

### EE320 Placement test

1. Attempt all.
2. Submit your work (in .pdf) on the Moodle. The required format of your filename is **studentID\_PT**
3. You will get **TWO** bonus points if you submit this placement test by the deadline.
4. **This placement test is due on Friday 14<sup>th</sup>, at 11 AM. Late submission will not be accepted.**

1. Suppose that market demand is given by  $P = 10 - Q^2$  and the market supply is given by  $Q = a + P$ , where  $P$  is the unit price,  $Q$  is the quantity of output, and  $a$  is the coefficient in the supply equation.
  - 1.1) Graph the market demand and market supply curve in a P-Q diagram. Set the value of  $a$  equal to  $-14$ .
  - 1.2) Solve for the market equilibrium quantity ( $Q^*$ ) and price ( $P^*$ ) when  $a = -14$ . Show your work.
  - 1.3) If " $a$ " increases to  $-12$ , what would happen to the market equilibrium quantity and price? State the qualitative predictions without redoing the algebra.

demand equation :  $P = 10 - Q^2$

$$Q = 0 \rightarrow P = 10 - 0$$

$$P = 10$$

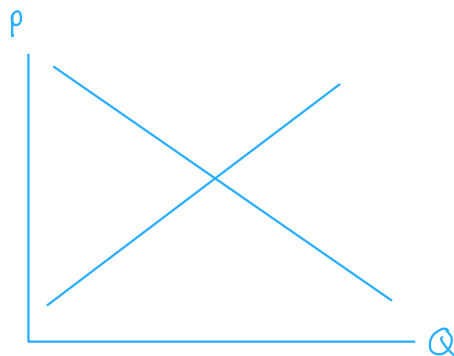
$$\therefore (0, 10)$$

supply equation :  $Q = -14 + P$

$$Q = 0 \rightarrow 0 = -14 + P$$

$$P = 14$$

$$\therefore (0, 14)$$



2. Suppose that the revenue function is given by  $R(Q) = \ln(Q^2 + 1) + 3\left(\frac{Q}{Q+1}\right)$ ,  $Q \geq 0$ . Use the derivative technique and calculate the marginal revenue function. Is the revenue function an increasing or decreasing function?

$$R(Q) = \ln(Q^2 + 1) + 3\left(\frac{Q}{Q+1}\right)$$

$$R'(Q) = \left(\ln(Q^2 + 1)\right)' + \left(\frac{3Q}{Q+1}\right)'$$

$$= \frac{d}{dQ} (\ln(Q^2 + 1)) + \frac{d}{dQ} \left(\frac{3Q}{Q+1}\right)$$

$$= \frac{2Q}{Q^2 + 1} + \frac{3}{(Q+1)^2}$$

3. Suppose that the profit function is given by  $\pi(Q) = -\frac{1}{3}Q^3 - Q^2 + 8Q - 1$  where  $Q$  is the level of output. Use the calculus and solve for the level of profit-maximizing output. Confirm your answer with the second derivative.

$$\pi(Q) = -\frac{1}{3}Q^3 - Q^2 + 8Q - 1$$

$$\pi(Q)' = -1Q^2 - 2Q + 8$$

$$= 2Q - 2$$

4. Suppose that  $A = \begin{bmatrix} 8 & 9 \\ 10 & 11 \end{bmatrix}$ ,  $B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$  and  $C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ , calculate the following object. Show your work.

4.1  $A + B$

NO Answer

4.2  $A * B$

$$\begin{bmatrix} 8 & 9 \\ 10 & 11 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} 8+36 & 16+45 & 24+54 \\ 10+44 & 20+55 & 30+66 \end{bmatrix} = \begin{bmatrix} 44 & 61 & 78 \\ 54 & 75 & 96 \end{bmatrix}$$

4.3  $\det(A)$

$$8 \cdot 11 = 88$$

$$9 \cdot 10 = 90$$

$$88 - 90 = -2$$

4.4  $\det(B)$

NO Answer

4.5  $\det(C)$

$$\begin{vmatrix} 1 & 2 & 3 & | & 1 & 2 \\ 4 & 5 & 6 & | & 4 & 5 \\ 7 & 8 & 9 & | & 7 & 8 \end{vmatrix} = 0$$

5. Suppose that  $U(x, y) = x^a y^b + \ln\left(\frac{x}{x+y}\right)$ . Use the partial derivative technique, calculate  $\frac{\partial U}{\partial x}$  and  $\frac{\partial U}{\partial y}$ .

$$\begin{aligned}\frac{dH(Q)}{dQ} &= -Q^2 - 2Q + 8 = 0 \\ &= -(Q^2 + 2Q - 8) = 0 \\ &= (Q + 4)(Q - 2) = 0 \\ &\quad \begin{array}{l} \text{L} \rightarrow Q = -4 \\ \quad \quad \quad \text{L} \rightarrow Q = 2 \end{array}\end{aligned}$$