

# FN 201 Business Finance


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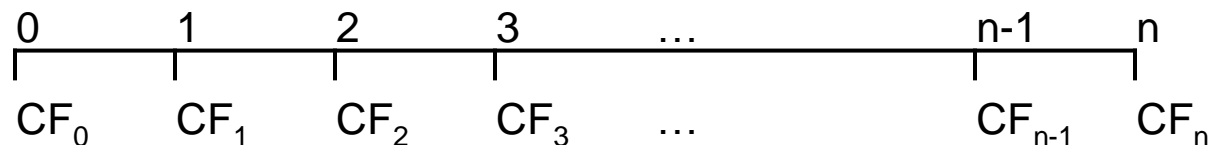


# Lecture 3: Time Value of Money

# Time value of money concept

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- ▶ The value of money differs at different point in time
- ▶ Value of 1 baht today is not the same as 1 baht one year from now, given non-zero interest rate.



- ▶ We need to know/calculate the equivalent value of two cash flows (CFs) at two different points in time to compare them

“One dollar today is worth more than one dollar tomorrow”

# The rules of Time travel

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- ▶ Some rules for shifting and combining money over time:
  - ▶ **Rule 1** only values at the same point in time can be compared or combined (added and subtracted)
  - ▶ **Rule 2** to move a cash flow forward in time, **compound it**

$$FV_n = PV_0 (1+r)^n$$

- ▶ **Rule 3** to move a cash flow backward in time, **discount it**

$$PV_0 = \frac{FV_n}{(1+r)^n}$$

# Interest rate

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- ▶ Other names for interest rate:
  - ▶ Required rate of return – returns required by investors or lenders to induce them to forgo current consumption
  - ▶ Discount rate
  - ▶ Opportunity cost
  - ▶ Cost of capital
  - ▶ Rate of return
  - ▶ Expected rate of return
- ▶ Compounding interest > interest earned on interest  
Simple annual rate + Compounding effect

# Example

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For the 1,000 baht which Bruce Wayne deposits with the financial institution, find the total amount that he has after 1 year under each of the following cases:

- ▶ 12% interest compounded annually
- ▶ 12% interest compounded semi-annually
- ▶ 12% interest compounded quarterly
- ▶ 12% interest compounded monthly

# SAR and EAR

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- ▶ Simple Annual Rate (SAR) or Annual Percentage Rate (APR)
  - ▶ Indicate a simple interest earned in one year ignoring the effect of compound interest.
- ▶ Effective Annual Rate (EAR)
  - ▶ The total amount of interest that will be earned over the course of one year (What investors really get)
  - ▶ Interest rate is annualized considering the compounding effect

$$\text{EAR} = \left(1 + \frac{\text{SAR}}{m}\right)^m - 1$$

$m$  = frequency of compounding



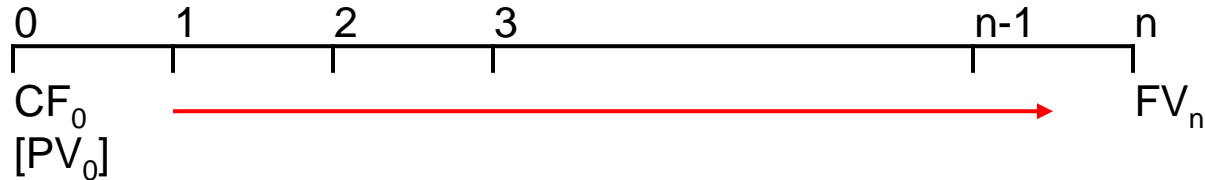
# Different concepts of TVM

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1. Single sum cash flows
2. Annuity cash flows
  - ▶ Ordinary Annuities = payment at the end of period
  - ▶ Annuities Due = Payment at the beginning of period
  - ▶ Growing Annuities
3. Perpetuity cash flows
  - ▶ Ordinary Perpetuity
  - ▶ Growing Perpetuity
4. Finite-life uneven series of cash flows
5. Continuous Compounding

# 1. Single sum cash flows – Future Value

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$$FV_n = PV_0 \cdot (1 + r)^n$$

## Example

- ▶ You invest your lottery winnings of 100,000 baht in a five-year deposit. The deposit pays 4 percent per year compounded annually. You can reinvest at this same interest rate for the entire investment horizon. How much will you have at the end of five years if there is no withdrawals.

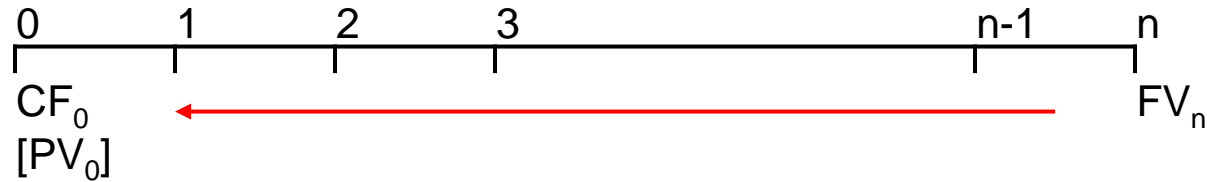
# Example

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- ▶ Mr. Han deposited 240,000 dollars in a saving account that pays 5 percent interest, compounded quarterly, planning to use it for retirement. Eighteen months later, he decided to close down the account and move to Europe.

# 1. Single sum cash flows – Present Value

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$$PV_0 = \frac{FV_n}{(1 + r)^n}$$

- ▶ If you need \$6,000 5 years from now, how much do you need to deposit in the first year, assuming 8 percent annual interest rate?

# 1. Single sum cash flows – using FN calculator

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## ▶ Texas Instrument BA II Plus

Clear TVM work

2ND

+

FV

Future Value

FV

Present Value

PV

Interest rate per period

I/Y

Number of periods

N

Compute

CPT

- ▶ Interest rate is entered as a percent, not a decimal  
i.e. for 10% enter 10 NOT .10

# 1. Example – Single sum

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- ▶ Andy needed to pay 500,000 baht for his parents' medical bill. However, he only had 20% cash of what the hospital billed him. Thus he borrowed the rest of the amount from a loan shark in which he was required to pay back a lump sum of 608,350 baht in three years from the borrowing date. What was the nominal interest rate that the loan shark charged him for?

# 1. Example – Single sum

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- ▶ Which amount is worth more at 14 percent: \$1,000 in hand today or \$2,000 due in 6 years?

# 1. Example – Single sum

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- ▶ Sales of Smoothie SME Co. were 10 million baht in 2005. If sales are expected to grow at 8 percent annually, in what year will the sales be 17.13824 million baht?

# 1. Example – Single sum

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- ▶ Mickey has been given an opportunity to receive \$20,000, 6 years from now. If he can earn 10 percent on his investment, what is the most he should pay for this opportunity? Assuming the opportunity will not default.

# Example

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- ▶ Barbie were sold at auction in 2000 by Christie's International PLC for \$16,500 each. At the time it was estimated that this represented a 18 percent annual rate of return. For this to be true, what was the Barbie have sold for new in 1961?

# Example

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- ▶ Homer is selling his house. Charles has offered Homer \$115,000. He will pay immediately. Kate has offered \$150,000 but he cannot pay until three years from today. The interest rate is 10 percent. Which offer should Homer choose?

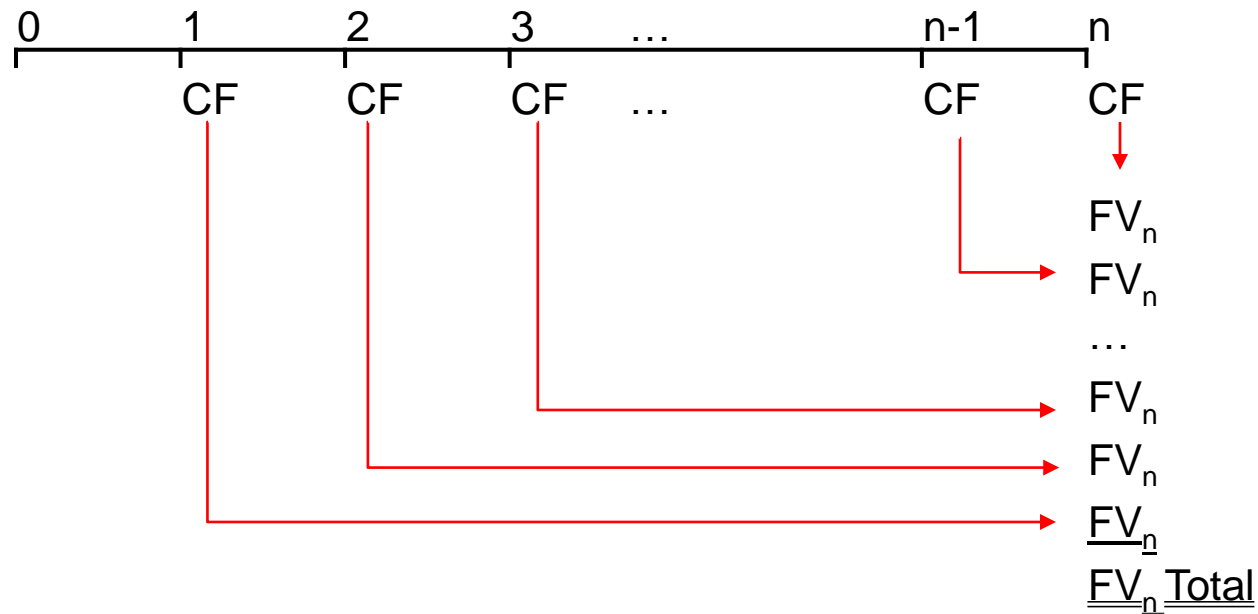
# Example

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- ▶ To the closest year, how long will it take your deposited money to be doubled, if it earns the following rates?
  - ▶ 7 percent
  - ▶ 10 percent
  - ▶ 18 percent
  - ▶ 100 percent

## 2. Annuity Cash Flows– FV of Ordinary annuity

- ▶ Series of equal finite-life cash flows at the end of each period
- ▶ Or series of equal finite-life cash flows, with the first cash flows made one period (1) from today (0)



$$FV_n = CF \left[ \frac{(1 + i)^n - 1}{i} \right]$$

## 2. Examples – Ordinary annuity

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- ▶ Danny intends to collect money for retirement. If Danny deposits \$100 at the end of each period for the next 50 years into an account paying 7 percent interest, annually compounded, how much money will Danny have in the account in 50 years?

# Example

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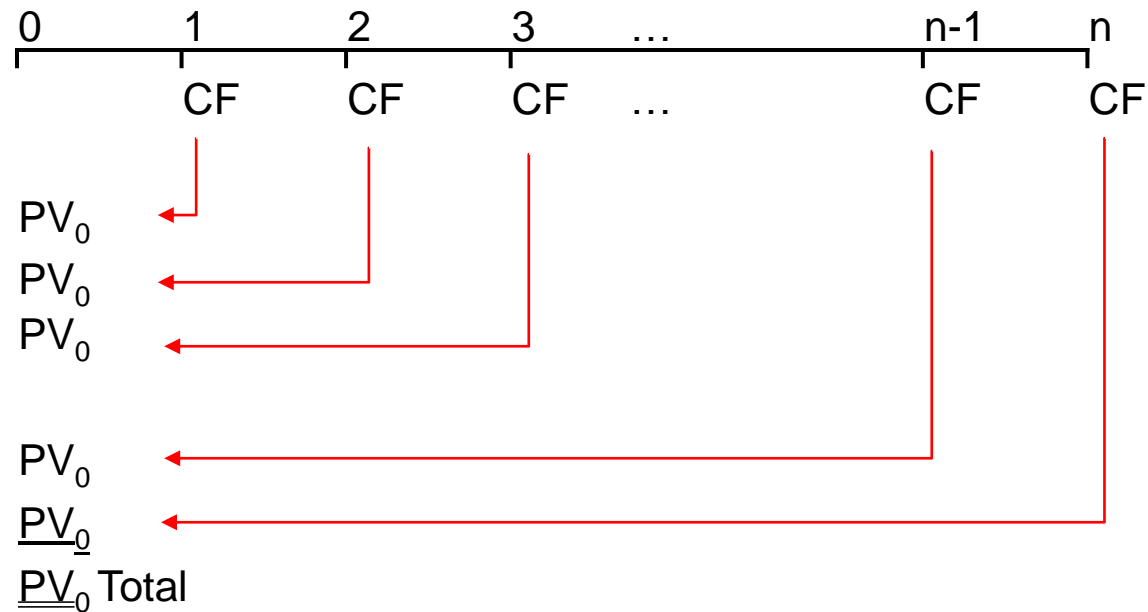
- ▶ Dre wants to have 100,000 pounds in his savings account five years from now; he is prepared to make equal deposits into the account at the end of each year. If the account pays 12 percent interest, annually compounded, what amount must he deposit each year?

## 2. Examples – Ordinary annuity

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- ▶ Ben wants to have 120,000 pounds in his saving account five years from now; he is preparing to make equal deposits into the account at the end of each year. If the account pays 12 percent interest, annually compounded, what amount must he deposit each year?
  
- ▶ What if the account pays 12 percent interest, quarterly compounded?
  
- ▶ What if he quarterly deposits the amount and the bank give him quarterly compounded rate?

## 2. Annuity Cash Flows– PV of Ordinary annuity



$$PV = \frac{CF}{(1+i)} + \frac{CF}{(1+i)^2} + \frac{CF}{(1+i)^3} + \dots + \frac{CF}{(1+i)^N} = \sum_{n=1}^N \frac{CF}{(1+i)^n}$$

$$\text{Annuity } PV_n = CF \left[ \frac{1 - \frac{1}{(1+i)^n}}{i} \right]$$

## 2. Examples – Ordinary annuity

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- ▶ Beginning three months from now, Peter wants to be able to withdraw 10,000 dollars each quarter from his bank account to cover his personal expenses over the next four years. If the account pays 16 percent interest, compounded quarterly, how much Peter need to have in his bank account today to meet his expense needs over the next four years?

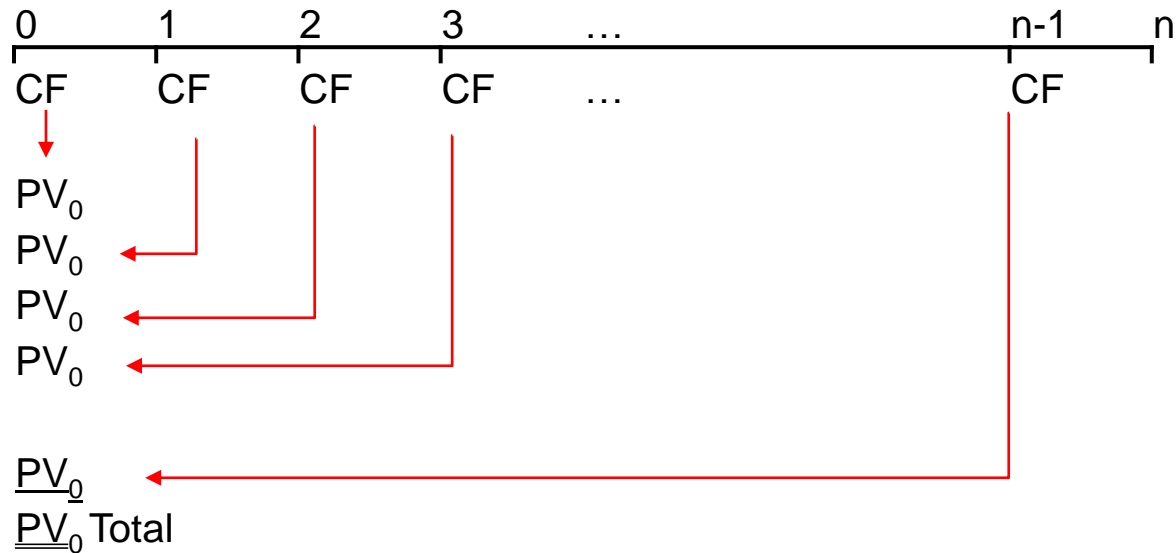
## 2. Examples – Ordinary annuity

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- ▶ You're asked to choose between 2 choices, which would you preferred?
  - ▶ 100,000 Yen at the end of each of the 12 subsequent months and to be deposited in an account paying a 15 percent, compounded monthly
  - ▶ 1,000,000 Yen now



## 2. Annuity Cash Flows— PV of annuity due



$$\text{Annuity due } PV_n = CF \left[ \frac{1 - \frac{1}{(1+i)^n}}{i} \right] (1+i)$$

## 2. Examples – annuity due

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- ▶ Goofy subscribed to a monthly released journal. His subscription is about to run out and he has a choice to renewing it by sending in 250 pounds a year for a regular rate or getting a lifetime subscription for the magazine by paying 3,000 pounds. His opportunity cost is 7 percent. How many years would he have to live to make the life time subscription the better buy? Payments for the regular subscription are made at the beginning of each year.



## 2. Examples – annuity due

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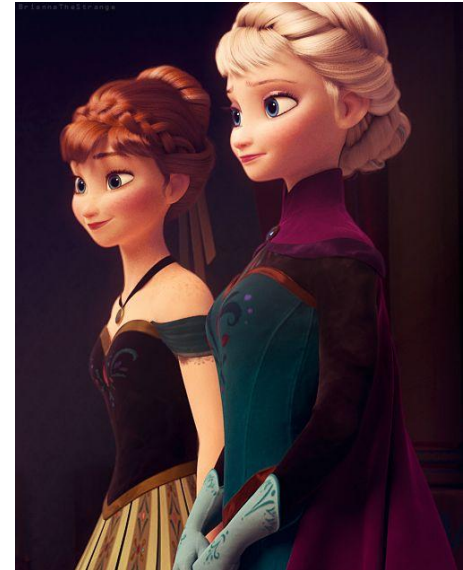
- ▶ Eric plans to marry Ariel in the near future. He saved 40% of his annual income in a time-deposit account, which earns 12 percent interest, compounded annually. The first and last deposits are made today and 1 year before the marriage, respectively. His annual income is constant at 100,000 dollars. If the dowry to be paid on the wedding day is 786,183 dollars, when will Eric marry Ariel?



## 2. Examples – annuity due

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- ▶ The Queen deposits 5,000,000 baht for the education of Elsa and Anna. The princesses are supposed to withdraw an equal amount of money from the account once a year for the next 12 years. The first withdrawal will be made today. After the last withdrawal is made, the account balance is zero. If the account earns an interest rate of 12 percent per annum, semi-annually compounded, how large is each withdrawal the two princesses can share?



## 2. Annuity cash flow – growing annuity

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- ▶ The present value of a growing annuity with the initial cash flow, growth rate  $g$  per period for a finite time.

$$PV = \frac{CF}{(i - g)} \left[ 1 - \left( \frac{1 + g}{1 + i} \right)^n \right]$$

### Example

Ellen considered saving \$10,000 per year for her retirement. Although \$10,000 is the most she can save in the first year, she expects her salary to increase each year so that she will be able to increase her saving by 5% per year. With this plan, if she earns 10% per year on her savings, how much will Ellen have saved at age 65 if she's currently 35 years old?

# Example

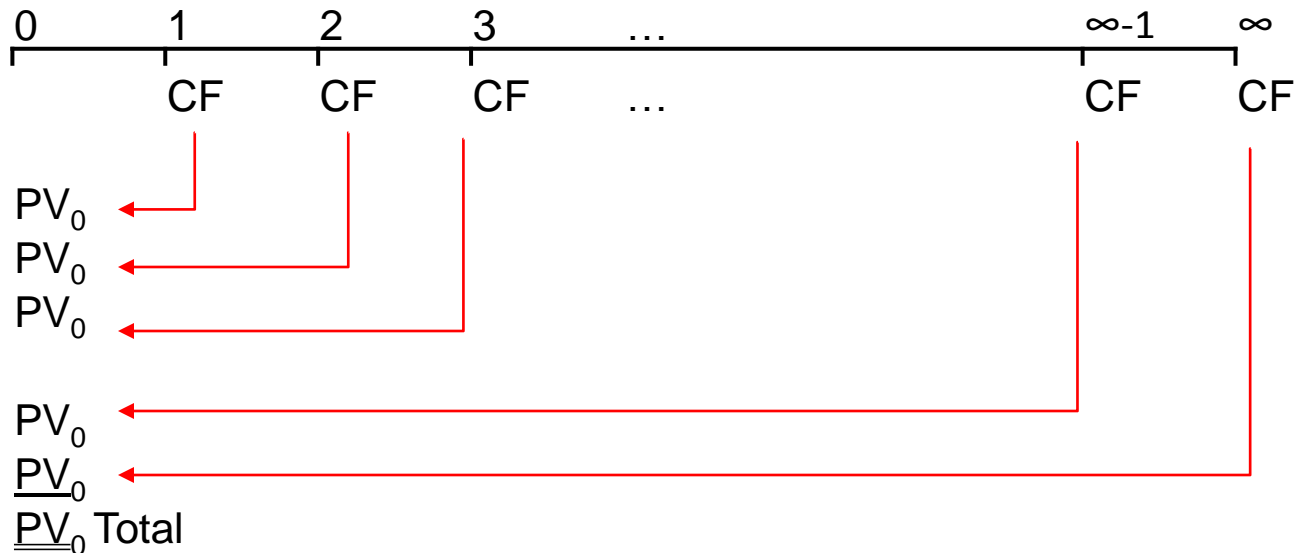
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- ▶ You want to begin saving for your retirement. You plan to contribute \$12,000 to the account at the end of this year. You anticipate you will be able to increase your annual contributions by 3% each year for the next 45 years. If your expected annual return is 8%, how much do you expect to have in your retirement account when you retire in 45 years?



### 3. Perpetuity – Ordinary

- ▶ Stream of all equal payments cash flows that never ends, with each made at a constant interval apart



$$PV_0 = \frac{CF}{(1+i)^1} + \frac{CF}{(1+i)^2} + \dots + \frac{CF}{(1+i)^\infty} = \boxed{\frac{CF}{i}}$$

### 3. Perpetuity - examples

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- ▶ Pocahontas is considering purchase 40 square yards of land for growing corn. This will produce cash flow of 25,000 baht per year indefinitely. If Pocahontas requires a return of 13 percent on this investment, what is the most that she should be willing to pay for the land?



### 3. Perpetuity - examples

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- ▶ You want to endow an annual MBA graduation party. You want the event to be a memorable one, so you budget \$30,000 per year forever for the party. If the university earns 8% per year on its investments, and if the first party is in one year's time, how much will you need to donate to endow the party?

### 3. Perpetuity - examples

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- ▶ Monsters, Inc.'s preferred stock has a 100-baht par value. It pays quarterly dividend of 6 percent per annum. If the required rate of return on this preferred stock is 12 percent, how much should it be the price for this stock?



### 3. Perpetuity - examples

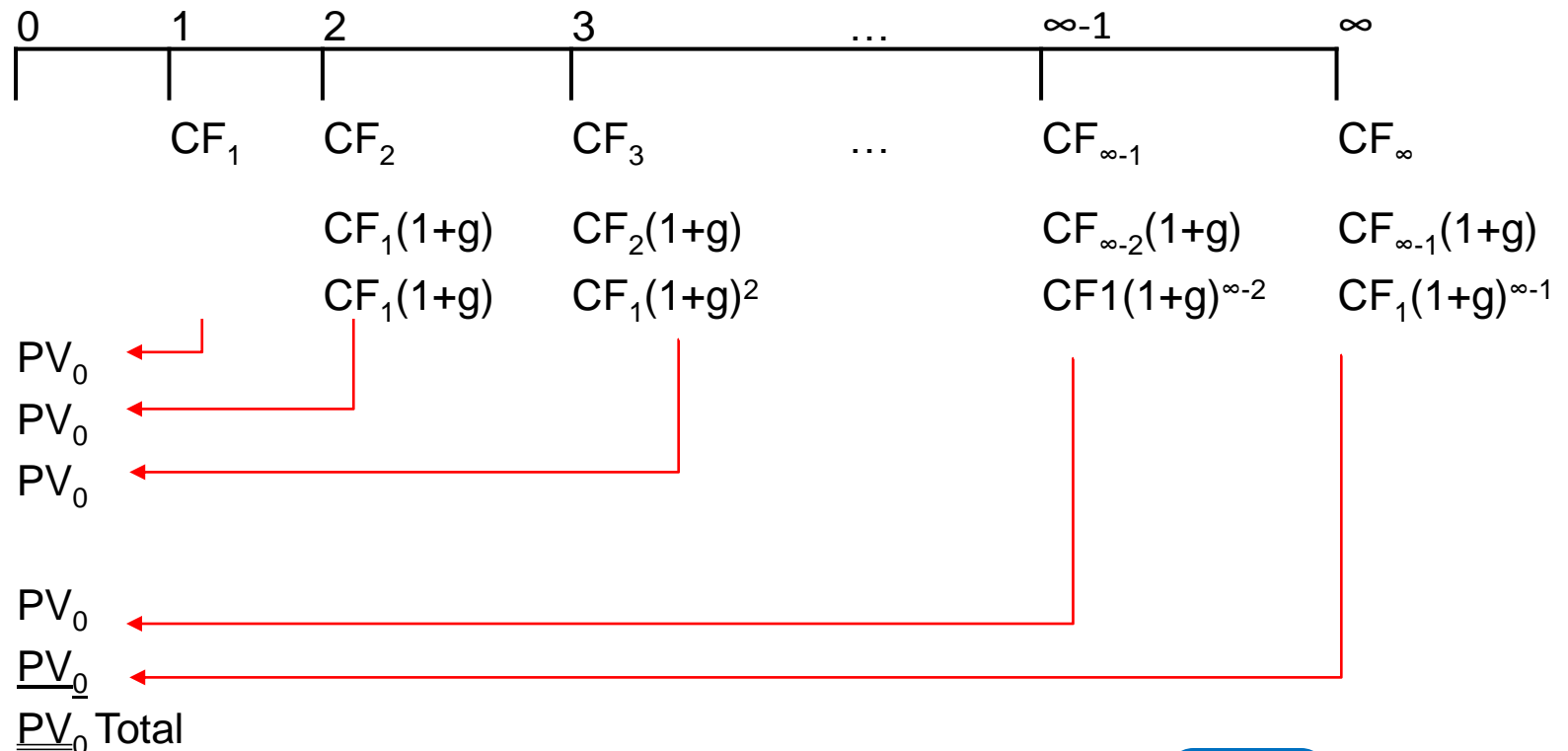
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- ▶ Aladdin would like to set aside an amount of cash in the specific account that aims to provide an even stream of donation to various foundations. If the account pays 5 percent interest, annually compounded, and Aladdin would like to donate 500 baht per year forever, starting the first donation today, what is the appropriate amount he needs to deposit today?



### 3. Perpetuity – growing perpetuity

- Series of equal perpetual cash flows, with each made at a constant interval apart; each value increases at a constant rate,  $g$  percent per year.



$$PV_0 = \frac{CF}{(1+i)^1} + \frac{CF(1+g)}{(1+i)^2} + \dots + \frac{CF(1+g)^{\infty-1}}{(1+i)^{\infty-1}} = \frac{CF}{i-g}$$

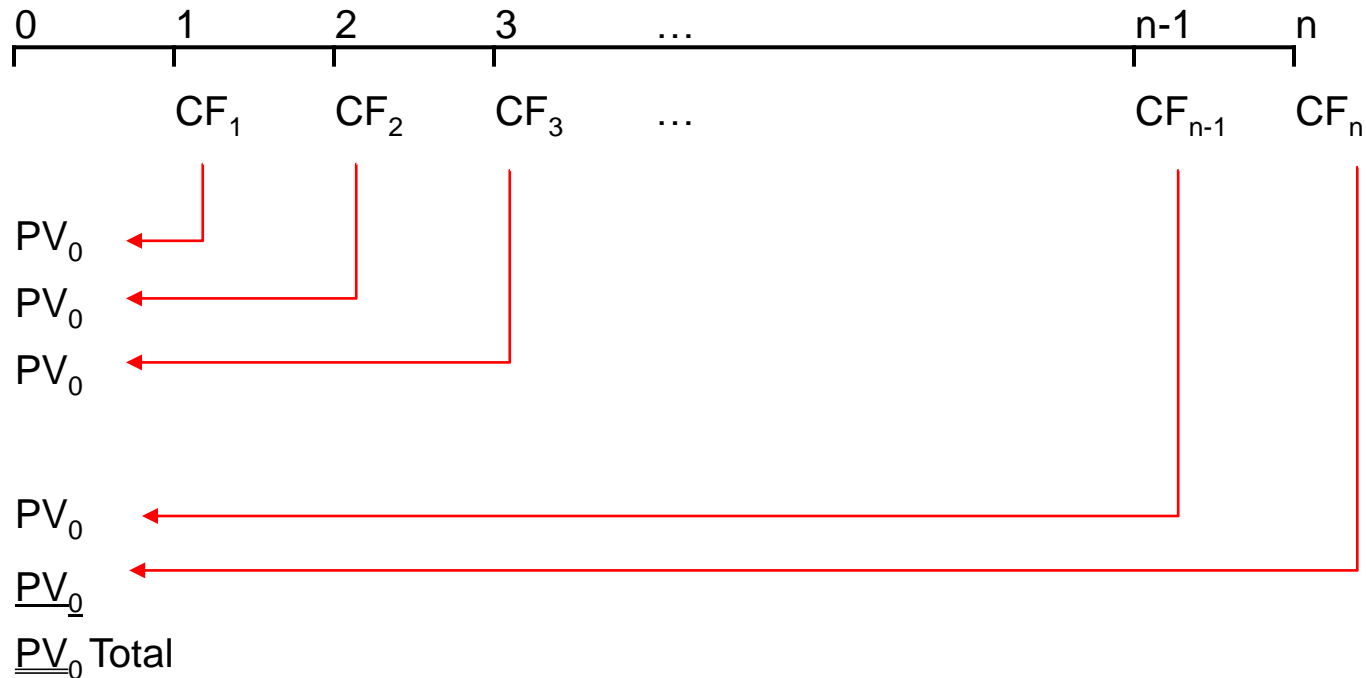
### 3. Perpetuity – growing perpetuity example

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- ▶ Andy invests in common stock of ABC firm which expected to pay 15 baht of dividend per share for next year. The ABC firm pledges to increase its dividend by 3.5% per year indefinitely. If Andy requires a 12.5% return on this investment, what is the maximum price Andy should pay for this stock?

## 4. Finite-life uneven series of cash flows - PV

- ▶ Series of unequal, unspecified pattern, finite-life cash flows



## 4. Finite-life uneven series of cash flows - Examples

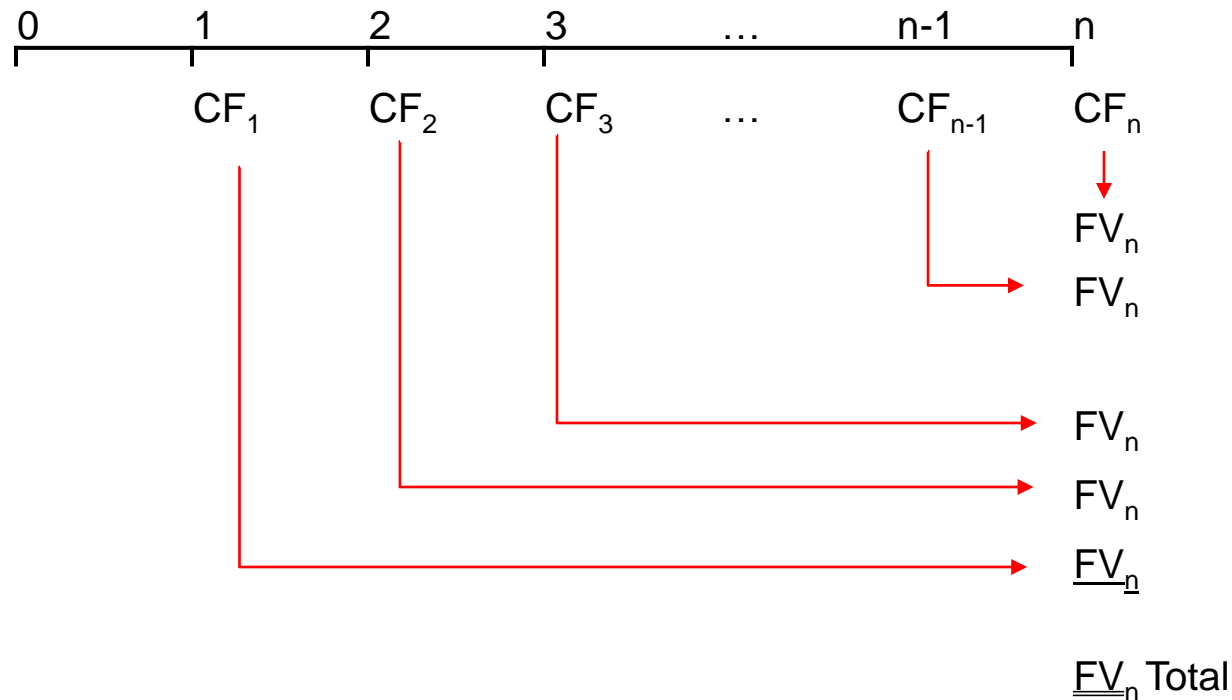
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- ▶ You are given the following cash flows. What is the present value if the discount rate is 12 percent?

<b>Year 0</b>	<b>Year 1</b>	<b>Year 2</b>	<b>Year 3</b>	<b>Year 4</b>	<b>Year 5</b>	<b>Year 6</b>
\$0	\$4,100	\$2,000	\$2,000	\$2,000	0	-\$1,400

## 4. Finite-life uneven series of cash flows - FV

- ▶ Series of unequal, unspecified pattern, finite-life cash flows



## 4. Finite-life uneven series of cash flows - Examples

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- ▶ You are given the following cash flows. What is the future value if the discount rate is 12 percent?

Year 0	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
\$0	\$4,100	\$2,000	\$2,000	\$2,000	0	-\$1,400

## 4. Finite-life uneven series of cash flows - Examples

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- ▶ Find the future value of the following cash flow stream, discounted at 11.75 percent: year 1, \$30,000; year 2, \$9,000; year 3 to 20, \$5,400.

## 5. Continuous compounding

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- ▶ Exponential function is applied for continuous compounding

$$FV = PV(e^{rt})$$

### Example

- ▶ You will require \$700 in 5 years. If you earn 5% interest continuously compounded on your funds, how much will you need to invest today in order to reach your savings goal?
  
- ▶ If you earn 6 percent per annum continuous compounding, how long will it take an account with \$100 to be double?

# Pure discount loan

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- ▶ Loan that the borrower receives money today and repays a single lump sum some time in the future
- ▶ Example When the US government borrows money on a short-term basis (a year or less), it does so by selling what are called “Treasury Bills” or “T-Bill” for short. A T-bill is a promise by the government to repay a fixed amount at some time in the future. If a T-bill promises to repay \$10,000 in 12 months, and the market interest rate is 7 percent, how much will the bill sell for in the market.

# Example

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- ▶ Antonio is discussing the loan with Shylock to borrow \$12,000 for one year. The interest rate is 12 percent. Both agree that the interest on the loan will be  $0.12 \times \$12,000 = \$1,440$ . So Shylock deducts this interest amount from the loan up front, called the “discount interest” and give Antonio \$10,560. What’s wrong here?

# Loans

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- ▶ Interest-Only Loan - loan that the borrower receive money today and pays interest each period and repay the entire principal at some point in the future.
  
- ▶ Amortized loan – loan that the borrower receives money today and repays in equal payments over its life.
  - ▶ Schedule showing precisely how a loan will be repaid
  - ▶ Break-down the required payment on each specified date, showing how much constitutes interest and how much constitutes repayment of principal.

# Example

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- ▶ Prepare an amortization schedule for a 3-year loan of \$30,000. The interest rate is 8 percent per year, and the loan calls for equal annual payments. How much interest is paid in the third year? How much total interest is paid over the life of the loan?

<b>Year</b>	<b>Beginning Balance</b>	<b>Total payment</b>	<b>Interest paid</b>	<b>Principal paid</b>	<b>Ending balance</b>
<b>Total</b>					

# Reference

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Ross, Stephen A., Randolph W. Westerfield, Jeffrey F. Jaffe, and Bradford D. Jordan, Core Principles and Applications of Corporate Finance, 3th edition, 2010.

Berk and DeMarzo, Corporate Finance, 3th edition, 2013.

Chatuporn Tangkathach, Time Value of Money, 2006.

