using Bayes' rule, and must be left undefined in the entire range of probabilities $\mu \in [0,1]$.

b) Given the labor union's beliefs, which is the labor union's optimal action, after observing every possible inflation announcement from the monetary authority?

After observing **Low inflation**, let us evaluate the Labor Union's expected utility from responding with H and with L, respectively.

$$EU_{Labor}(H|Low\ Inflation) = -100 * .06 + 0 * .04 = -60$$
$$EU_{Labor}(L|Low\ Inflation) = 0 * 0.6 + (-100) * .04 = -40$$

Therefore, after observing Low inflation, the Labor Union prefers L, since it yields a higher expected utility.

Let us now move to the case in which the Labor Union observes **High inflation**. We know that this occurs off-the-equilibrium path, but we must find which the optimal response by the Labor Union is if it ever observes such announcement from the monetary authority.

$$EU_{Labor}(H|High) = -100\mu + 0 * (1-\mu) = -100\mu$$
$$EU_{Labor}(L|High) = 0 * \mu + (-100)(1-\mu) = -100 + 100 * \mu$$

Hence, the Labor union chooses H after observing a High inflation announcement if

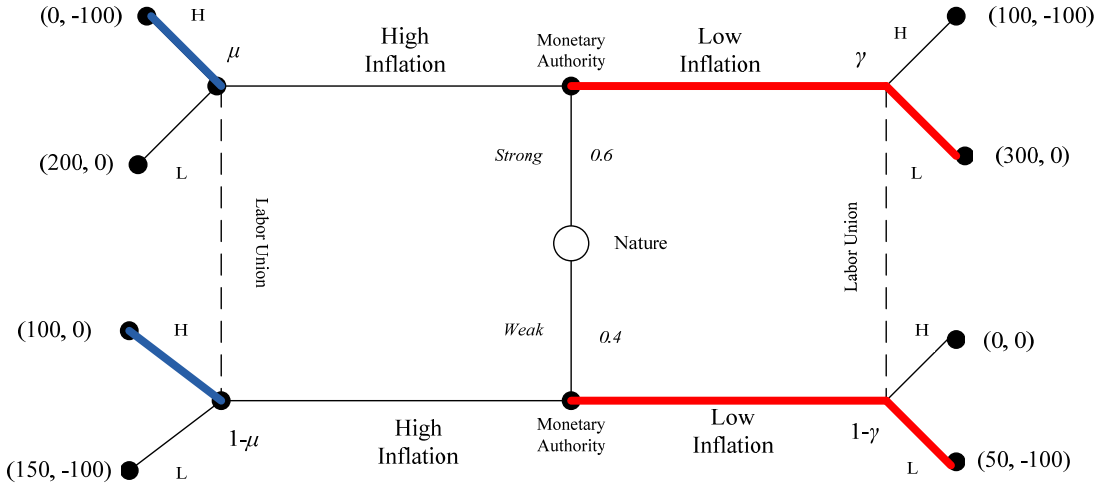
$$-\mu > -1 + \mu, \text{ or } \frac{1}{2} > \mu$$

We will then need to divide our following analysis into two cases:

- ***Case 1:*** $\mu < \frac{1}{2}$, where the Labor Union responds with H after observing High inflation.
- ***Case 2:*** $\mu > \frac{1}{2}$, where the Labor Union responds with L after observing High inflation.

CASE 1: $\mu < \frac{1}{2}$

Hence, in the case that μ<1/2, we have that the Labor Union (responder) chooses H in the out-of-equilibrium event that the Monetary Authority announces a High Inflation forecast. The next figure helps you illustrate this case, i.e., shade the H branch for the Labor Union in the left-hand side of the figure.

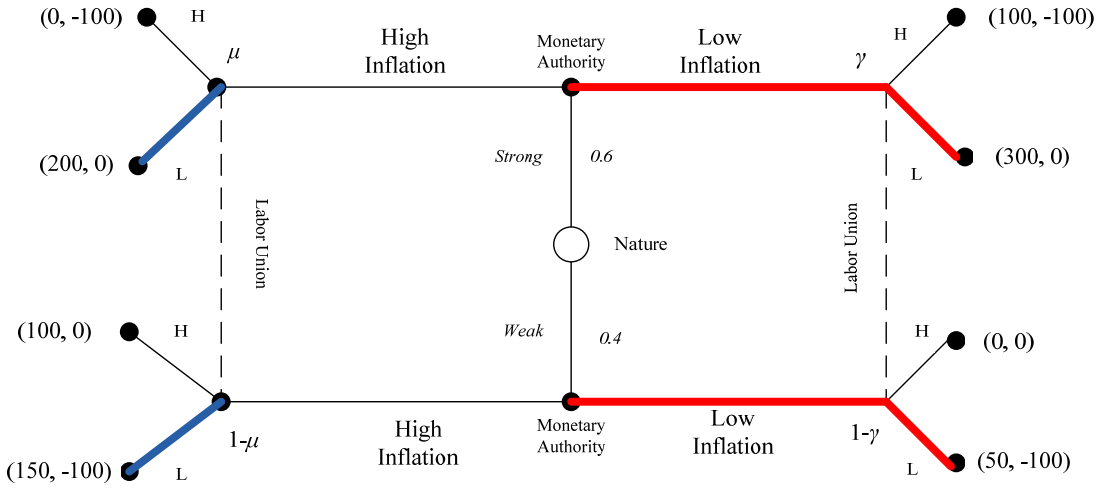


Can this pooling strategy profile be sustained as a PBE?

- **No**, since the Weak type of monetary authority prefers to deviate towards High inflation, given that 100>50.

CASE 2: $\mu > \frac{1}{2}$

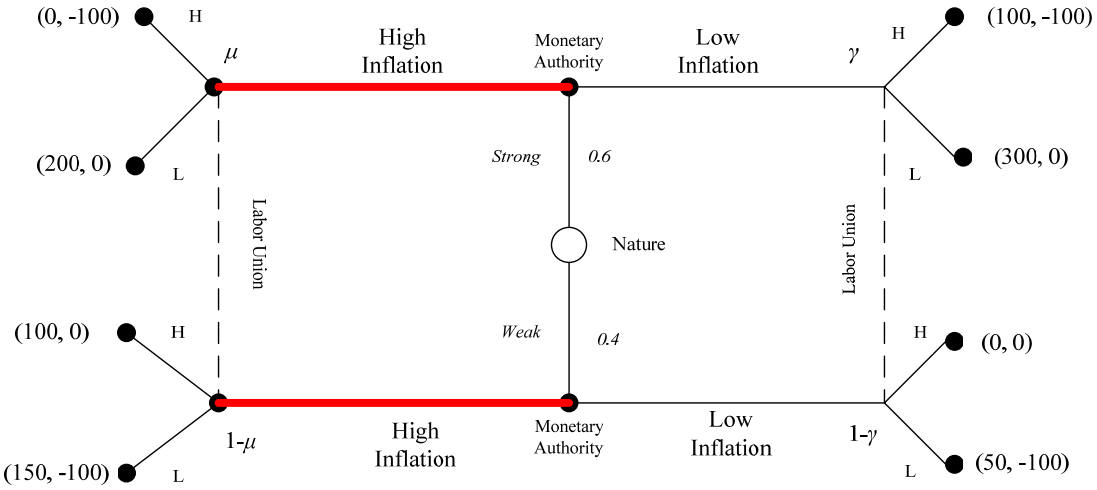
And in the case that μ>1/2, we have that the Labor Union (responder) chooses L in the out-of-equilibrium event that the Monetary Authority announces a High Inflation forecast. The next figure helps you illustrate this case, i.e., shade the L branch for the Labor Union in the left-hand side of the figure.



Can this pooling strategy profile be sustained as a PBE?

- **No**, since the Weak type of monetary authority prefers to deviate towards High inflation, given that 150>50.

4. *Pooling PBE with both types of Monetary authorities forecast a High inflation : (High, High)*



a) Labor Union's beliefs (responder beliefs) in this pooling PBE

Using Bayes' rule, the Labor Union's beliefs after observing High inflation (in equilibrium) are

$$\mu = \frac{p * 1}{p * 1 + (1-p) * 1} = \frac{p}{p + 1 - p} = p = 0.6$$

And after observing Low inflation (off-the-equilibrium), beliefs are

$$\gamma = \frac{p * 0}{p * 0 + (1-p) * 0} = \frac{0}{0}$$

Thus, off-the-equilibrium beliefs cannot be specified using Bayes' rule, and must be left undefined in the entire range of probabilities $\gamma \in [0,1]$.

b) Given the labor union's beliefs, which is the labor union's optimal action, after observing every possible inflation announcement from the monetary authority?

- After observing **High inflation**, let us evaluate the Labor Union's expected utility from responding with H and with L, respectively.

$$EU_{Labor}(H|High\ inflation) = -100 - 0.6 + 0 * 0.4 = -60$$
$$EU_{Labor}(L|High\ inflation) = 0 * 0.6 + (-100) * 0.4 = -40$$

Therefore, after observing High inflation, the Labor Union prefers L, since it yields a higher expected utility.

Let us now move to the case in which the Labor Union observes **Low inflation**. We know that this occurs off-the-equilibrium path, but we must find which the optimal response by the Labor Union is if it ever observes such announcement from the monetary authority.

$$EU_{Labor}(H|Low\ inflation) = -100\gamma + 0 * (1-\gamma) = -100\gamma$$
$$EU_{Labor}(L|Low\ inflation) = 0 * \gamma + (-100)(1-\gamma) = -100 + 100\gamma$$

Hence, the Labor union chooses H after observing a Low inflation announcement if

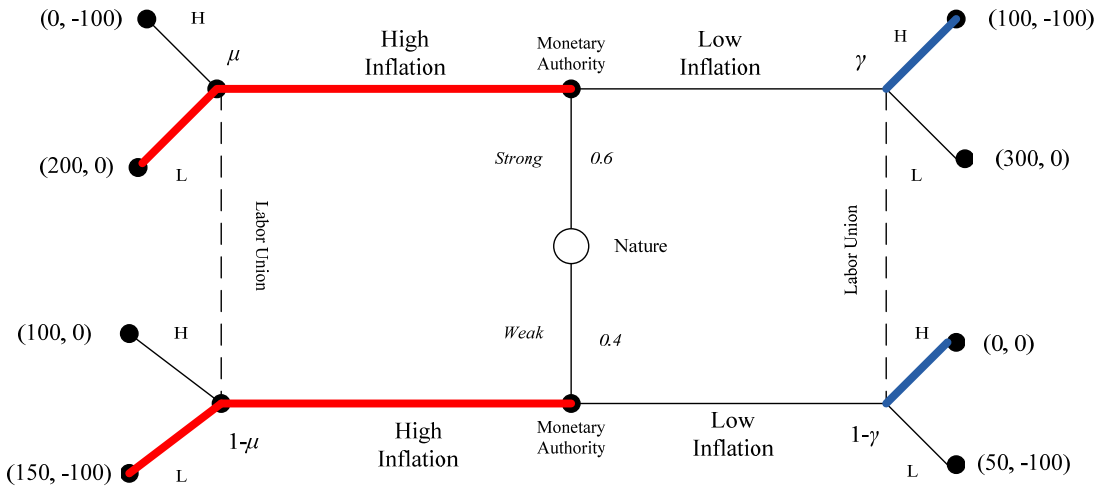
$$-\gamma > -1 + \gamma, \text{ or } \frac{1}{2} > \gamma$$

We will then need to divide our following analysis into two cases:

- ***Case 1:*** $\gamma < \frac{1}{2}$, where the Labor Union responds with H after observing Low inflation.
- ***Case 2:*** $\gamma > \frac{1}{2}$, where the Labor Union responds with L after observing Low inflation.

CASE 1: $\gamma < \frac{1}{2}$

Hence, in the case that γ<1/2, we have that the Labor Union (responder) chooses H in the out-of-equilibrium event that the Monetary Authority announces a Low Inflation forecast. The next figure helps you illustrate this case, i.e., shade the H branch for the Labor Union in the right-hand side of the figure.



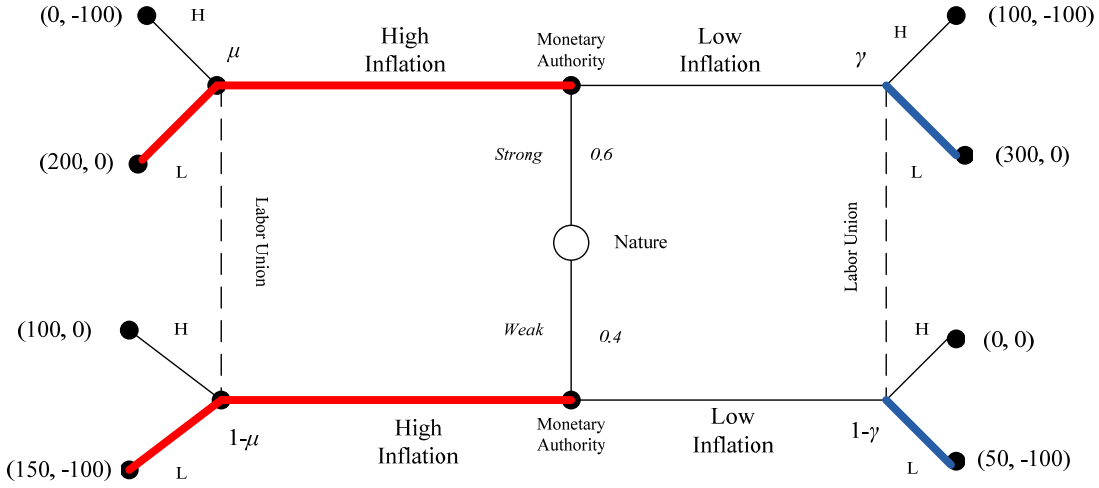
Can this pooling strategy profile be sustained as a PBE?

- **Yes:**
  - The Strong type of monetary authority obtains 200 by choosing a High inflation announcement (as prescribed in this strategy profile), but obtains only 100 if it deviates towards Low inflation (which is responded with H as indicated in the figure).
  - Similarly, the Weak type of monetary authority obtains 150 by choosing a High inflation announcement (as prescribed in this strategy profile), but obtains only 0 if it deviates towards Low inflation (which is responded with H as indicated in the figure).

  Since no type of first mover (monetary authority) has incentives to deviate from the prescribed pooling strategy, we can claim that this strategy profile can be sustained as a PBE when off-the-equilibrium beliefs satisfy $\gamma < \frac{1}{2}$.

CASE 2: $\gamma > \frac{1}{2}$

And in the case that γ>1/2, we have that the Labor Union (responder) chooses L in the out-of-equilibrium event that the Monetary Authority announces a High Inflation forecast. The next figure helps you illustrate this case, i.e., shade the L branch for the Labor Union in the right-hand side of the figure.



Can this pooling strategy profile be sustained as a PBE?

- **No**, since the Strong type of monetary authority has incentives to deviate towards Low inflation, given that 150>50.

1. จากรูปด้านล่างถ้า Nature เลือกค่า $x = 0$ ด้วยความน่าจะเป็น 2/3 และ $x=100$ ด้วยความน่าจะเป็น 1/3 โดยผู้เล่น 1 ทราบการเลือกของ Nature ขณะที่ผู้เล่น 2 ไม่ทราบ

		Player 2	
		a	b
Player 1	A	50,100	x,0
	B	0,0	300,300

จงวาด extensive form game และ Bayesian normal form พร้อมทั้งหา Bayesian Nash Equilibrium

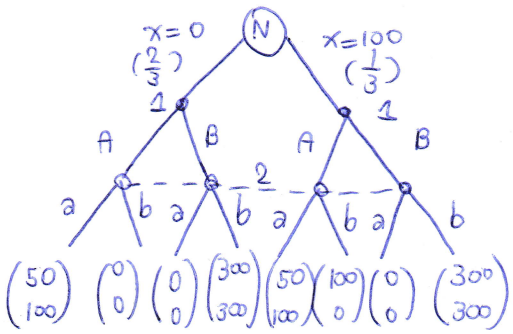
Pl1 \ Pl2	a	b
A	50,100	0,0
B	0,0	300,300

Prob = $\frac{2}{3}$

Pl1 \ Pl2	a	b
A	50,100	100,0
B	0,0	300,300

Prob = $\frac{1}{3}$

1) Extensive form game



- Pl1 has 2 information sets (observe nature's action)

⇒ Pl1 available strategies : $\{A^{\circ}A^{100}, A^{\circ}B^{100}, B^{\circ}A^{100}, B^{\circ}B^{100}\}$

- Pl2 has 1 information set (not observe nature's action & simultaneously move)

⇒ Pl2 available strategies : $\{a, b\}$

2) Bayesian normal form

Pl1 \ Pl2	a	b
$A^{\circ}A^{100}$	50,100	$\frac{100}{3}$,0
$A^{\circ}B^{100}$	$\frac{100}{3}$, $\frac{200}{3}$	100,100
$B^{\circ}A^{100}$	$\frac{50}{3}$, $\frac{100}{3}$	$\frac{700}{3}$, 200
$B^{\circ}B^{100}$	0,0	300,300

$(A^{\circ}A^{100}, a)$ – Pl1 pay off = 50
– Pl2 pay off = 100

$(A^{\circ}A^{100}, b)$ – Pl1 pay off = $(0)(\frac{2}{3})+(100)(\frac{1}{3})$
– Pl2 pay off = 0

$(A^{\circ}B^{100}, a)$ – pl1 pay off = $(50)(\frac{2}{3})$
– Pl2 pay off = $(100)(\frac{2}{3})$

$(A^{\circ}B^{100}, b)$ – pl1 pay off = $(300)(\frac{1}{3})$
– pl2 pay off = $(300)(\frac{1}{3})$

$(B^{\circ}A^{100}, a)$ – pl1 pay off = $(50)(\frac{1}{3})$
– pl2 pay off = $(100)(\frac{1}{3})$

$(B^{\circ}A^{100}, b)$ – pl1 pay off = $300(\frac{2}{3}) + 100(\frac{1}{3}) = \frac{700}{3}$
– pl2 pay off = $300(\frac{2}{3})$

$(B^{\circ}B^{100}, a)$ – pl1 pay off = 0
– pl2 pay off = 0

$(B^{\circ}B^{100}, b)$ – pl1 pay off = 300
– pl2 pay off = 300

3) Bayesian Nash Equilibrium

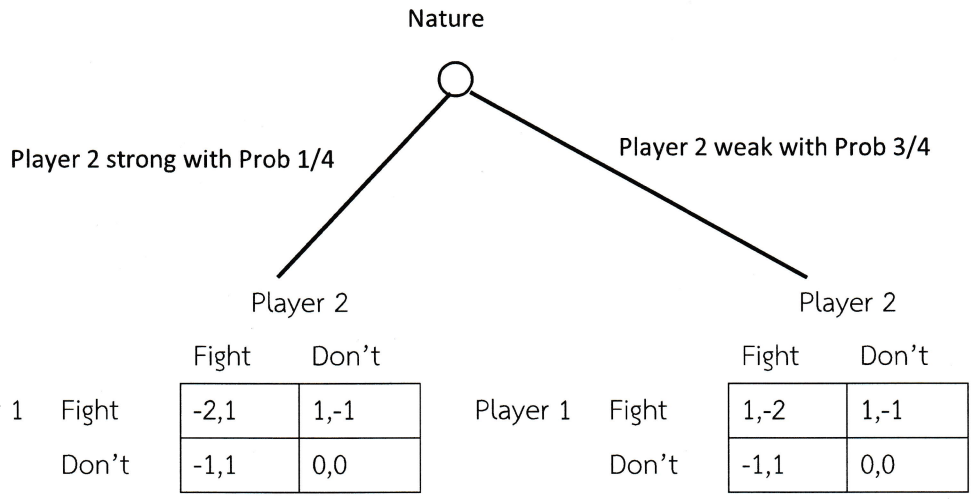
BNE = $\{(A^{\circ}, A^{100}), a), ((B^{\circ}, B^{100}), b)\}$

4. ผู้เล่นสองรายกำลังตัดสินใจกลยุทธ์ทางธุรกิจว่าจะ สู้ (Fight) หรือ ไม่สู้ (Don't) โดยผู้เล่น 2 ทราบประเภท (type) ของตนเองว่าเป็น strong หรือ weak แต่ผู้เล่น 1 ไม่ทราบประเภทของผู้เล่น 2 แต่ทราบการกระจายของประเภทว่า ผู้เล่น 2 มีโอกาสที่จะเป็น strong ด้วยความน่าจะเป็น ¼ และเป็น weak ด้วยความน่าจะเป็น ¾ ดังแสดงในภาพประกอบ ซึ่งผู้เล่นต้องทำการตัดสินใจกลยุทธ์พร้อมกัน

   ก. จงแสดงสถานการณ์ดังกล่าวด้วย extensive form game

   ข. จงหา Bayesian Nash Equilibrium ของเกมนี้

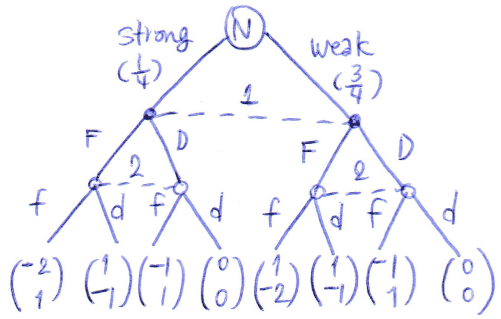
Note ☆ Fight is the dominant strategy of strong P12
☆ If P12 is weak, fight is the dominant strategy of P11



Player 1		Player 2: Fight	Player 2: Don't
	Fight	-2,1	1,-1
	Don't	-1,1	0,0

Player 1		Player 2: Fight	Player 2: Don't
	Fight	1,-2	1,-1
	Don't	-1,1	0,0

1) Extensive form game



- P11 has 1 information set (not observe P12's type)
⇒ P11 available strategies : {F, D}
- P12 has 2 information set (observe his own type & simultaneously move)
⇒ P12 available strategies : {$f^S f^W$, $f^S d^W$, $d^S f^W$, $d^S d^W$}

P11 \ P12	$f^S f^W$	$f^S d^W$	$d^S f^W$	$d^S d^W$
F	(1/4), -5/4	(1/4), (-1/2)	(1), -7/4	(1), -1
D	-1, (1)	-1/4, 1/4	-3/4, 3/4	0,0

(F, $f^S f^W$)
- P11 pay off = $(-2)(\frac{1}{4}) + (1)(\frac{3}{4}) = \frac{1}{4}$
- P12 pay off = $(1)(\frac{1}{4}) + (-2)(\frac{3}{4}) = -\frac{5}{4}$

(F, $f^S d^W$)
- P11 pay off = $(-2)(\frac{1}{4}) + (1)(\frac{3}{4}) = \frac{1}{4}$
- P12 pay off = $(1)(\frac{1}{4}) + (-1)(\frac{3}{4}) = -\frac{2}{4}$

(F, $d^S f^W$)
- P11 pay off = 1
- P12 pay off = $(-1)(\frac{1}{4}) + (-2)(\frac{3}{4}) = -\frac{7}{4}$

(D, $f^S d^W$) - P11 pay off = $-1(\frac{1}{4})$
- P12 pay off = $\frac{1}{4}$

(D, $d^S f^W$) - P11 pay off = $(-1)(\frac{3}{4})$
- P12 pay off = $\frac{3}{4}$

2) Bayesian Nash Equilibrium : BNE = {F, (f^S, d^W)}
☆ Fight is the dominant strategy of P11 because P12 is more likely to be weak (Prob = $\frac{3}{4}$).