Demand curve

$\boxed{P = 100 - 2Q}$, $MC = 10$, 4 firms in market

Let: q^c = q cartel ; $q^{\#}$ = q cheat ;

There're 4 firms in market $Q = q_1 + q_2 + q_3 + q_4$

$\boxed{Q = 4q}$

demand curve for each firm : $P = 100 - 2q$

$2q = 100 - P$

$\boxed{q = 50 - \frac{P}{2}}$

demand curve in market : $Q = 4q$

$Q = 4(50 - \frac{P}{2})$

$Q = 200 - 2P$

$\boxed{P = 100 - \frac{Q}{2}}$

Monopoly Market Profit ; Joint-Profit; Price, Quantity : $\Pi = PQ - TC$

$\Pi = (100 - \frac{Q}{2})Q - TC$

$\frac{d\Pi}{dQ} = 100 - Q - MC$ $\quad \therefore MC = 10$

$0 = 100 - Q - 10$

$\boxed{Q = 90} \rightarrow P = 100 - \frac{Q}{2}$; $\boxed{P = 55}$

$\Pi = 90(55) - 10(90)$

$\boxed{\Pi = 4950}$ Joint Profit #

If they split profit equally.

$Q = 4q$

$q = \frac{90}{4} = 22.5$ # units.

each firm will receive $\Pi_q = 55(22.5) - 10(22.5)$

$\boxed{\Pi_q = 1012.5}$ #.

If one firm decides to increase the quantity by 50%.

* 1 firm cheat $\Rightarrow q^{\#}$

$$Q = 3q + q^{\#}$$

$\uparrow 50\%\, q = 22.5 \times 1.5 = \underline{33.75 \text{ units}}\ q^{\#}$ $\quad Q = 22.5(3) + 33.75$

$\boxed{Q = 101.25}$

(new Market Price): $P = 100 - \frac{Q}{2}$

$$P = 100 - \frac{(3q + q^{\#})}{2}$$

$$P = 100 - \frac{(22.5(3) + 33.75)}{2}$$

$\boxed{P = 49.375}$ #

$\boxed{\pi \text{ of each firm :}}$

① $\pi^{cheat} = PQ - TC$

$= (49.375)(33.75) - 10(33.75)$

$\underline{\pi^{cheat} = 1328.9}$ #

② $\pi^{good} = PQ - TC$

$= (49.375)(22.5) - 10(22.5)$

$\underline{\pi^{good} = 885.9375}$ #

* Is it a Nash Equilibrium?

- Nash Equilibrium is the state that no player in game wants to more from current strategy. However, in this case there's still extra profit in cartel strategy: Incentive is still there for firm to cheat. Therefore it is not a Nash Equilibrium in joint-monopoly case.

(2) Answer:

Definition:

Economies of scope states that the average total cost of production ↓ as an increase in number of different goods produced.

In this case, the production of fuel and gasoline. can achelve economies of scope by using crude oil as input. It is more cost efficient than separate production.

Mathematically,

$$SC = \frac{[C(q_1, 0) + C(0, q_2) - C(q_1, q_2)]}{C(q_1, q_2)}$$

The economies of scope will exist if $SC > 0$

(4). There're various reasons for a firm to make M&A decision.

In this case, the purpose il to diversify the business.

If labor turnorer of firm A is costly, it is better to purchase firm B under the condition that the merger could help to lower the cost of labor turnorer. This action is aimed to acheire higher efficiency.

(6) The exists of economies of scope has no bearing on the relationship between price and marginal cost. It states only that 2 goods can be produced cheaper together.

The firm's AC will decrease as it produce more means that firm has reacked economies of scale.

As more products have been produced, MC must declines to lower than AC., If the Price = MC

Then $P = MC < AC \Rightarrow$ The firm is facing a loss.