0.1) Given that $Z = \frac{x^3 - y^3}{x^2 y^2}$, show that $x\frac{\partial Z}{\partial x} + y\frac{\partial Z}{\partial y} = -Z$

$$\frac{\partial z}{\partial x} = \frac{(x^2y^2)(3x^2) - \left[(x^3 - y^3)(2xy^2)\right]}{x^4y^4}$$

$$x\frac{\partial z}{\partial x} = \frac{3x^4y^2 - 2x^4y^2 + 2xy^5}{x^3y^4} = \frac{x^4y^2 + 2xy^5}{x^3y^4}$$

$$= \frac{x}{y^2} + \frac{2y}{x^2}$$

$$\frac{\partial z}{\partial y} = \frac{(x^2y^2)(-3y^2) - \left[(x^3 - y^3)(2x^2y)\right]}{x^4y^4}$$

$$y\frac{\partial z}{\partial x} = \frac{-3x^2y^4 - 2x^5y + 2x^2y^4}{x^4y^3} = \frac{-2x^5y - x^2y^4}{x^4y^3}$$

$$= \frac{-2x}{y^2} - \frac{y}{x^2}$$

$$\therefore \frac{x}{y^2} + \frac{2y}{x^2} - \frac{2x}{y^2} - \frac{y}{x^2}$$

$$= \frac{y}{x^2} - \frac{x}{y^2} = \frac{y^3 - x^3}{x^2y^2}$$

$$\therefore \frac{y^3 - x^3}{x^2y^2} = -z = \frac{x^3 - y^3}{x^2y^2}$$

0.2) Given that $Z = \frac{x-y}{x+y}$, use the total differential and calculate the change in Z when x= 1 and y =1. What would happen to Z if X increases by 2 units while Y decreases by 2 units?

$$dz = \frac{\partial z}{\partial x} \cdot dx + \frac{\partial z}{\partial y} \cdot dy$$

$$= \frac{(x+y)(1) - (x-y)(1)}{(x+y)^2} \cdot dx + \frac{(x+y)(-1) - (x-y)(1)}{(x+y)^2} \cdot dy$$

when $x = 1$, $y = 1$, $dx = 2$, $dy = -2$

$$\therefore \quad dz = \frac{(1+1)(1) - (1-1)(1)}{(1+1)^2} \cdot 2 + \frac{(1+1)(-1) - (1-1)(1)}{(1+1)^2} \cdot (-2)$$

$$= \frac{2-0}{4} \cdot 2 + \frac{(-2)-0}{4} \cdot (-2)$$

$$= 1 + 1 = 2$$

0.3) If $z = 2x^2y + 3xy + y^2$ where $x = r^2 + 2rs$ and $y = 2r - 4s$, then by means of the chain rule, (0.3a) find ∂z/∂s and ∂z/∂r ; (0.3b) evaluate when r = 1 and s = 0

(a)

$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial s} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial s}$$

$$= (4xy + 3y)(2r) + (2x^2 + 3x + 2y)(-4)$$

$$\frac{\partial z}{\partial r} = \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial r} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial r}$$

$$= (4xy + 3y)(2r + 2s) + (2x^2 + 3x + 2y)(2)$$

(b) $x = r^2 + 2rs$ when $r = 1$, $s = 0$

$$\therefore x = 1 + 2(1)(0) = 1$$

$y = 2r - 4s$ when $r = 1$, $s = 0$

$$\therefore y = 2(1) - 4(0) = 2$$

$$\therefore \frac{\partial z}{\partial s} = (4(1)(2) + 3(2))(2(1)) + (2(1)^2 + 3(1) + 2(2))(-4)$$

$$= (14 \times 2) + (9 \times (-4)) = 28 - 36 = -8$$

$$\therefore \frac{\partial z}{\partial r} = (4(1)(2) + 3(2))(2(1) + 2(0)) + (2(1)^2 + 3(1) + 2(2))(2)$$

$$= (14 \times 2) + (9 \times 2) = 28 + 18 = 46$$

0.4) For $2x^2 + 3y^2 + 2z^2 = 16$, evaluate ∂z/∂y when x = 1, y = 2, z = -1.

$$f(x,y,z) = 2x^2 + 3y^2 + 2z^2 - 16$$

$$\frac{\partial z}{\partial y} = \frac{-\frac{\partial f}{\partial y}}{\frac{\partial f}{\partial z}}$$

$$= \frac{-\frac{\partial(2x^2+3y^2+2z^2-16)}{\partial y}}{-\frac{\partial(2x^2+3y^2+2z^2-16)}{\partial z}}$$

$$= \frac{-6y}{4z}$$

when $y=2,\ z=-1$; $\frac{\partial z}{\partial y} = \frac{-6(2)}{4(-1)}$

$$= \frac{12}{4} = 3$$

0.5) Given that $ln\,(x+y+z)+xyz = ze^{x+y+z}$, evaluate ∂z/∂x when x = 0, y = 1, z = 0.

$$f(x,y,z) = \ln(x+y+z) + xyz - ze^{x+y+z}$$

$$\frac{\partial z}{\partial x} = \frac{-\frac{\partial f}{\partial x}}{\frac{\partial f}{\partial z}}$$

$$= \frac{-\left[\left(\frac{1}{x+y+z}\right) + yz - (1)(ze^{x+y+z})\right]}{\left(\frac{1}{x+y+z}\right) + xy - (1)(ze^{x+y+z})}$$

when $x=0,\ y=1,\ z=0$; $$= \frac{-\left[\left(\frac{1}{0+1+0}\right) + 1(0) - (1)(0 \cdot e^{0+1+0})\right]}{\left(\frac{1}{0+1+0} + (0)1 - (1)(0 \cdot e^{0+1+0})\right)}$$

$$= \frac{-1}{1} = -1$$

Question 1: Suppose that the demand of a product is given by,

$$Q_x = 100 - 4P_x - \frac{50}{\sqrt{P_y}} + 0.5I^2.$$

where P_x is the price of good X, P_y is the price of good Y, and I is the level of income.

Consider the following problems

1.1) What is the relationship between good x and good y? Are they substitute product/complementary product? Show your results using the partial derivative

$$Q_x = 100 - 4P_x - \frac{50}{\sqrt{P_y}} + 0.5I^2$$

$$\frac{\partial Q_x}{\partial P_y} = -50\left(-\frac{1}{2}\right)P_y^{-\frac{3}{2}} = \frac{25}{P_y^{3/2}}$$

$\therefore$ Relation ship between good x and good y are

Substitute because when $P_y \uparrow \Rightarrow Q_x \uparrow$

Means that people will buy more good X when Price of good Y increase.

1.2) Is the product X considered an inferior product?

$$Q_x = 100 - 4P_x - \frac{50}{\sqrt{P_y}} + 0.5I^2$$

$$\frac{\partial Q_x}{\partial I} = I$$

$\therefore$ product X is not inferior good

because as Income increase the quantity of

good X is increase

1.3) What is the level of quantity demanded if $P_x = 10, P_y = 25$ and $I = 10$?

$$Q_x = 100 - 4P_x - \frac{50}{\sqrt{P_y}} + 0.5I^2$$

$$= 100 - 4(10) - \frac{50}{\sqrt{25}} + 0.5(10)^2$$

$$= 100 - 40 - \frac{\cancel{50}^{1}}{\cancel{5}} + 50$$

$$Q_x = 100$$

$\underline{\underline{\text{Ans}}}$

1.4) Calculate the own-price elasticity of demand, and evaluate the value when $P_x = 10, P_y =$ 25 and $I = 10$.

$$\varepsilon_{p_x} = \frac{\partial Q}{\partial P_x} \cdot \frac{P_x}{Q}$$

$$= -4 \cdot \frac{10}{100} = -0.4$$

$\therefore |\varepsilon_{p_y}| = 0.4$ Elastic

1.5) Calculate the cross-price elasticity of demand when $P_x = 10, P_y = 25$ and $I = 10$.

$$\varepsilon_c = \frac{\partial Q_x}{\partial P_y} \cdot \frac{P_y}{Q_x}$$

$$= \frac{25}{P_y^{3/2}} \cdot \frac{25}{100} = \frac{25}{25^{3/2}} \cdot \frac{\cancel{25}^{1}}{\cancel{100}_{4}} = \frac{1}{5} \cdot \frac{1}{4} = \frac{1}{20}$$

$$= 0.05$$

$\therefore |\varepsilon_c| = |0.05| = 0.05$ inelastic

1.6) Calculate income elasticity of demand when $P_x = 10, P_y = 25$ and $I = 10$. Is the product a necessary or luxurious product?

$$\varepsilon_I = \frac{\partial Q_x}{\partial I} \cdot \frac{I}{Q_x}$$

$$= I \cdot \frac{10}{100} = 10 \cdot \frac{10}{100} = 1$$

$\therefore$ This product is necessary product because as income increase 1 quantity of product is increase

Question 3: Suppose that the preference set of a household can be given by

$$U(x,y) = x^{1/2} + y^{1/2},$$

where x is the amount of consumption on good-x, and y is the amount of consumption on good-y. Consider the following problems.

3.1) Calculate the marginal utility of good x and good y, respectively.

$$MU_x = \frac{\partial u}{\partial x} = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$$

$$MU_y = \frac{\partial u}{\partial y} = \frac{1}{2}y^{-1/2} = \frac{1}{2\sqrt{y}}$$

3.2) Does the utility function satisfy with the law of diminishing marginal utility?

Since $MU_x = \frac{1}{2\sqrt{x}} > 0$ and $MU_y = \frac{1}{2\sqrt{y}} > 0$. The utility function does not satisfy with the law of diminishing marginal utility.

3.3) Does the marginal utility curve of good x shift up when the consumer consumes more units of good-y?

$$\frac{\partial MU_x}{\partial y} = \frac{\partial}{\partial y}\frac{1}{2}y^{-1/2} = -\frac{1}{4}y^{-3/2} < 0$$

Since $\frac{\partial MU_x}{\partial y} < 0$, MU_x decreases as y increase.

Hence, the MU_x curve shift down when the consumer consumes more of good-y.

3.4) What is the level of the household utility when the consumer consumes 1 unit of good-x and 2 units of good-y?

$$u(x,y) = x^{1/2} + y^{1/2} = 1^{1/2} + 2^{1/2} = 2.414$$

3.5) Following (3.4), use the total differential to calculate the change in the level of utility under which the consumer increases the consumption on good x by 3 units and reduces the consumption on good y by 1 unit.

$$du = \frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy = \frac{1}{2\sqrt{x}}dx + \frac{1}{2\sqrt{y}}dy = \frac{3}{2\sqrt{x}} - \frac{1}{2\sqrt{y}}$$

3.6) Derive the MRS and show that MRS is decreasing in x.

$$MRS = -\frac{MU_x}{MU_y} = -\frac{\frac{\partial u}{\partial x}}{\frac{\partial u}{\partial y}} = -\frac{\frac{1}{2\sqrt{x}}}{\frac{1}{2\sqrt{y}}} = -\frac{\sqrt{y}}{\sqrt{x}}$$

As x increases, MRS will decrease.

As x increases, y decreases and MRS decreases.

Hence, MRS is decreasing in x

Members

6204640376
6204640681
6204640921