

to_on

2021-06-10

Question 1

```
## Attaching package: 'MTS'

## The following object is masked from 'package:TTR':
##
##      VMA

getSymbols("IPDCONGD",src="FRED")

## 'getSymbols' currently uses auto.assign=TRUE by default, but will
## use auto.assign=FALSE in 0.5-0. You will still be able to use
## 'loadSymbols' to automatically load data. getOption("getSymbols.env")
## and getOption("getSymbols.auto.assign") will still be checked for
## alternate defaults.
##
## This message is shown once per session and may be disabled by setting
## options("getSymbols.warning4.0"=FALSE). See ?getSymbols for details.

## [1] "IPDCONGD"

dim(IPDCONGD)

## [1] 892  1

tail(IPDCONGD)

##           IPDCONGD
## 2020-11-01 104.8970
## 2020-12-01 105.3158
## 2021-01-01 107.6347
## 2021-02-01 100.0964
## 2021-03-01 102.5666
## 2021-04-01 100.7979

getSymbols("IPNCONGD",src="FRED")

## [1] "IPNCONGD"

dim(IPNCONGD)

## [1] 892  1

getSymbols("IPBUSEQ",src="FRED")

## [1] "IPBUSEQ"

dim(IPBUSEQ)

## [1] 892  1
```

```

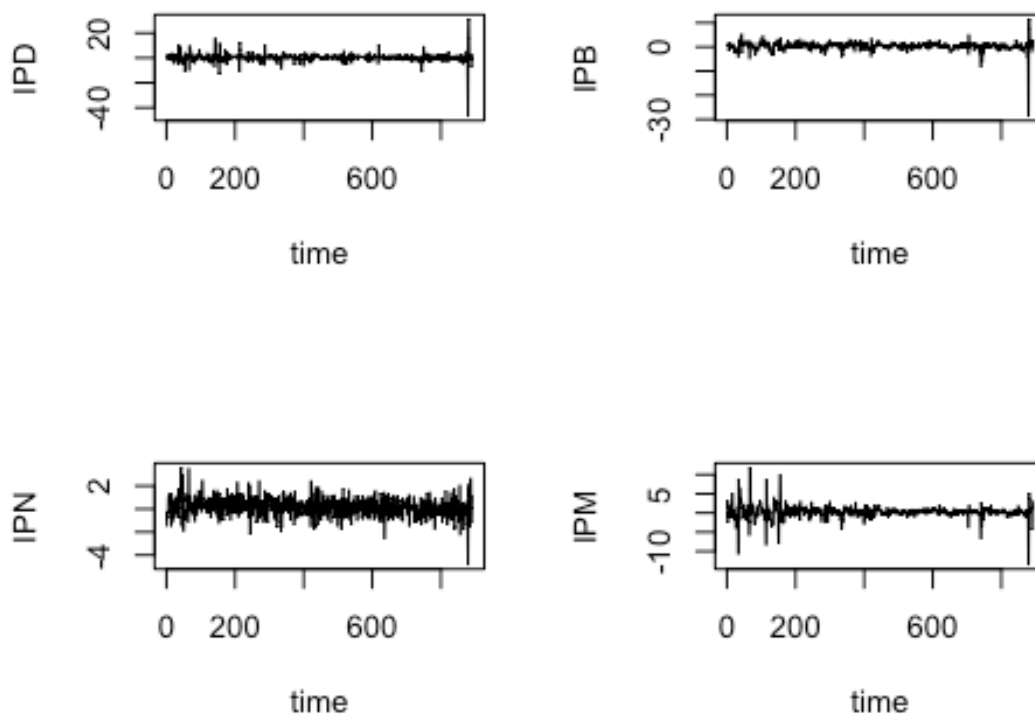
getSymbols("IPMAT",src="FRED")
## [1] "IPMAT"

dim(IPMAT)
## [1] 988  1

IP = cbind(as.numeric(IPDCONGD),as.numeric(IPNCONGD),as.numeric(IPBUSEQ),as.n
umeric(IPMAT[-c(1:96)]))
dim(IP)
## [1] 892  4

colnames(IP)<-c("IPD","IPN","IPB","IPM")
#1.1
IP=log(IP)
zt=diffM(IP)*100
MTSplot(zt)

```



```

#1.2
VARorder(zt)
## selected order: aic = 6
## selected order: bic = 2

```

```

## selected order: hq = 3
## Summary table:
##      p    AIC    BIC    HQ    M(p) p-value
## [1,]  0 2.8153 2.8153 2.8153  0.0000 0.0000
## [2,]  1 2.6470 2.7330 2.6798 178.2331 0.0000
## [3,]  2 2.5125 2.6846 2.5783 147.9623 0.0000
## [4,]  3 2.4570 2.7151 2.5556  79.0713 0.0000
## [5,]  4 2.4563 2.8005 2.5879  31.4646 0.0117
## [6,]  5 2.4045 2.8348 2.5689  75.1412 0.0000
## [7,]  6 2.3832 2.8995 2.5805  48.7671 0.0000
## [8,]  7 2.3984 3.0008 2.6287  17.5535 0.3507
## [9,]  8 2.4168 3.1053 2.6799  14.8136 0.5383
## [10,] 9 2.4224 3.1970 2.7184  25.4492 0.0623
## [11,] 10 2.4268 3.2874 2.7557  26.3997 0.0487
## [12,] 11 2.4327 3.3793 2.7945  24.9845 0.0701
## [13,] 12 2.4258 3.4585 2.8205  35.4558 0.0034
## [14,] 13 2.4266 3.5454 2.8542  28.9289 0.0244

m1=VAR(zt,6)

## Constant term:
## Estimates:  0.2040297 0.1573752 0.1125976 0.1053311
## Std.Error:  0.1218999 0.03035209 0.06279805 0.05838232
## AR coefficient matrix
## AR( 1 )-matrix
##      [,1] [,2] [,3] [,4]
## [1,] 0.0420 0.2551 -0.0533 0.36428
## [2,] 0.0333 -0.1817  0.0139 0.00113
## [3,] 0.0182  0.0786  0.0321 0.20610
## [4,] 0.0212  0.1377  0.0492 0.18413
## standard error
##      [,1] [,2] [,3] [,4]
## [1,] 0.0503 0.1472 0.1004 0.0859
## [2,] 0.0125 0.0366 0.0250 0.0214
## [3,] 0.0259 0.0758 0.0517 0.0442
## [4,] 0.0241 0.0705 0.0481 0.0411
## AR( 2 )-matrix
##      [,1] [,2] [,3] [,4]
## [1,] -0.23369 0.2624 -0.09357 0.12629
## [2,]  0.00238 -0.0443 -0.00387 -0.00966
## [3,] -0.13653 0.0929  0.12781 0.10189
## [4,]  0.00752 0.1451  0.06589 -0.10850
## standard error
##      [,1] [,2] [,3] [,4]
## [1,] 0.0500 0.1497 0.0995 0.0856
## [2,] 0.0124 0.0373 0.0248 0.0213
## [3,] 0.0258 0.0771 0.0512 0.0441
## [4,] 0.0239 0.0717 0.0476 0.0410
## AR( 3 )-matrix
##      [,1] [,2] [,3] [,4]

```

```

## [1,] -0.1374 0.1874 -0.0237 0.12870
## [2,] 0.0226 0.0283 -0.0339 -0.00202
## [3,] -0.1140 0.1101 0.1308 0.13163
## [4,] -0.0127 0.2006 0.0167 0.00225
## standard error
##      [,1] [,2] [,3] [,4]
## [1,] 0.0513 0.1500 0.1005 0.0867
## [2,] 0.0128 0.0373 0.0250 0.0216
## [3,] 0.0264 0.0773 0.0518 0.0447
## [4,] 0.0246 0.0718 0.0481 0.0415
## AR( 4 )-matrix
##      [,1] [,2] [,3] [,4]
## [1,] -0.04052 0.2162 -0.1612 0.20930
## [2,] 0.00597 0.0843 -0.0284 -0.00111
## [3,] -0.02123 0.1047 -0.0745 0.12391
## [4,] 0.04583 0.0957 -0.0388 0.01972
## standard error
##      [,1] [,2] [,3] [,4]
## [1,] 0.0514 0.1501 0.0995 0.0864
## [2,] 0.0128 0.0374 0.0248 0.0215
## [3,] 0.0265 0.0773 0.0513 0.0445
## [4,] 0.0246 0.0719 0.0477 0.0414
## AR( 5 )-matrix
##      [,1] [,2] [,3] [,4]
## [1,] -0.01366 0.08590 -0.24443 0.063
## [2,] -0.00685 0.04387 -0.00165 0.044
## [3,] -0.02941 -0.00504 -0.04715 0.108
## [4,] 0.05632 0.02542 -0.02394 -0.159
## standard error
##      [,1] [,2] [,3] [,4]
## [1,] 0.0494 0.1508 0.0976 0.0847
## [2,] 0.0123 0.0375 0.0243 0.0211
## [3,] 0.0255 0.0777 0.0503 0.0436
## [4,] 0.0237 0.0722 0.0467 0.0406
## AR( 6 )-matrix
##      [,1] [,2] [,3] [,4]
## [1,] 0.1364 -0.0678 -0.3126 0.0537
## [2,] 0.0198 0.0239 0.0240 -0.0172
## [3,] 0.0122 -0.1053 0.0270 0.0320
## [4,] 0.0521 -0.0438 -0.0266 -0.0106
## standard error
##      [,1] [,2] [,3] [,4]
## [1,] 0.0494 0.1471 0.0977 0.0857
## [2,] 0.0123 0.0366 0.0243 0.0213
## [3,] 0.0254 0.0758 0.0503 0.0442
## [4,] 0.0237 0.0704 0.0468 0.0411
##
## Residuals cov-mtx:
##      [,1] [,2] [,3] [,4]
## [1,] 9.5558929 0.8261288 3.5834514 2.2928640

```

```

## [2,] 0.8261288 0.5924362 0.4307313 0.3083878
## [3,] 3.5834514 0.4307313 2.5360441 1.2557605
## [4,] 2.2928640 0.3083878 1.2557605 2.1919322
##
## det(SSE) = 8.698098
## AIC = 2.378593
## BIC = 2.894939
## HQ = 2.575936

#MTSdiag(m1)
m2=refVAR(m1,thres=1.645)

## Constant term:
## Estimates: 0.2985377 0.157489 0.1542269 0.1081254
## Std.Error: 0.1122942 0.02740226 0.0568169 0.05440154
## AR coefficient matrix
## AR( 1 )-matrix
##      [,1] [,2] [,3] [,4]
## [1,] 0.0000 0.226 0 0.417
## [2,] 0.0358 -0.166 0 0.000
## [3,] 0.0340 0.000 0 0.225
## [4,] 0.0395 0.145 0 0.200
## standard error
##      [,1] [,2] [,3] [,4]
## [1,] 0.00000 0.1370 0 0.0701
## [2,] 0.00836 0.0348 0 0.0000
## [3,] 0.01982 0.0000 0 0.0413
## [4,] 0.01832 0.0687 0 0.0381
## AR( 2 )-matrix
##      [,1] [,2] [,3] [,4]
## [1,] -0.213 0.000 0.0000 0.000
## [2,] 0.000 0.000 0.0000 0.000
## [3,] -0.129 0.000 0.1374 0.105
## [4,] 0.000 0.141 0.0737 -0.100
## standard error
##      [,1] [,2] [,3] [,4]
## [1,] 0.0337 0.0000 0.0000 0.0000
## [2,] 0.0000 0.0000 0.0000 0.0000
## [3,] 0.0251 0.0000 0.0484 0.0431
## [4,] 0.0000 0.0698 0.0360 0.0390
## AR( 3 )-matrix
##      [,1] [,2] [,3] [,4]
## [1,] -0.131 0.000 0.000 0.138
## [2,] 0.000 0.000 0.000 0.000
## [3,] -0.113 0.000 0.149 0.134
## [4,] 0.000 0.185 0.000 0.000
## standard error
##      [,1] [,2] [,3] [,4]
## [1,] 0.0379 0.0000 0.0000 0.0804
## [2,] 0.0000 0.0000 0.0000 0.0000

```

```

## [3,] 0.0245 0.0000 0.0478 0.0439
## [4,] 0.0000 0.0654 0.0000 0.0000
## AR( 4 )-matrix
##      [,1] [,2] [,3] [,4]
## [1,] 0.0000 0.0000 -0.1674 0.187
## [2,] 0.0000 0.0616 0.0000 0.000
## [3,] 0.0000 0.0000 -0.0827 0.117
## [4,] 0.0443 0.0000 0.0000 0.000
## standard error
##      [,1] [,2] [,3] [,4]
## [1,] 0.0000 0.0000 0.0746 0.0811
## [2,] 0.0000 0.0329 0.0000 0.0000
## [3,] 0.0000 0.0000 0.0389 0.0422
## [4,] 0.0163 0.0000 0.0000 0.0000
## AR( 5 )-matrix
##      [,1] [,2] [,3] [,4]
## [1,] 0.0000 0 -0.228 0.0000
## [2,] 0.0000 0 0.000 0.0305
## [3,] -0.0407 0 0.000 0.1049
## [4,] 0.0524 0 0.000 -0.1676
## standard error
##      [,1] [,2] [,3] [,4]
## [1,] 0.0000 0 0.0658 0.0000
## [2,] 0.0000 0 0.0000 0.0169
## [3,] 0.0198 0 0.0000 0.0417
## [4,] 0.0177 0 0.0000 0.0379
## AR( 6 )-matrix
##      [,1] [,2] [,3] [,4]
## [1,] 0.1592 0 -0.326 0
## [2,] 0.0255 0 0.000 0
## [3,] 0.0000 0 0.000 0
## [4,] 0.0391 0 0.000 0
## standard error
##      [,1] [,2] [,3] [,4]
## [1,] 0.04560 0 0.0869 0
## [2,] 0.00807 0 0.0000 0
## [3,] 0.00000 0 0.0000 0
## [4,] 0.01600 0 0.0000 0
##
## Residuals cov-mtx:
##      [,1] [,2] [,3] [,4]
## [1,] 9.672303 0.8232940 3.6137189 2.3002478
## [2,] 0.823294 0.6005646 0.4296673 0.3098108
## [3,] 3.613719 0.4296673 2.5704582 1.2639624
## [4,] 2.300248 0.3098108 1.2639624 2.2059227
##
## det(SSE) = 9.263687
## AIC = 2.3114
## BIC = 2.515787
## HQ = 2.389515

```

```
#MTSdiag(m2)
```

```
#1.3
```

```
detach("package:MTS", unload = TRUE)
require(vars)
```

```
## Loading required package: vars
```

```
## Loading required package: MASS
```

```
## Loading required package: strucchange
```

```
## Loading required package: sandwich
```

```
## Loading required package: urca
```

```
## Loading required package: lmtest
```

```
varfit1=VAR(zt, p=6)
summary(varfit1)
```

```
##
```

```
## VAR Estimation Results:
```

```
## =====
```

```
## Endogenous variables: IPD, IPN, IPB, IPM
```

```
## Deterministic variables: const
```

```
## Sample size: 885
```

```
## Log Likelihood: -5980.216
```

```
## Roots of the characteristic polynomial:
```

```
## 0.8157 0.8157 0.7766 0.7766 0.745 0.745 0.7311 0.7311 0.7043 0.6979 0.6979
0.6954 0.6954 0.6882 0.6882 0.6726 0.6726 0.6682 0.6335 0.6335 0.6274 0.6274
0.6055 0.09344
```

```
## Call:
```

```
## VAR(y = zt, p = 6)
```

```
##
```

```
##
```

```
## Estimation results for equation IPD:
```

```
## =====
```

```
## IPD = IPD.l1 + IPN.l1 + IPB.l1 + IPM.l1 + IPD.l2 + IPN.l2 + IPB.l2 + IPM.l
2 + IPD.l3 + IPN.l3 + IPB.l3 + IPM.l3 + IPD.l4 + IPN.l4 + IPB.l4 + IPM.l4 + I
PD.l5 + IPN.l5 + IPB.l5 + IPM.l5 + IPD.l6 + IPN.l6 + IPB.l6 + IPM.l6 + const
```

```
##
```

```
##      Estimate Std. Error t value Pr(>|t|)
```

```
## IPD.l1  0.04198    0.05031   0.834  0.40426
```

```
## IPN.l1  0.25508    0.14719   1.733  0.08345 .
```

```
## IPB.l1 -0.05333    0.10037  -0.531  0.59528
```

```
## IPM.l1  0.36428    0.08589   4.241 2.46e-05 ***
```

```
## IPD.l2 -0.23369    0.04999  -4.675 3.42e-06 ***
```

```
## IPN.l2  0.26244    0.14966   1.754  0.07987 .
```

```
## IPB.l2 -0.09357    0.09945  -0.941  0.34702
```

```
## IPM.l2  0.12629    0.08559   1.476  0.14041
```

```

## IPD.l3 -0.13740    0.05127   -2.680   0.00750 **
## IPN.l3  0.18739    0.14996    1.250   0.21181
## IPB.l3 -0.02369    0.10051   -0.236   0.81374
## IPM.l3  0.12870    0.08668    1.485   0.13800
## IPD.l4 -0.04052    0.05137   -0.789   0.43041
## IPN.l4  0.21624    0.15008    1.441   0.15000
## IPB.l4 -0.16121    0.09950   -1.620   0.10557
## IPM.l4  0.20930    0.08643    2.422   0.01566 *
## IPD.l5 -0.01366    0.04943   -0.276   0.78237
## IPN.l5  0.08590    0.15079    0.570   0.56905
## IPB.l5 -0.24443    0.09756   -2.505   0.01242 *
## IPM.l5  0.06298    0.08469    0.744   0.45732
## IPD.l6  0.13638    0.04939    2.761   0.00588 **
## IPN.l6 -0.06783    0.14705   -0.461   0.64474
## IPB.l6 -0.31255    0.09766   -3.201   0.00142 **
## IPM.l6  0.05373    0.08573    0.627   0.53100
## const  0.20403    0.12190    1.674   0.09454 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 3.136 on 860 degrees of freedom
## Multiple R-squared: 0.1296, Adjusted R-squared: 0.1053
## F-statistic: 5.336 on 24 and 860 DF, p-value: 6.753e-15
##
##
## Estimation results for equation IPN:
## =====
## IPN = IPD.l1 + IPN.l1 + IPB.l1 + IPM.l1 + IPD.l2 + IPN.l2 + IPB.l2 + IPM.l
2 + IPD.l3 + IPN.l3 + IPB.l3 + IPM.l3 + IPD.l4 + IPN.l4 + IPB.l4 + IPM.l4 + I
PD.l5 + IPN.l5 + IPB.l5 + IPM.l5 + IPD.l6 + IPN.l6 + IPB.l6 + IPM.l6 + const
##
##          Estimate Std. Error t value Pr(>|t|)
## IPD.l1  0.033278    0.012527   2.656  0.00804 **
## IPN.l1 -0.181707    0.036648  -4.958 8.57e-07 ***
## IPB.l1  0.013927    0.024990   0.557  0.57748
## IPM.l1  0.001125    0.021386   0.053  0.95805
## IPD.l2  0.002378    0.012447   0.191  0.84850
## IPN.l2 -0.044255    0.037265  -1.188  0.23534
## IPB.l2 -0.003874    0.024762  -0.156  0.87571
## IPM.l2 -0.009664    0.021310  -0.454  0.65029
## IPD.l3  0.022594    0.012765   1.770  0.07708 .
## IPN.l3  0.028276    0.037340   0.757  0.44910
## IPB.l3 -0.033857    0.025026  -1.353  0.17645
## IPM.l3 -0.002022    0.021584  -0.094  0.92538
## IPD.l4  0.005968    0.012790   0.467  0.64090
## IPN.l4  0.084347    0.037369   2.257  0.02425 *
## IPB.l4 -0.028420    0.024776  -1.147  0.25166
## IPM.l4 -0.001115    0.021521  -0.052  0.95870
## IPD.l5 -0.006847    0.012308  -0.556  0.57818

```



```

## IPN.l15  0.043870    0.037546    1.168    0.24295
## IPB.l15 -0.001649    0.024293   -0.068    0.94591
## IPM.l15  0.043986    0.021087    2.086    0.03728 *
## IPD.l16  0.019753    0.012297    1.606    0.10858
## IPN.l16  0.023908    0.036615    0.653    0.51396
## IPB.l16  0.023983    0.024316    0.986    0.32425
## IPM.l16 -0.017243    0.021347   -0.808    0.41945
## const   0.157375    0.030352    5.185 2.69e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 0.7808 on 860 degrees of freedom
## Multiple R-Squared: 0.06574, Adjusted R-squared: 0.03967
## F-statistic: 2.522 on 24 and 860 DF,  p-value: 8.207e-05
##
##
## Estimation results for equation IPB:
## =====
## IPB = IPD.l11 + IPN.l11 + IPB.l11 + IPM.l11 + IPD.l12 + IPN.l12 + IPB.l12 + IPM.l
2 + IPD.l13 + IPN.l13 + IPB.l13 + IPM.l13 + IPD.l14 + IPN.l14 + IPB.l14 + IPM.l4 + I
PD.l15 + IPN.l15 + IPB.l15 + IPM.l15 + IPD.l16 + IPN.l16 + IPB.l16 + IPM.l6 + const
##
##      Estimate Std. Error t value Pr(>|t|)
## IPD.l11  0.018176    0.025918    0.701  0.48332
## IPN.l11  0.078606    0.075825    1.037  0.30017
## IPB.l11  0.032099    0.051705    0.621  0.53489
## IPM.l11  0.206097    0.044246    4.658 3.70e-06 ***
## IPD.l12 -0.136527    0.025753   -5.301 1.46e-07 ***
## IPN.l12  0.092895    0.077101    1.205  0.22860
## IPB.l12  0.127808    0.051233    2.495  0.01280 *
## IPM.l12  0.101888    0.044090    2.311  0.02107 *
## IPD.l13 -0.114010    0.026411   -4.317 1.77e-05 ***
## IPN.l13  0.110077    0.077256    1.425  0.15457
## IPB.l13  0.130807    0.051778    2.526  0.01171 *
## IPM.l13  0.131626    0.044656    2.948  0.00329 **
## IPD.l14 -0.021233    0.026462   -0.802  0.42254
## IPN.l14  0.104686    0.077316    1.354  0.17609
## IPB.l14 -0.074467    0.051260   -1.453  0.14667
## IPM.l14  0.123911    0.044526    2.783  0.00551 **
## IPD.l15 -0.029409    0.025465   -1.155  0.24848
## IPN.l15 -0.005043    0.077681   -0.065  0.94826
## IPB.l15 -0.047148    0.050262   -0.938  0.34848
## IPM.l15  0.107847    0.043630    2.472  0.01363 *
## IPD.l16  0.012195    0.025442    0.479  0.63185
## IPN.l16 -0.105321    0.075756   -1.390  0.16481
## IPB.l16  0.027047    0.050309    0.538  0.59097
## IPM.l16  0.032039    0.044167    0.725  0.46840
## const   0.112598    0.062798    1.793  0.07332 .
## ---

```

```

## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 1.615 on 860 degrees of freedom
## Multiple R-Squared: 0.1681, Adjusted R-squared: 0.1448
## F-statistic: 7.238 on 24 and 860 DF, p-value: < 2.2e-16
##
##
## Estimation results for equation IPM:
## =====
## IPM = IPD.l1 + IPN.l1 + IPB.l1 + IPM.l1 + IPD.l2 + IPN.l2 + IPB.l2 + IPM.l
2 + IPD.l3 + IPN.l3 + IPB.l3 + IPM.l3 + IPD.l4 + IPN.l4 + IPB.l4 + IPM.l4 + I
PD.l5 + IPN.l5 + IPB.l5 + IPM.l5 + IPD.l6 + IPN.l6 + IPB.l6 + IPM.l6 + const
##
##      Estimate Std. Error t value Pr(>|t|)
## IPD.l1  0.021225   0.024096   0.881  0.37864
## IPN.l1  0.137728   0.070493   1.954  0.05105 .
## IPB.l1  0.049229   0.048069   1.024  0.30606
## IPM.l1  0.184129   0.041135   4.476 8.62e-06 ***
## IPD.l2  0.007522   0.023942   0.314  0.75346
## IPN.l2  0.145057   0.071680   2.024  0.04331 *
## IPB.l2  0.065891   0.047631   1.383  0.16691
## IPM.l2 -0.108503   0.040990  -2.647  0.00827 **
## IPD.l3 -0.012702   0.024554  -0.517  0.60508
## IPN.l3  0.200617   0.071824   2.793  0.00534 **
## IPB.l3  0.016737   0.048137   0.348  0.72816
## IPM.l3  0.002253   0.041516   0.054  0.95673
## IPD.l4  0.045827   0.024601   1.863  0.06283 .
## IPN.l4  0.095746   0.071880   1.332  0.18320
## IPB.l4 -0.038801   0.047656  -0.814  0.41576
## IPM.l4  0.019716   0.041395   0.476  0.63399
## IPD.l5  0.056317   0.023675   2.379  0.01759 *
## IPN.l5  0.025416   0.072219   0.352  0.72498
## IPB.l5 -0.023939   0.046727  -0.512  0.60857
## IPM.l5 -0.159069   0.040562  -3.922 9.49e-05 ***
## IPD.l6  0.052073   0.023653   2.201  0.02797 *
## IPN.l6 -0.043805   0.070429  -0.622  0.53412
## IPB.l6 -0.026646   0.046772  -0.570  0.56903
## IPM.l6 -0.010648   0.041061  -0.259  0.79546
## const  0.105331   0.058382   1.804  0.07156 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 1.502 on 860 degrees of freedom
## Multiple R-Squared: 0.1301, Adjusted R-squared: 0.1058
## F-statistic: 5.358 on 24 and 860 DF, p-value: 5.578e-15
##
##
##

```

```

## Covariance matrix of residuals:
##      IPD      IPN      IPB      IPM
## IPD 9.8337 0.8501 3.6876 2.3595
## IPN 0.8501 0.6097 0.4433 0.3174
## IPB 3.6876 0.4433 2.6098 1.2923
## IPM 2.3595 0.3174 1.2923 2.2557
##
## Correlation matrix of residuals:
##      IPD      IPN      IPB      IPM
## IPD 1.0000 0.3472 0.7279 0.5010
## IPN 0.3472 1.0000 0.3514 0.2706
## IPB 0.7279 0.3514 1.0000 0.5326
## IPM 0.5010 0.2706 0.5326 1.0000

varfit2<-restrict(varfit1,thresh=1.645)
summary(varfit2)

##
## VAR Estimation Results:
## =====
## Endogenous variables: IPD, IPN, IPB, IPM
## Deterministic variables: const
## Sample size: 885
## Log Likelihood: -6008.093
## Roots of the characteristic polynomial:
## 0.7922 0.7922 0.7889 0.7889 0.7355 0.7355 0.7336 0.7336 0.7264 0.7264 0.68
09 0.6809 0.573 0.573 0.5636 0.5636 0.5618 0.5618 0.5546 0.5546 0.2921      0
0      0
## Call:
## VAR(y = zt, p = 6)
##
##
## Estimation results for equation IPD:
## =====
## IPD = IPN.l1 + IPM.l1 + IPD.l2 + IPD.l3 + IPM.l3 + IPB.l4 + IPM.l4 + IPB.l
5 + IPD.l6 + IPB.l6 + const
##
##      Estimate Std. Error t value Pr(>|t|)
## IPN.l1  0.22550    0.13698   1.646 0.100080
## IPM.l1  0.41737    0.07015   5.950 3.88e-09 ***
## IPD.l2 -0.21280    0.03365  -6.323 4.08e-10 ***
## IPD.l3 -0.13134    0.03791  -3.464 0.000558 ***
## IPM.l3  0.13768    0.08038   1.713 0.087078 .
## IPB.l4 -0.16739    0.07461  -2.243 0.025117 *
## IPM.l4  0.18714    0.08114   2.307 0.021315 *
## IPB.l5 -0.22831    0.06580  -3.470 0.000547 ***
## IPD.l6  0.15924    0.04560   3.492 0.000503 ***
## IPB.l6 -0.32615    0.08691  -3.753 0.000186 ***
## const   0.29854    0.11229   2.659 0.007992 **
## ---

```

```

## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 3.13 on 874 degrees of freedom
## Multiple R-Squared: 0.1235, Adjusted R-squared: 0.1124
## F-statistic: 11.19 on 11 and 874 DF, p-value: < 2.2e-16
##
##
## Estimation results for equation IPN:
## =====
## IPN = IPD.l1 + IPN.l1 + IPN.l4 + IPM.l5 + IPD.l6 + const
##
##           Estimate Std. Error t value Pr(>|t|)
## IPD.l1  0.035802    0.008362   4.282 2.06e-05 ***
## IPN.l1 -0.166227    0.034823  -4.774 2.12e-06 ***
## IPN.l4  0.061584    0.032901   1.872  0.06157 .
## IPM.l5  0.030534    0.016868   1.810  0.07060 .
## IPD.l6  0.025487    0.008071   3.158  0.00164 **
## const   0.157489    0.027402   5.747 1.25e-08 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 0.7776 on 879 degrees of freedom
## Multiple R-Squared: 0.09067, Adjusted R-squared: 0.08446
## F-statistic: 14.61 on 6 and 879 DF, p-value: 6.039e-16
##
##
## Estimation results for equation IPB:
## =====
## IPB = IPD.l1 + IPM.l1 + IPD.l2 + IPB.l2 + IPM.l2 + IPD.l3 + IPB.l3 + IPM.l
3 + IPB.l4 + IPM.l4 + IPD.l5 + IPM.l5 + const
##
##           Estimate Std. Error t value Pr(>|t|)
## IPD.l1  0.03402     0.01982   1.717  0.08639 .
## IPM.l1  0.22486     0.04129   5.445 6.73e-08 ***
## IPD.l2 -0.12949     0.02506  -5.168 2.93e-07 ***
## IPB.l2  0.13741     0.04837   2.841  0.00460 **
## IPM.l2  0.10522     0.04305   2.444  0.01473 *
## IPD.l3 -0.11282     0.02455  -4.596 4.95e-06 ***
## IPB.l3  0.14926     0.04783   3.121  0.00186 **
## IPM.l3  0.13392     0.04393   3.048  0.00237 **
## IPB.l4 -0.08267     0.03886  -2.128  0.03365 *
## IPM.l4  0.11748     0.04220   2.784  0.00548 **
## IPD.l5 -0.04073     0.01977  -2.061  0.03961 *
## IPM.l5  0.10487     0.04167   2.517  0.01202 *
## const   0.15423     0.05682   2.714  0.00677 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##

```

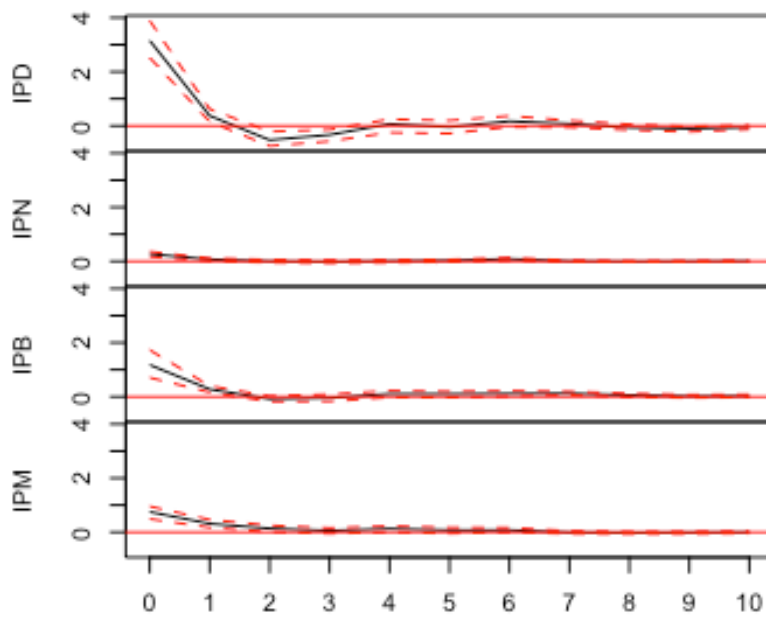
```

##
## Residual standard error: 1.615 on 872 degrees of freedom
## Multiple R-Squared: 0.1838, Adjusted R-squared: 0.1716
## F-statistic: 15.1 on 13 and 872 DF, p-value: < 2.2e-16
##
##
## Estimation results for equation IPM:
## =====
## IPM = IPD.l1 + IPN.l1 + IPM.l1 + IPN.l2 + IPB.l2 + IPM.l2 + IPN.l3 + IPD.l
4 + IPD.l5 + IPM.l5 + IPD.l6 + const
##
##      Estimate Std. Error t value Pr(>|t|)
## IPD.l1  0.03951    0.01832   2.157  0.03126 *
## IPN.l1  0.14507    0.06866   2.113  0.03489 *
## IPM.l1  0.19985    0.03806   5.251 1.90e-07 ***
## IPN.l2  0.14065    0.06977   2.016  0.04411 *
## IPB.l2  0.07372    0.03603   2.046  0.04108 *
## IPM.l2 -0.10017    0.03898  -2.570  0.01034 *
## IPN.l3  0.18455    0.06539   2.822  0.00488 **
## IPD.l4  0.04426    0.01628   2.718  0.00670 **
## IPD.l5  0.05241    0.01771   2.959  0.00317 **
## IPM.l5 -0.16758    0.03786  -4.426 1.08e-05 ***
## IPD.l6  0.03905    0.01600   2.441  0.01485 *
## const   0.10813    0.05440   1.988  0.04717 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 1.495 on 873 degrees of freedom
## Multiple R-Squared: 0.1432, Adjusted R-squared: 0.1314
## F-statistic: 12.16 on 12 and 873 DF, p-value: < 2.2e-16
##
##
## Covariance matrix of residuals:
##      IPD      IPN      IPB      IPM
## IPD 9.9535 0.8472 3.7188 2.3671
## IPN 0.8472 0.6180 0.4422 0.3188
## IPB 3.7188 0.4422 2.6452 1.3007
## IPM 2.3671 0.3188 1.3007 2.2700
##
## Correlation matrix of residuals:
##      IPD      IPN      IPB      IPM
## IPD 1.0000 0.3416 0.7247 0.4980
## IPN 0.3416 1.0000 0.3458 0.2692
## IPB 0.7247 0.3458 1.0000 0.5308
## IPM 0.4980 0.2692 0.5308 1.0000

impresp=irf(varfit2)
plot(impresp)

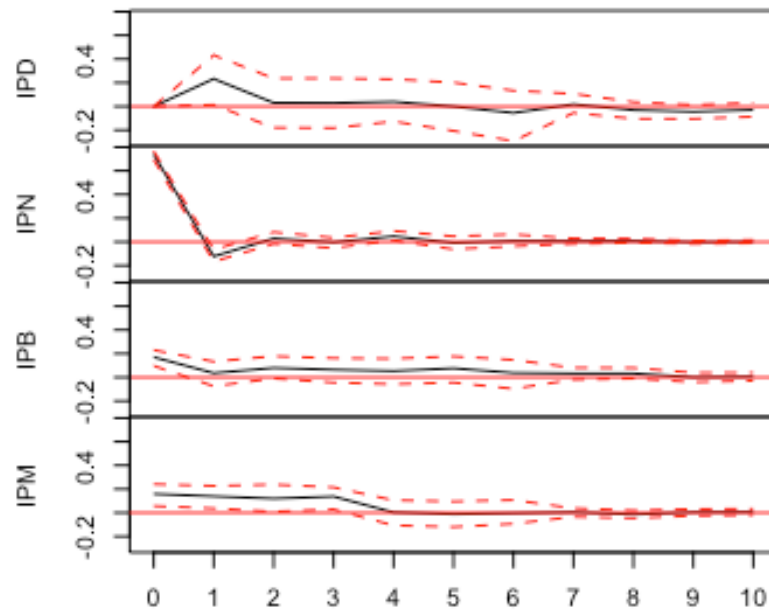
```

Orthogonal Impulse Response from IPD



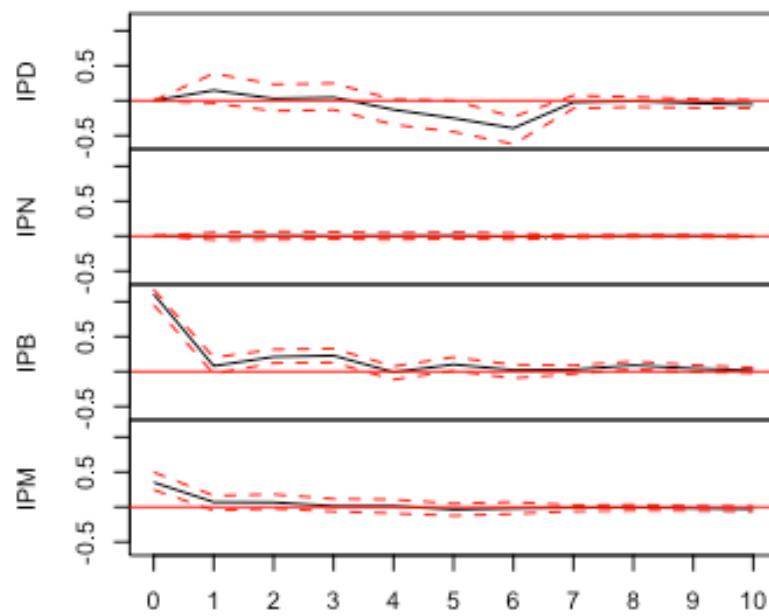
95 % Bootstrap CI, 100 runs

Orthogonal Impulse Response from IPN



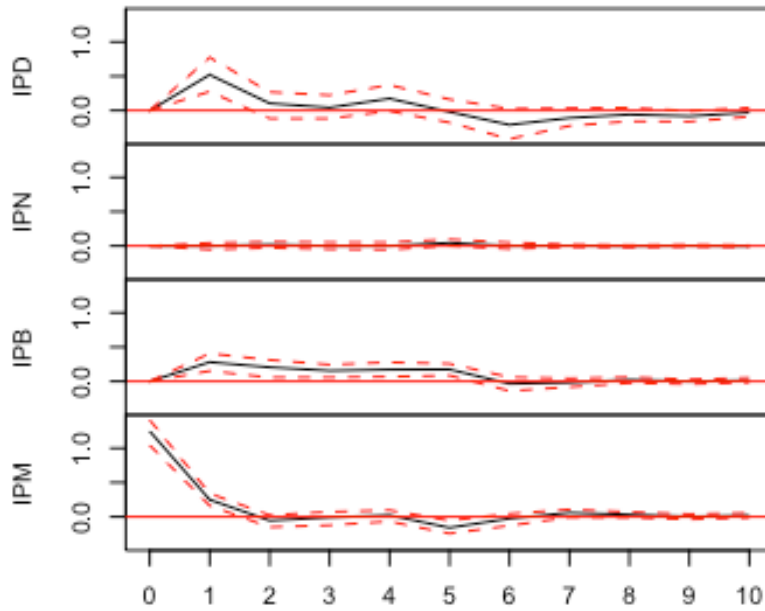
95 % Bootstrap CI, 100 runs

Orthogonal Impulse Response from IPB



95 % Bootstrap CI, 100 runs

Orthogonal Impulse Response from IPM



95 % Bootstrap CI, 100 runs

#1.4

```
fevd(varfit2,n.ahead = 20)
```

```
## $IPD
##          IPD          IPN          IPB          IPM
## [1,] 1.0162791 0.000000000 0.000000000 0.000000000
## [2,] 0.9824672 0.005279578 0.002094763 0.02642407
## [3,] 0.9821815 0.005224575 0.002119878 0.02674088
## [4,] 0.9821024 0.005246297 0.002295769 0.02662441
## [5,] 0.9777281 0.005367907 0.003850941 0.02931819
## [6,] 0.9720542 0.005335837 0.009677250 0.02918178
## [7,] 0.9546833 0.005497666 0.023514152 0.03253440
## [8,] 0.9536307 0.005517148 0.023527725 0.03355381
## [9,] 0.9532684 0.005598469 0.023513264 0.03384959
## [10,] 0.9524733 0.005766881 0.023590530 0.03439857
## [11,] 0.9521017 0.005841247 0.023828008 0.03445752
## [12,] 0.9513875 0.005886896 0.024532248 0.03441912
## [13,] 0.9506049 0.005933864 0.025296225 0.03438856
## [14,] 0.9504630 0.005937495 0.025395872 0.03442671
## [15,] 0.9503586 0.005942617 0.025446425 0.03447517
## [16,] 0.9502519 0.005949491 0.025477382 0.03454385
## [17,] 0.9501491 0.005960875 0.025501861 0.03461060
## [18,] 0.9500726 0.005970263 0.025537864 0.03464160
```

```

## [19,] 0.9500432 0.005976508 0.025559279 0.03464327
## [20,] 0.9500347 0.005979302 0.025564934 0.03464336
##
## $IPN
##          IPD          IPN          IPB          IPM
## [1,] 0.1192644 0.9028287 0.000000e+00 0.0000000000
## [2,] 0.1230410 0.8989813 0.000000e+00 0.0000000000
## [3,] 0.1228147 0.8986023 4.411844e-05 0.0005565253
## [4,] 0.1233267 0.8980870 4.414190e-05 0.0005568213
## [5,] 0.1229838 0.8984253 4.815606e-05 0.0005580230
## [6,] 0.1236989 0.8945437 1.017092e-04 0.0036410180
## [7,] 0.1333677 0.8847696 1.886358e-04 0.0036016444
## [8,] 0.1333756 0.8846578 2.666894e-04 0.0036268538
## [9,] 0.1334806 0.8845419 2.705794e-04 0.0036328385
## [10,] 0.1334950 0.8845124 2.740176e-04 0.0036443219
## [11,] 0.1335318 0.8843872 3.399585e-04 0.0036660101
## [12,] 0.1335199 0.8842869 4.480334e-04 0.0036691312
## [13,] 0.1334896 0.8840691 6.765002e-04 0.0036868716
## [14,] 0.1334886 0.8840617 6.827616e-04 0.0036889731
## [15,] 0.1334948 0.8840410 6.891013e-04 0.0036968741
## [16,] 0.1335076 0.8840086 7.018609e-04 0.0037035652
## [17,] 0.1335088 0.8839842 7.186509e-04 0.0037096221
## [18,] 0.1335096 0.8839629 7.346745e-04 0.0037140199
## [19,] 0.1335089 0.8839469 7.488110e-04 0.0037163215
## [20,] 0.1335108 0.8839409 7.514547e-04 0.0037178379
##
## $IPB
##          IPD          IPN          IPB          IPM
## [1,] 0.5325818 0.01108068 0.4702910 0.000000000
## [2,] 0.5293160 0.01090062 0.4453601 0.02845625
## [3,] 0.5136922 0.01271750 0.4455026 0.04216594
## [4,] 0.5002567 0.01375608 0.4508443 0.04924366
## [5,] 0.4967367 0.01452222 0.4444657 0.05840314
## [6,] 0.4910426 0.01615129 0.4396056 0.06735104
## [7,] 0.4927291 0.01652144 0.4375295 0.06738367
## [8,] 0.4948519 0.01677905 0.4354249 0.06712004
## [9,] 0.4939130 0.01704939 0.4362062 0.06700722
## [10,] 0.4936640 0.01703781 0.4365126 0.06696083
## [11,] 0.4936918 0.01704095 0.4364431 0.06699960
## [12,] 0.4933146 0.01707793 0.4362472 0.06753642
## [13,] 0.4931959 0.01708218 0.4361324 0.06776605
## [14,] 0.4932051 0.01709971 0.4360667 0.06780539
## [15,] 0.4932007 0.01712719 0.4360418 0.06780754
## [16,] 0.4932093 0.01713123 0.4360306 0.06780621
## [17,] 0.4932157 0.01713148 0.4360178 0.06781230
## [18,] 0.4932175 0.01713146 0.4360146 0.06781371
## [19,] 0.4932175 0.01713145 0.4360139 0.06781442
## [20,] 0.4932158 0.01713140 0.4360125 0.06781757
##
## $IPM

```

```
##          IPD          IPN          IPB          IPM
## [1,] 0.2517357 0.01127683 0.05524346 0.6968603
## [2,] 0.2734694 0.01841036 0.05313009 0.6702015
## [3,] 0.2766705 0.02394181 0.05405900 0.6605860
## [4,] 0.2754787 0.03131310 0.05366630 0.6548557
## [5,] 0.2801549 0.03110672 0.05338635 0.6506701
## [6,] 0.2788247 0.03076806 0.05328942 0.6524390
## [7,] 0.2803882 0.03069026 0.05332377 0.6509225
## [8,] 0.2800315 0.03065320 0.05328888 0.6513507
## [9,] 0.2800787 0.03070404 0.05325974 0.6512827
## [10,] 0.2801056 0.03070312 0.05338510 0.6511310
## [11,] 0.2799832 0.03070097 0.05367349 0.6509665
## [12,] 0.2798723 0.03068822 0.05402914 0.6507339
## [13,] 0.2797382 0.03068052 0.05420299 0.6507017
## [14,] 0.2796877 0.03067623 0.05422886 0.6507306
## [15,] 0.2796930 0.03068644 0.05423886 0.6507051
## [16,] 0.2797058 0.03069953 0.05425403 0.6506641
## [17,] 0.2797163 0.03070452 0.05427397 0.6506287
## [18,] 0.2797265 0.03070513 0.05430037 0.6505914
## [19,] 0.2797282 0.03070593 0.05432320 0.6505660
## [20,] 0.2797264 0.03070585 0.05433225 0.6505588
```

#1.5

```
detach("package:vars", unload = TRUE)
require(MTS)
```

```
## Loading required package: MTS
```

```
##
```

```
## Attaching package: 'MTS'
```

```
## The following object is masked from 'package:TTR':
```

```
##
```

```
##      VMA
```

```
VARpred(m2,6,879)
```

```
## orig 879
```

```
## Forecasts at origin: 879
```

```
##          IPD          IPN          IPB          IPM
## [1,] -1.9893 -0.672682 -3.6293 -5.4249
## [2,] 10.9914 0.195877 0.1568 -2.9138
## [3,] 6.6928 0.429555 -1.4738 -1.3429
## [4,] 1.7880 -0.001829 -1.4463 -2.1403
## [5,] 2.0648 -0.661563 -2.9263 -1.0580
## [6,] 0.4245 -1.016649 -2.6670 -0.4851
## Standard Errors of predictions:
##      [,1] [,2] [,3] [,4]
## [1,] 3.110 0.7750 1.603 1.485
## [2,] 3.185 0.7872 1.652 1.545
## [3,] 3.229 0.7880 1.681 1.557
```

IPM

```
1 step [ -5.4249 - 1.96 (1.485)
2 step [ -2.9138 - 1.96 (1.545)
3 step [ -1.3429 - 1.96 (1.557)
4 step [ -2.1403 - 1.96 (1.564)
5 step [ -1.0580 - 1.96 (1.569)
6 step [ -0.4851 - 1.96 (1.579)
```

```
## [4,] 3.246 0.7882 1.704 1.564
## [5,] 3.254 0.7896 1.717 1.569
## [6,] 3.264 0.7914 1.733 1.579
## Root mean square errors of predictions:
##      [,1] [,2] [,3] [,4]
## [1,] 3.154 0.7859 1.626 1.506
## [2,] 3.435 0.8288 1.811 1.736
## [3,] 3.377 0.7906 1.779 1.601
## [4,] 3.306 0.7890 1.784 1.588
## [5,] 3.282 0.7946 1.759 1.588
## [6,] 3.298 0.7975 1.788 1.614
## Observations, predicted values, errors, and MSE
##      time      obs      fcst      err      obs      fcst      err      obs      fcst
## case  880 18.0079 -1.9893 19.9972  1.7436 -0.6727  2.4163  9.8203 -3.6293
## case  881 31.6900 10.9914 20.6986  2.1309  0.1959  1.9350 11.7652  0.1568
## case  882 15.0623  6.6928  8.3695  1.0549  0.4296  0.6253  7.4632 -1.4738
## case  883 -0.9751  1.7880 -2.7630  1.2688 -0.0018  1.2706  2.5593 -1.4463
## case  884 -1.3476  2.0648 -3.4123 -1.5321 -0.6616 -0.8705 -0.1420 -2.9263
## case  885  0.8809  0.4245  0.4564  0.0960 -1.0166  1.1127  0.8422 -2.6670
##      err      obs      fcst      err
## case 13.4495 -1.0442 -5.4249  4.3807
## case 11.6084  5.1887 -2.9138  8.1025
## case  8.9369  3.6564 -1.3429  4.9993
## case  4.0056  0.6973 -2.1403  2.8376
## case  2.7842  0.0761 -1.0580  1.1341
## case  3.5092  1.1026 -0.4851  1.5877
```

Question 2

```
getSymbols("ETH-USD",from="2015-01-01",to="2021-05-24")

## 'getSymbols' currently uses auto.assign=TRUE by default, but will
## use auto.assign=FALSE in 0.5-0. You will still be able to use
## 'loadSymbols' to automatically load data. getOption("getSymbols.env")
## and getOption("getSymbols.auto.assign") will still be checked for
## alternate defaults.
##
## This message is shown once per session and may be disabled by setting
## options("getSymbols.warning4.0"=FALSE). See ?getSymbols for details.

## Warning: ETH-USD contains missing values. Some functions will not work if
## objects contain missing values in the middle of the series. Consider using
## na.omit(), na.approx(), na.fill(), etc to remove or replace them.

## [1] "ETH-USD"

rate=diff(log(as.numeric(`ETH-USD`[,6])))
ETHlogreturn=na.omit(rate)
#2.1
plot(`ETH-USD`,type='l')
```

Question 1

1.1 From the diagram, we can see that all of them may have relationship to each other because they all have high and low volatility in the same period.

$$1.2 \quad \hat{IPD}_t = 0.2985377 + 0.226 IPN_{t-1} + 0.417 IPM_{t-1} - 0.213 \hat{IPD}_{t-2} - 0.131 IPD_{t-3} + 0.138 IPM_{t-3} - 0.1674 IPB_{t-4} + 0.187 IPM_{t-4} \\ (0.1121442) \quad (0.1370) \quad (0.0701) \quad (0.0337) \quad (0.0379) \quad (0.0804) \quad (0.0746) \quad (0.0811) \\ - 0.228 IPB_{t-5} + 0.1592 IPD_{t-6} - 0.326 IPB_{t-6} \\ (0.0658) \quad (0.0456) \quad (0.0819)$$

$$\hat{IPN}_t = 0.157489 + 0.0358 IPD_{t-1} - 0.166 IPN_{t-1} + 0.0616 IPN_{t-4} + 0.0305 IPM_{t-5} + 0.0255 IPD_{t-6} \\ (0.02740216) \quad (0.00876) \quad (0.0348) \quad (0.0329) \quad (0.0119) \quad (0.00807)$$

$$\hat{IPB}_t = 0.1542269 + 0.0340 IPD_{t-1} + 0.225 IPM_{t-1} - 0.129 IPD_{t-2} + 0.1374 IPB_{t-2} + 0.105 IPM_{t-2} - 0.113 IPD_{t-3} + 0.149 IPB_{t-3} \\ (0.0568167) \quad (0.01922) \quad (0.0413) \quad (0.0251) \quad (0.0484) \quad (0.0431) \quad (0.0245) \quad (0.0478) \\ + 0.134 IPM_{t-3} - 0.0827 IPB_{t-4} + 0.117 IPM_{t-4} - 0.0407 IPD_{t-5} + 0.1049 IPM_{t-5} \\ (0.0439) \quad (0.0389) \quad (0.0422) \quad (0.0195) \quad (0.0417)$$

$$\hat{IPM}_t = 0.1081254 + 0.0395 IPD_{t-1} + 0.145 IPN_{t-1} + 0.2 IPM_{t-1} + 0.141 IPN_{t-2} + 0.0737 IPB_{t-2} - 0.1 IPM_{t-2} + 0.185 IPN_{t-2} \\ (0.0540154) \quad (0.01832) \quad (0.0687) \quad (0.0381) \quad (0.0698) \quad (0.0360) \quad (0.039) \quad (0.0654) \\ + 0.0443 IPD_{t-4} + 0.0524 IPD_{t-5} - 0.1676 IPM_{t-5} + 0.0391 IPD_{t-6} \\ (0.0163) \quad (0.0177) \quad (0.0379) \quad (0.016)$$

1.3 IPD = durable consumer good index IPB = business equipment index

IPN = nondurable consumer goods index IPM = material index

Shock from IPD

- the shock of IPD have positive impact to durable consumer good index (IPD) for 1 period and have negative impact to itself in period 2 and 3 before die down in period 4.
- the shock of IPD also have positive impact to IPN, IPB and IPM with different magnitude and magnitude of effect to IPN is the least.

Shock from IPN

- The shock from IPN have positive impact to IPD for 2 period before creating negative impact to IPD in period 6 and die down in period 7.
- The shock from IPN have positive impact to itself from initial until creating negative impact in period 1 and die down.
- The shock from IPN have slightly positive impact to IPB and IPM.

Shock from IPB

- The shock from IPB have negative impact to IPD from period 4 to 7.
- The shock from IPB doesn't have any impact to IPN
- The shock from IPB have positive impact to itself and IPM from initial to period 4 and from initial to period 3, respectively.

Shock from IPM

- The shock from IPM have positive impact to IPD from period 1 to 5 and have negative impact to IPD until period 8.
- The shock from IPM does not have any impact to IPN.
- The shock from IPM have positive impact to IPB from period 1 to 6.
- The shock from IPM have positive impact to itself for first 2 period.

1.4

For IPD

Majority impact come from ε_{IPD} itself with more than 90% and around 0.5% come from ε_{IPN} , around 2% come from ε_{IPB} and around 2-3% come from ε_{IPM} that make us forecast error

For IPN

Majority impact come from ε_{IPN} itself around 88-90% and from ε_{IPD} around 12-13% and the rest come from ε_{IPB} and ε_{IPM} that make us forecast error.

For IPB

Majority impact come from ε_{IPB} itself around 43-47% and from ε_{IPD} around 49-53% and the rest come from ε_{IPN} (1-2%) and ε_{IPM} (4-6%) that make us forecast error.

For IPM

Majority impact come from ε_{IPM} itself around 65-69% and from ε_{IPD} around 27-28%. The rest come from ε_{IPN} (1-3%) and ε_{IPB} (5%) that make us forecast error.

1.5

IPD

1 step	$[-1.9893 - 1.96(3.110), -1.9893 + 1.96(3.110)]$
2 step	$[10.9914 - 1.96(3.185), 10.9914 + 1.96(3.185)]$
3 step	$[6.6928 - 1.96(3.229), 6.6928 + 1.96(3.229)]$
4 step	$[1.7880 - 1.96(3.246), 1.7880 + 1.96(3.246)]$
5 step	$[2.0648 - 1.96(3.254), 2.0648 + 1.96(3.254)]$
6 step	$[0.4245 - 1.96(3.264), 0.4245 + 1.96(3.264)]$

IPN

1 step	$[-0.672682 - 1.96(0.7750), -0.672682 + 1.96(0.7750)]$
2 step	$[0.195877 - 1.96(0.7872), 0.195877 + 1.96(0.7872)]$
3 step	$[0.429555 - 1.96(0.7880), 0.429555 + 1.96(0.7880)]$
4 step	$[-0.001829 - 1.96(0.7882), -0.001829 + 1.96(0.7882)]$
5 step	$[-0.661563 - 1.96(0.7896), -0.661563 + 1.96(0.7896)]$
6 step	$[-1.016649 - 1.96(0.7914), -1.016649 + 1.96(0.7914)]$

IPB

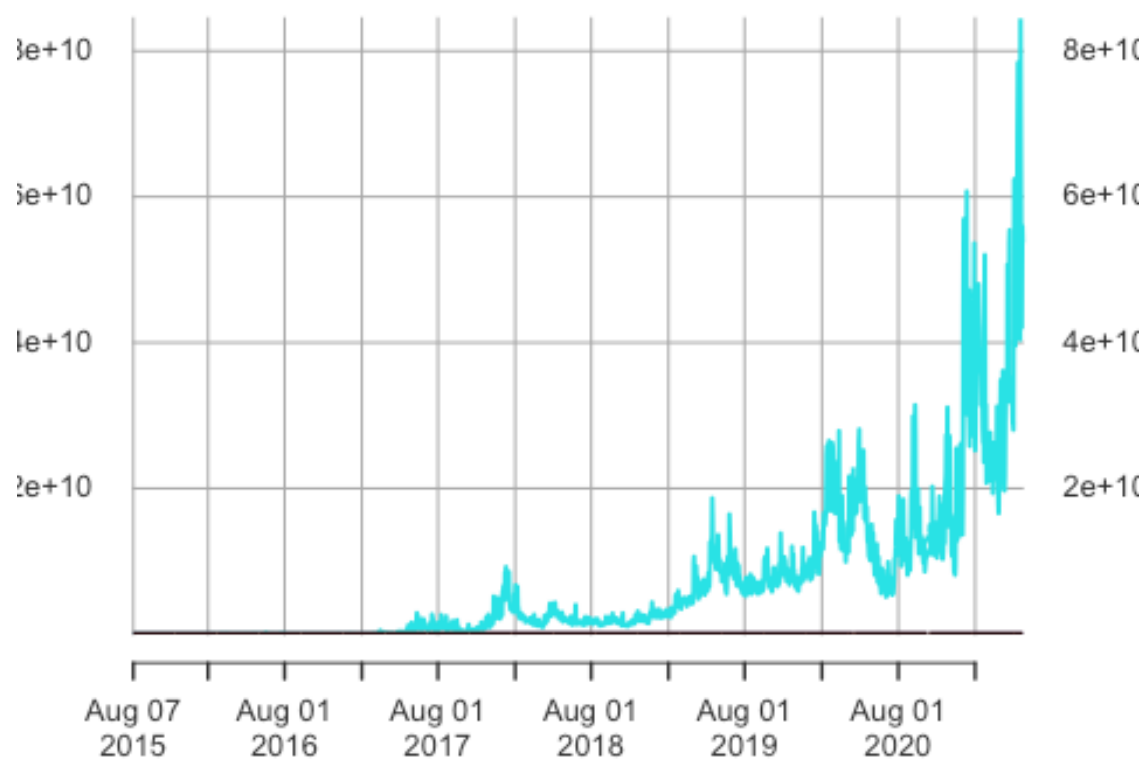
1 step	$[-3.6293 - 1.96(1.603), -3.6293 + 1.96(1.603)]$
2 step	$[0.1568 - 1.96(1.652), 0.1568 + 1.96(1.652)]$
3 step	$[-1.4738 - 1.96(1.681), -1.4738 + 1.96(1.681)]$
4 step	$[-1.4413 - 1.96(1.704), -1.4413 + 1.96(1.704)]$
5 step	$[-2.9263 - 1.96(1.717), -2.9263 + 1.96(1.717)]$
6 step	$[-2.6670 - 1.96(1.733), -2.6670 + 1.96(1.733)]$

IPM

1 step	$[-5.4249 - 1.96(1.485), -5.4249 + 1.96(1.485)]$
2 step	$[-2.9138 - 1.96(1.545), -2.9138 + 1.96(1.545)]$
3 step	$[-1.7429 - 1.96(1.557), -1.7429 + 1.96(1.557)]$
4 step	$[-2.1403 - 1.96(1.564), -2.1403 + 1.96(1.564)]$
5 step	$[-1.0580 - 1.96(1.569), -1.0580 + 1.96(1.569)]$
6 step	$[-0.4851 - 1.96(1.579), -0.4851 + 1.96(1.579)]$

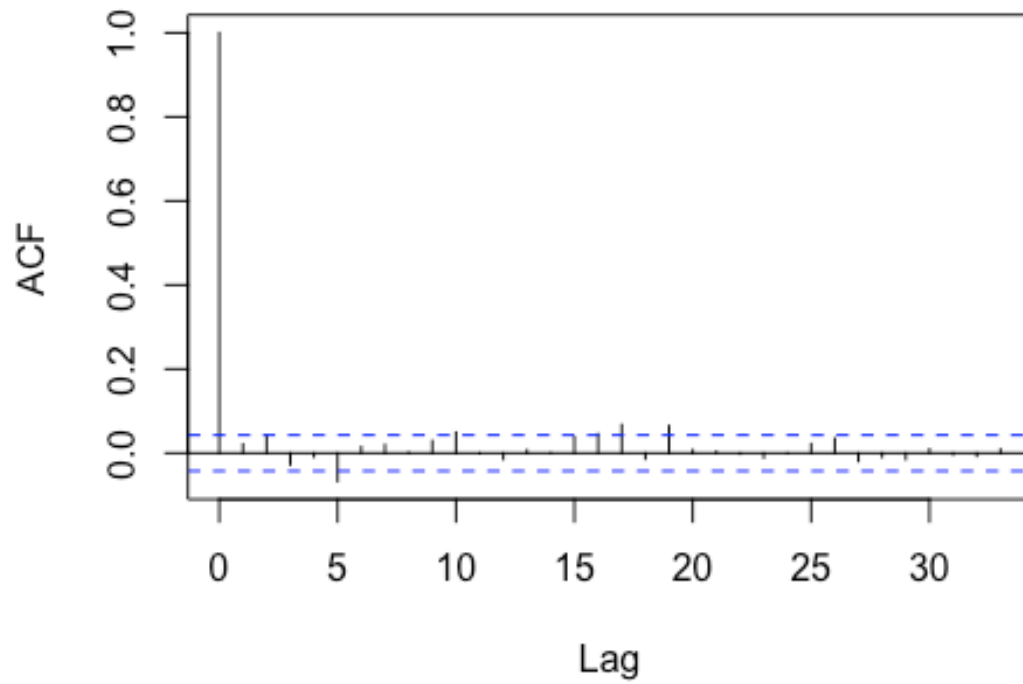
ETH-USD

2015-08-07 / 2021-05-24



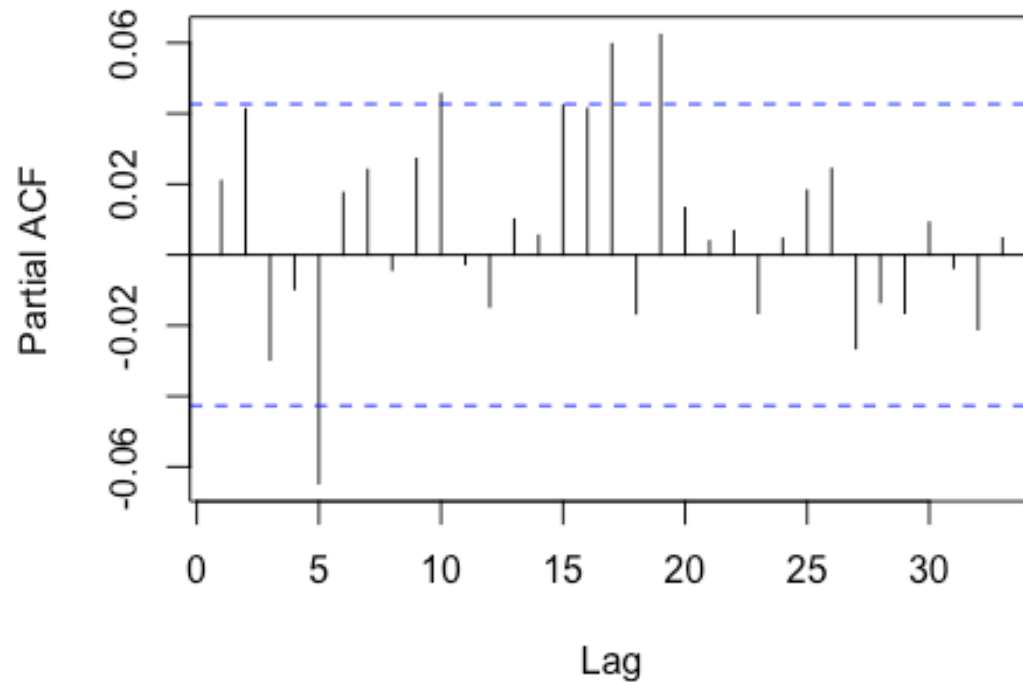
acf(ETHlogreturn)

Series ETHlogreturn



```
pacf(ETHlogreturn)
```


Series ETHlogreturn

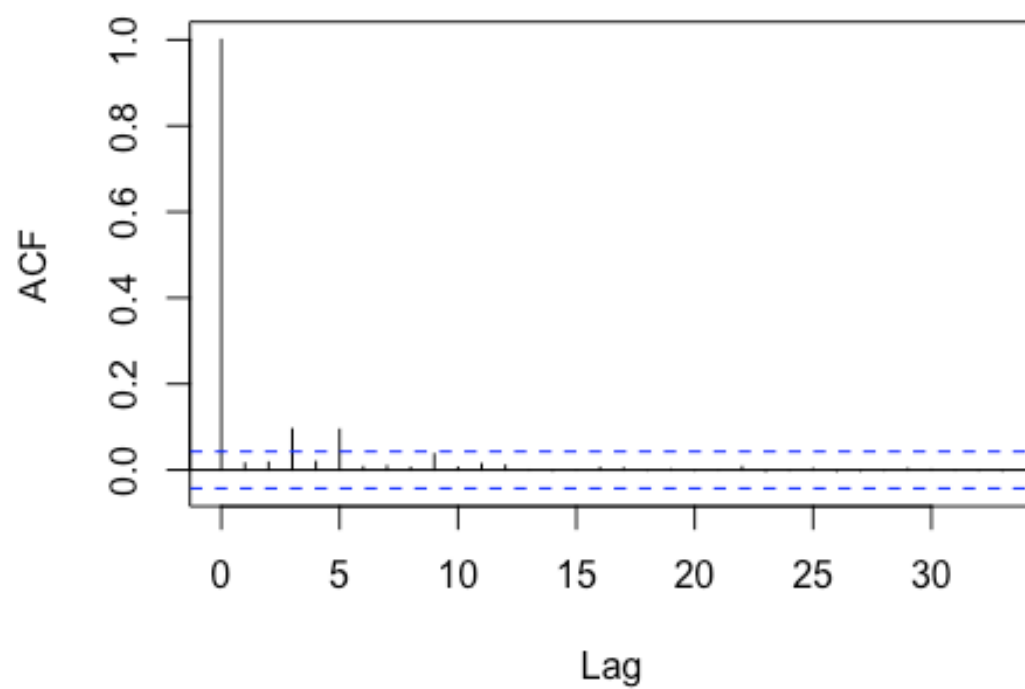


```
t.test(ETHlogreturn)

##
##  One Sample t-test
##
## data:  ETHlogreturn
## t = 2.1366, df = 2109, p-value = 0.03274
## alternative hypothesis: true mean is not equal to 0
## 95 percent confidence interval:
##  0.0002613857 0.0061011761
## sample estimates:
##  mean of x
## 0.003181281

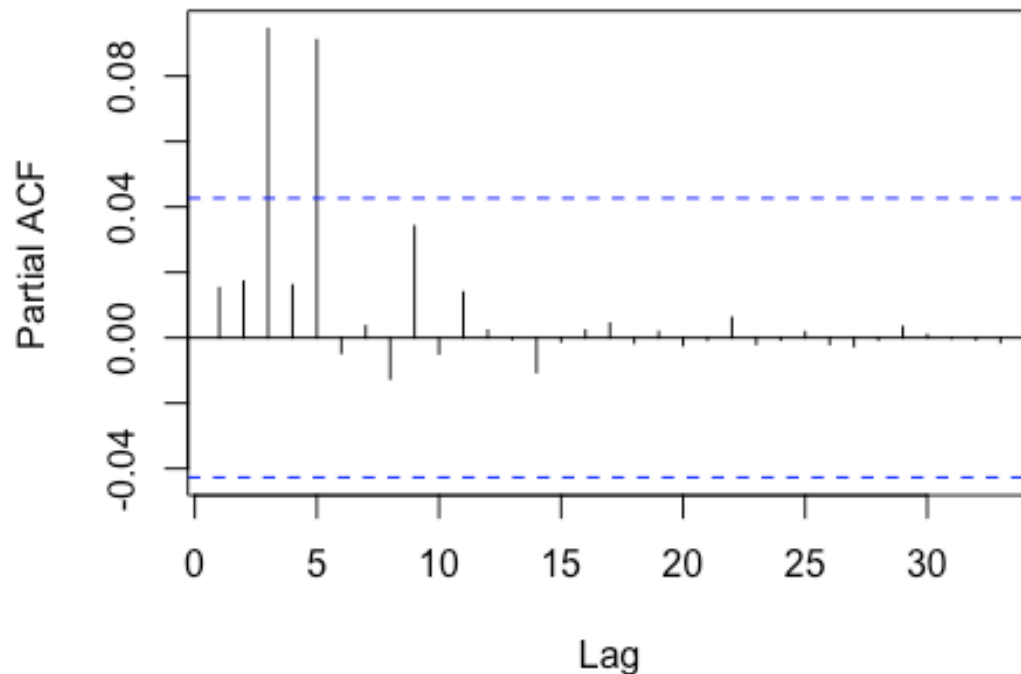
acf(ETHlogreturn^2)
```

Series ETHlogreturn^2



```
pacf(ETHlogreturn^2)
```

Series ETHlogreturn^2



```
Box.test(ETHlogreturn^2,lag=10,type = 'Ljung')
```

```
##
```

```
## Box-Ljung test
```

```
##
```

```
## data: ETHlogreturn^2
```

```
## X-squared = 42.837, df = 10, p-value = 5.319e-06
```

```
m1=garchFit(~garch(1,1),data=ETHlogreturn,trace=F)
```

```
## Warning: Using formula(x) is deprecated when x is a character vector of length > 1.
```

```
## Consider formula(paste(x, collapse = " ")) instead.
```

```
summary(m1)
```

```
##
```

```
## Title:
```

```
## GARCH Modelling
```

```
##
```

```
## Call:
```

```
## garchFit(formula = ~garch(1, 1), data = ETHlogreturn, trace = F)
```

```
##
```

```
## Mean and Variance Equation:
```

```

## data ~ garch(1, 1)
## <environment: 0x7fc979b6ecb0>
## [data = ETHlogreturn]
##
## Conditional Distribution:
## norm
##
## Coefficient(s):
##      mu      omega      alpha1      beta1
## 0.00181437 0.00030506 0.22581488 0.73102249
##
## Std. Errors:
## based on Hessian
##
## Error Analysis:
##      Estimate Std. Error t value Pr(>|t|)
## mu      1.814e-03  1.145e-03   1.584   0.113
## omega   3.051e-04  5.238e-05   5.824 5.74e-09 ***
## alpha1  2.258e-01  3.316e-02   6.810 9.73e-12 ***
## beta1   7.310e-01  3.061e-02  23.879 < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
## 2921.7      normalized: 1.384692
##
## Description:
## Wed Jun  9 16:37:06 2021 by user:
##
##
## Standardised Residuals Tests:
##
##      Jarque-Bera Test  R      Chi^2  235016.8  0
##      Shapiro-Wilk Test  R      W      0.843771  0
##      Ljung-Box Test     R      Q(10)  18.23277  0.05116175
##      Ljung-Box Test     R      Q(15)  22.71407  0.09040232
##      Ljung-Box Test     R      Q(20)  37.32252  0.01070214
##      Ljung-Box Test     R^2    Q(10)  0.1873262 0.99999999
##      Ljung-Box Test     R^2    Q(15)  0.2821231 1
##      Ljung-Box Test     R^2    Q(20)  0.3662152 1
##      LM Arch Test       R      TR^2   8.72732  0.7260276
##
## Information Criterion Statistics:
##      AIC      BIC      SIC      HQIC
## -2.765593 -2.754873 -2.765600 -2.761667

m2=garchFit(~garch(1,1),data=ETHlogreturn,cond.dist='std',trace=F)

```

```
## Warning: Using formula(x) is deprecated when x is a character vector of length > 1.
```

```
## Consider formula(paste(x, collapse = " ")) instead.
```

```
summary(m2)
```

```
##
```

```
## Title:
```

```
## GARCH Modelling
```

```
##
```

```
## Call:
```

```
## garchFit(formula = ~garch(1, 1), data = ETHlogreturn, cond.dist = "std",  
## trace = F)
```

```
##
```

```
## Mean and Variance Equation:
```

```
## data ~ garch(1, 1)
```

```
## <environment: 0x7fc979394a28>
```

```
## [data = ETHlogreturn]
```

```
##
```

```
## Conditional Distribution:
```

```
## std
```

```
##
```

```
## Coefficient(s):
```

```
##          mu          omega        alpha1        beta1        shape  
## 0.00091922 0.00027738 0.28888895 0.74732147 2.93594522
```

```
##
```

```
## Std. Errors:
```

```
## based on Hessian
```

```
##
```

```
## Error Analysis:
```

```
##      Estimate Std. Error t value Pr(>|t|)  
## mu      0.0009192 0.0008166   1.126 0.260301  
## omega   0.0002774 0.0000723   3.837 0.000125 ***  
## alpha1 0.2888889 0.0563659   5.125 2.97e-07 ***  
## beta1   0.7473215 0.0303087  24.657 < 2e-16 ***  
## shape   2.9359452 0.2251239  13.041 < 2e-16 ***
```

```
## ---
```

```
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
##
```

```
## Log Likelihood:
```

```
## 3290.666 normalized: 1.559557
```

```
##
```

```
## Description:
```

```
## Wed Jun 9 16:37:07 2021 by user:
```

```
##
```

```
##
```

```
## Standardised Residuals Tests:
```

```
##      Statistic p-Value  
## Jarque-Bera Test R Chi^2 330248.5 0  
## Shapiro-Wilk Test R W 0.8305545 0
```

```

## Ljung-Box Test      R      Q(10) 17.83585 0.05779657
## Ljung-Box Test      R      Q(15) 22.21607 0.1022622
## Ljung-Box Test      R      Q(20) 35.66518 0.01683461
## Ljung-Box Test      R^2    Q(10) 0.1809104 1
## Ljung-Box Test      R^2    Q(15) 0.2710454 1
## Ljung-Box Test      R^2    Q(20) 0.3454955 1
## LM Arch Test        R      TR^2  9.82396 0.6314013
##
## Information Criterion Statistics:
##      AIC      BIC      SIC      HQIC
## -3.114375 -3.100976 -3.114387 -3.109469

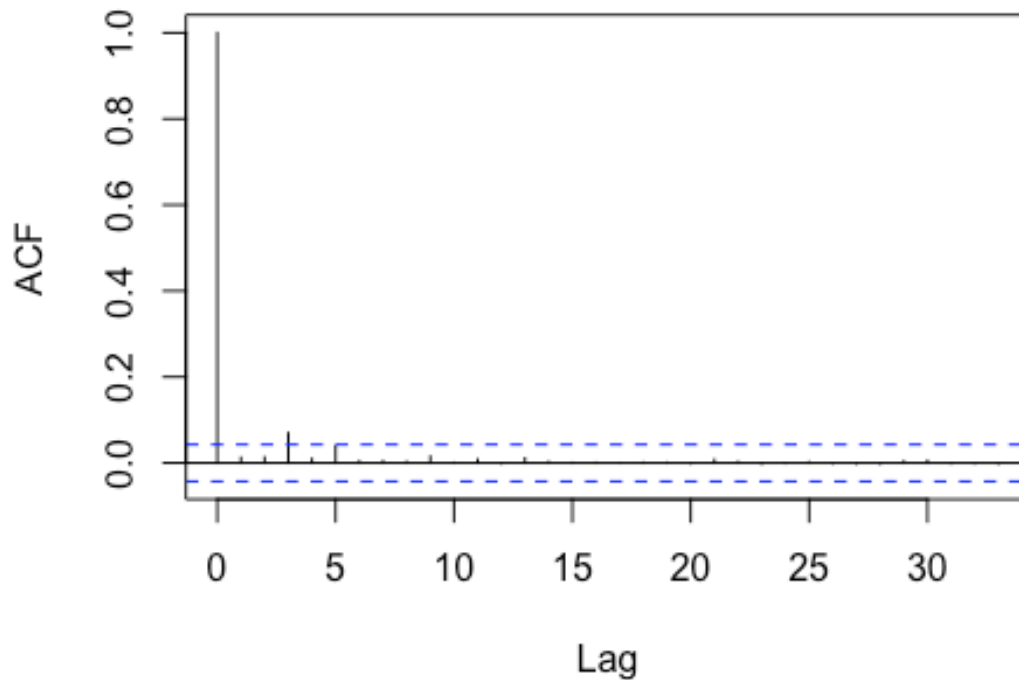
m3=auto.arima(ETHlogreturn)
summary(m3)

## Series: ETHlogreturn
## ARIMA(5,0,5) with non-zero mean
##
## Coefficients:
##      ar1      ar2      ar3      ar4      ar5      ma1      ma2      ma3
##      -0.1825 -0.2961 0.0234 -0.4489 -0.7065 0.2044 0.3502 -0.0420
## s.e.  0.1245 0.0794 0.0954 0.0786 0.1271 0.1312 0.0842 0.1018
##      ma4      ma5      mean
##      0.4456 0.6186 0.0032
## s.e. 0.0803 0.1386 0.0015
##
## sigma^2 estimated as 0.004614: log likelihood=2685.9
## AIC=-5347.79 AICc=-5347.64 BIC=-5279.94
##
## Training set error measures:
##      ME      RMSE      MAE      MPE      MAPE      MA
SE
## Training set -1.441276e-05 0.06774656 0.04212343 78.67127 204.4328 0.69458
22
##      ACF1
## Training set -0.003782915

acf(m3$residuals^2)

```

Series m3\$residuals^2



```
Box.test(m3$residuals^2,lag=10,type = 'Ljung')

##
## Box-Ljung test
##
## data: m3$residuals^2
## X-squared = 15.141, df = 10, p-value = 0.127

m4=garchFit(~garch(1,2),data=ETHlogreturn,cond.dist='std',trace=F)

## Warning: Using formula(x) is deprecated when x is a character vector of length > 1.
## Consider formula(paste(x, collapse = " ")) instead.

summary(m4)

##
## Title:
## GARCH Modelling
##
## Call:
## garchFit(formula = ~garch(1, 2), data = ETHlogreturn, cond.dist = "std",
## trace = F)
##
```

```

## Mean and Variance Equation:
## data ~ garch(1, 2)
## <environment: 0x7fc979970c00>
## [data = ETHlogreturn]
##
## Conditional Distribution:
## std
##
## Coefficient(s):
##      mu      omega      alpha1      beta1      beta2      shape
## 0.00089697 0.00026597 0.30577762 0.67601667 0.06819523 2.91776683
##
## Std. Errors:
## based on Hessian
##
## Error Analysis:
##      Estimate Std. Error t value Pr(>|t|)
## mu      8.970e-04  8.146e-04   1.101 0.270874
## omega  2.660e-04  7.951e-05   3.345 0.000823 ***
## alpha1 3.058e-01  7.412e-02   4.125 3.7e-05 ***
## beta1  6.760e-01  2.046e-01   3.304 0.000953 ***
## beta2  6.820e-02  1.692e-01   0.403 0.686871
## shape  2.918e+00  2.200e-01  13.263 < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
## 3289.918      normalized: 1.559203
##
## Description:
## Wed Jun  9 16:37:09 2021 by user:
##
## Standardised Residuals Tests:
##
##      Jarque-Bera Test  R      Chi^2 315496.4 0
##      Shapiro-Wilk Test  R      W      0.8280633 0
##      Ljung-Box Test    R      Q(10) 15.07738 0.1292662
##      Ljung-Box Test    R      Q(15) 19.14171 0.2073839
##      Ljung-Box Test    R      Q(20) 33.11988 0.03273253
##      Ljung-Box Test    R^2 Q(10) 20.51501 0.02474113
##      Ljung-Box Test    R^2 Q(15) 20.61074 0.1497339
##      Ljung-Box Test    R^2 Q(20) 20.68499 0.4158778
##      LM Arch Test      R      TR^2 10.2514 0.5939156
##
## Information Criterion Statistics:
##      AIC      BIC      SIC      HQIC
## -3.112719 -3.096640 -3.112735 -3.106831

m5=garchFit(~garch(5,0),data=ETHlogreturn,cond.dist='std',trace=F)

```



```

## Warning: Using formula(x) is deprecated when x is a character vector of length > 1.
## Consider formula(paste(x, collapse = " ")) instead.

summary(m5)

##
## Title:
## GARCH Modelling
##
## Call:
## garchFit(formula = ~garch(5, 0), data = ETHlogreturn, cond.dist = "std",
## trace = F)
##
## Mean and Variance Equation:
## data ~ garch(5, 0)
## <environment: 0x7fc977ac4a08>
## [data = ETHlogreturn]
##
## Conditional Distribution:
## std
##
## Coefficient(s):
##      mu      omega      alpha1      alpha2      alpha3      alpha4
## 0.00068415 0.00151061 0.33633976 0.24404973 0.20796997 0.18127655
##      alpha5      shape
## 0.19545918 2.83419557
##
## Std. Errors:
## based on Hessian
##
## Error Analysis:
##      Estimate Std. Error t value Pr(>|t|)
## mu      0.0006842 0.0008230 0.831 0.405829
## omega 0.0015106 0.0002658 5.684 1.31e-08 ***
## alpha1 0.3363398 0.0877557 3.833 0.000127 ***
## alpha2 0.2440497 0.0696305 3.505 0.000457 ***
## alpha3 0.2079700 0.0688542 3.020 0.002524 **
## alpha4 0.1812766 0.0607164 2.986 0.002830 **
## alpha5 0.1954592 0.0630001 3.103 0.001919 **
## shape 2.8341956 0.2144659 13.215 < 2e-16 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
## 3280.91 normalized: 1.554933
##
## Description:
## Wed Jun 9 16:37:09 2021 by user:
##

```

```
##
## Standardised Residuals Tests:
##
##           Statistic p-Value
## Jarque-Bera Test   R      Chi^2 147170.5 0
## Shapiro-Wilk Test  R      W      0.8553714 0
## Ljung-Box Test     R      Q(10) 15.93493 0.1015104
## Ljung-Box Test     R      Q(15) 19.4331 0.1947742
## Ljung-Box Test     R      Q(20) 34.17784 0.02494653
## Ljung-Box Test     R^2  Q(10) 13.34737 0.2048933
## Ljung-Box Test     R^2  Q(15) 13.4776 0.5654594
## Ljung-Box Test     R^2  Q(20) 13.60017 0.8501761
## LM Arch Test       R      TR^2 12.48429 0.4076102
##
## Information Criterion Statistics:
##           AIC      BIC      SIC      HQIC
## -3.102284 -3.080845 -3.102313 -3.094433

m6=garchFit(~garch(3,0),data=ETHlogreturn,cond.dist='std',trace=F)

## Warning: Using formula(x) is deprecated when x is a character vector of length > 1.
## Consider formula(paste(x, collapse = " ")) instead.

summary(m6)

##
## Title:
## GARCH Modelling
##
## Call:
## garchFit(formula = ~garch(3, 0), data = ETHlogreturn, cond.dist = "std",
## trace = F)
##
## Mean and Variance Equation:
## data ~ garch(3, 0)
## <environment: 0x7fc97900e6d8>
## [data = ETHlogreturn]
##
## Conditional Distribution:
## std
##
## Coefficient(s):
##           mu      omega      alpha1      alpha2      alpha3      shape
## 0.0010220 0.0022809 0.4012062 0.3017895 0.2775118 2.7320298
##
## Std. Errors:
## based on Hessian
##
## Error Analysis:
##           Estimate Std. Error t value Pr(>|t|)
## mu      0.0010220 0.0008399 1.217 0.223713
```

```
## omega 0.0022809 0.0003987 5.722 1.06e-08 ***
## alpha1 0.4012062 0.1065742 3.765 0.000167 ***
## alpha2 0.3017895 0.0840833 3.589 0.000332 ***
## alpha3 0.2775118 0.0846087 3.280 0.001038 **
## shape 2.7320298 0.2015884 13.553 < 2e-16 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
## 3264.491 normalized: 1.547152
##
## Description:
## Wed Jun 9 16:37:10 2021 by user:
##
##
## Standardised Residuals Tests:
##
## Statistic p-Value
## Jarque-Bera Test R Chi^2 215047.7 0
## Shapiro-Wilk Test R W 0.8281301 0
## Ljung-Box Test R Q(10) 11.60007 0.312713
## Ljung-Box Test R Q(15) 15.70265 0.4020844
## Ljung-Box Test R Q(20) 32.21015 0.0411099
## Ljung-Box Test R^2 Q(10) 2.426389 0.9919055
## Ljung-Box Test R^2 Q(15) 2.519227 0.9998664
## Ljung-Box Test R^2 Q(20) 2.886809 0.9999971
## LM Arch Test R TR^2 10.49032 0.5730271
##
## Information Criterion Statistics:
## AIC BIC SIC HQIC
## -3.088617 -3.072538 -3.088633 -3.082729

predict(m6,10)

## meanForecast meanError standardDeviation
## 1 0.001021973 0.1602262 0.1602262
## 2 0.001021973 0.1727443 0.1727443
## 3 0.001021973 0.1897456 0.1897456
## 4 0.001021973 0.1812615 0.1812615
## 5 0.001021973 0.1860361 0.1860361
## 6 0.001021973 0.1899298 0.1899298
## 7 0.001021973 0.1905687 0.1905687
## 8 0.001021973 0.1932418 0.1932418
## 9 0.001021973 0.1955342 0.1955342
## 10 0.001021973 0.1974039 0.1974039
```

Question 3

```
##
## Attaching package: 'MTS'
```

Question 2

To identify the optimal model for Ethereum. Firstly, we pull the data from Yahoo Finance. Then, we turn the series into log return and construct ACF. The warn that we need to change price to return is that it is not weakly stationary.

From ACF figure, we can see that there is no pattern in figure which can implied that AR, MA, ARMA model may not suitable for mean equation. The mean equation of this series may be $r_t = \mu + a_t$

Then, we check that μ is equal to zero or not. After, use t-test and obtain p-value equal to 0.03274 which less than $\alpha = 0.05$. So, we can reject $H_0: \mu = 0$ at 0.05 level of significant. That mean, mean of log return of ETH is not equal to zero at 95% CI.

③ most of the standardized residual test show that the model is adequate.

Next, we check ARCH effect in the series by using Ljung-Box test

$H_0: \rho_1 = \rho_2 = \dots = \rho_m$ or There is no ARCH effect

$H_1: \exists \rho_m \neq 0; m = 1, 2, 3, \dots, 10$ or There is ARCH effect

p-value = 5.319e-06

Since p-value is less than $\alpha = 0.05$. Then, we reject H_0 at 0.05 level of significant.

That is, there is ARCH effect in the residual term in the series.

After that, we construct PACF to find optimal lag

We try to construct 6 model and we choose model b that may be the most suitable model in this case.

Because ① the sum of parameter of variance eq. < 1 ② it change distribution to student-t ③ all of parameter is significant in variance eq.

④ BIC, AIC criterion have value very close to the model that get the highest value

The fitted model: mean eq.: $\hat{r}_t = 0.0010220 + a_t$
(0.0008399)

Variance eq.: $\hat{\sigma}_t^2 = 0.0022809 + 0.4012062 a_{t-1}^2 + 0.3017895 a_{t-2}^2 + 0.2775118 a_{t-3}^2$
(0.0003987) (0.10657+2) (0.8840833) (0.0846087)

with $a_t \sim t_{2.7320298}$

Finally, we check the adequacy of the model from $\hat{a}_t$ (standardized residual term)

Checking distribution of model by Jarque-Bera test

and get p-value < 0.05 which mean that the distribution is not normal which suitable to the model that we use student-t distribution.

```

## Standardised Residuals Tests:
##
## Jarque-Bera Test  R    Chi^2 215047.7 0
## Shapiro-Wilk Test R    W      0.8281301 0
## Ljung-Box Test    R    Q(10) 11.60007 0.312713
## Ljung-Box Test    R    Q(15) 15.70265 0.4020844
## Ljung-Box Test    R    Q(20) 32.21015 0.0411099
## Ljung-Box Test    R^2  Q(10) 2.426389 0.9919055
## Ljung-Box Test    R^2  Q(15) 2.519227 0.9998664
## Ljung-Box Test    R^2  Q(20) 2.886809 0.9999971
## LM Arch Test      R    TR^2  10.49032 0.5730271
##

```

test linear dependency in $\hat{a}_t$ by Ljung-Box test.

From the result, we get p-value > 0.05 for $m = 10, 15$.

Then, there is no serial correlation in $\hat{a}_t$ or mean equation is adequate with 95% CI.

test non-linear dependent in $\hat{a}_t$

According to Ljung-Box test of $\hat{a}_t^2$ ($m = 10, 15, 20$), we get p-value > 0.05. Then, there is no serial correlation among $\hat{a}_t^2$ and variance equation is adequate with 95% CI

test ARCH effect by LM ARCH test. Then, we get p-value > 0.05. Therefore there is no ARCH effect left in this model.

In conclusion, we can use this model to forecast the log return and its volatility

mean ± 1.96 (SD)

```

## The following object is masked from 'package:TTR':
##
##      VMA

library(fUnitRoots)
getSymbols("CLVMNACSCAB1GQUK",src="FRED")

## 'getSymbols' currently uses auto.assign=TRUE by default, but will
## use auto.assign=FALSE in 0.5-0. You will still be able to use
## 'loadSymbols' to automatically load data. getOption("getSymbols.env")
## and getOption("getSymbols.auto.assign") will still be checked for
## alternate defaults.
##
## This message is shown once per session and may be disabled by setting
## options("getSymbols.warning4.0"=FALSE). See ?getSymbols for details.

## [1] "CLVMNACSCAB1GQUK"

dim(CLVMNACSCAB1GQUK)

## [1] 183    1

tail(CLVMNACSCAB1GQUK)

##              CLVMNACSCAB1GQUK
## 2019-04-01          470665.5
## 2019-07-01          473006.0
## 2019-10-01          473087.6
## 2020-01-01          458804.7
## 2020-04-01          372766.5
## 2020-07-01          432422.1

getSymbols("NAEXKP01CAQ189S",src="FRED")

## [1] "NAEXKP01CAQ189S"

dim(NAEXKP01CAQ189S)

## [1] 241    1

tail(NAEXKP01CAQ189S)

##              NAEXKP01CAQ189S
## 2019-10-01    528528250000
## 2020-01-01    518324000000
## 2020-04-01    459052500000
## 2020-07-01    499863000000
## 2020-10-01    511481250000
## 2021-01-01    519664950000

getSymbols("GDPC1",src="FRED")

## [1] "GDPC1"

```

```

dim(GDPC1)

## [1] 297 1

tail(GDPC1)

##          GDPC1
## 2019-10-01 19253.96
## 2020-01-01 19010.85
## 2020-04-01 17302.51
## 2020-07-01 18596.52
## 2020-10-01 18794.43
## 2021-01-01 19088.06

x=cbind(as.numeric(CLVMNACSCAB1GQUK[21:146,]),as.numeric(NAEXKP01CAQ189S[77:202,]),as.numeric(GDPC1[133:258,]))
zt=log(x)
#ccm(zt)
colnames(zt)<-c("UKrealgdp","Canadarealgdp","USArealgdp")
#3.1
VARorder(zt)

## selected order: aic = 2
## selected order: bic = 2
## selected order: hq = 2
## Summary table:
##      p      AIC      BIC      HQ      M(p) p-value
## [1,] 0 -19.1870 -19.1870 -19.1870 0.0000 0.0000
## [2,] 1 -30.8581 -30.6555 -30.7758 1281.8143 0.0000
## [3,] 2 -31.7835 -31.3783 -31.6189 112.7073 0.0000
## [4,] 3 -31.7746 -31.1669 -31.5277 13.7316 0.1322
## [5,] 4 -31.7090 -30.8986 -31.3798 7.6844 0.5662
## [6,] 5 -31.6656 -30.6526 -31.2540 9.5949 0.3843
## [7,] 6 -31.5926 -30.3771 -31.0988 6.5361 0.6853
## [8,] 7 -31.5344 -30.1163 -30.9583 7.6607 0.5687
## [9,] 8 -31.4368 -29.8160 -30.7783 3.9549 0.9144
## [10,] 9 -31.4173 -29.5940 -30.6765 10.4266 0.3171
## [11,] 10 -31.4933 -29.4674 -30.6702 17.8361 0.0371
## [12,] 11 -31.4227 -29.1942 -30.5174 5.6773 0.7717
## [13,] 12 -31.4948 -29.0637 -30.5071 16.2242 0.0623
## [14,] 13 -31.4542 -28.8205 -30.3842 7.4155 0.5939

detach("package:MTS", unload = TRUE)
require(vars)

## Loading required package: vars
## Loading required package: MASS
## Loading required package: strucchange
## Loading required package: sandwich

```

```

## Loading required package: urca

##
## Attaching package: 'urca'

## The following objects are masked from 'package:fUnitRoots':
##
##      punitroot, qunitroot, unitrootTable

## Loading required package: lmtest

varfit=VAR(zt,p=2)
summary(varfit)

##
## VAR Estimation Results:
## =====
## Endogenous variables: UKrealgdp, Canadarealgdp, USArealgdp
## Deterministic variables: const
## Sample size: 124
## Log Likelihood: 1414.364
## Roots of the characteristic polynomial:
## 0.9856 0.9563 0.8217 0.5202 0.5202 0.1523
## Call:
## VAR(y = zt, p = 2)
##
##
## Estimation results for equation UKrealgdp:
## =====
## UKrealgdp = UKrealgdp.l1 + Canadarealgdp.l1 + USArealgdp.l1 + UKrealgdp.l2
+ Canadarealgdp.l2 + USArealgdp.l2 + const
##
##              Estimate Std. Error t value Pr(>|t|)
## UKrealgdp.l1    1.40140    0.08166  17.161 < 2e-16 ***
## Canadarealgdp.l1 0.03900    0.08309   0.469   0.640
## USArealgdp.l1    0.11577    0.09334   1.240   0.217
## UKrealgdp.l2    -0.49628    0.08134  -6.101 1.4e-08 ***
## Canadarealgdp.l2 -0.03246    0.08465  -0.383   0.702
## USArealgdp.l2    -0.04250    0.09416  -0.451   0.653
## const           0.34393    0.44609   0.771   0.442
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 0.005656 on 117 degrees of freedom
## Multiple R-Squared: 0.9995, Adjusted R-squared: 0.9994
## F-statistic: 3.565e+04 on 6 and 117 DF, p-value: < 2.2e-16
##
##
## Estimation results for equation Canadarealgdp:
## =====

```

```

## Canadarealgdp = UKrealgdp.l1 + Canadarealgdp.l1 + USArealgdp.l1 + UKrealgdp.l2 + Canadarealgdp.l2 + USArealgdp.l2 + const
##
##              Estimate Std. Error t value Pr(>|t|)
## UKrealgdp.l1      0.28589    0.08432   3.391 0.000952 ***
## Canadarealgdp.l1  1.20185    0.08579  14.009 < 2e-16 ***
## USArealgdp.l1     0.35327    0.09638   3.665 0.000373 ***
## UKrealgdp.l2     -0.26496    0.08399  -3.155 0.002042 **
## Canadarealgdp.l2 -0.24763    0.08741  -2.833 0.005432 **
## USArealgdp.l2    -0.33122    0.09723  -3.407 0.000902 ***
## const             0.74342    0.46060   1.614 0.109218
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 0.00584 on 117 degrees of freedom
## Multiple R-Squared: 0.9994, Adjusted R-squared: 0.9994
## F-statistic: 3.394e+04 on 6 and 117 DF, p-value: < 2.2e-16
##
## Estimation results for equation USArealgdp:
## =====
## USArealgdp = UKrealgdp.l1 + Canadarealgdp.l1 + USArealgdp.l1 + UKrealgdp.l2 + Canadarealgdp.l2 + USArealgdp.l2 + const
##
##              Estimate Std. Error t value Pr(>|t|)
## UKrealgdp.l1      0.36850    0.08432   4.370 2.7e-05 ***
## Canadarealgdp.l1  0.17407    0.08580   2.029 0.04474 *
## USArealgdp.l1     1.04262    0.09639  10.817 < 2e-16 ***
## UKrealgdp.l2     -0.26915    0.08399  -3.204 0.00174 **
## Canadarealgdp.l2 -0.25543    0.08741  -2.922 0.00417 **
## USArealgdp.l2    -0.06237    0.09723  -0.641 0.52250
## const             1.08791    0.46063   2.362 0.01984 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 0.00584 on 117 degrees of freedom
## Multiple R-Squared: 0.9996, Adjusted R-squared: 0.9995
## F-statistic: 4.507e+04 on 6 and 117 DF, p-value: < 2.2e-16
##
##
## Covariance matrix of residuals:
##              UKrealgdp Canadarealgdp USArealgdp
## UKrealgdp      3.199e-05    1.229e-06    5.799e-06
## Canadarealgdp  1.229e-06    3.411e-05    1.441e-05
## USArealgdp     5.799e-06    1.441e-05    3.411e-05
##
## Correlation matrix of residuals:

```



```

##          UKrealgdp Canadarealgdp USArealgdp
## UKrealgdp      1.0000      0.0372      0.1756
## Canadarealgdp  0.0372      1.0000      0.4225
## USArealgdp     0.1756      0.4225      1.0000

varfit1 <- restrict(varfit, thresh=1.645)
summary(varfit1)

##
## VAR Estimation Results:
## =====
## Endogenous variables: UKrealgdp, Canadarealgdp, USArealgdp
## Deterministic variables: const
## Sample size: 124
## Log Likelihood: 1411.917
## Roots of the characteristic polynomial:
##      1      1 0.816 0.5505 0.5505 0.2299
## Call:
## VAR(y = zt, p = 2)
##
##
## Estimation results for equation UKrealgdp:
## =====
## UKrealgdp = UKrealgdp.l1 + USArealgdp.l1 + UKrealgdp.l2 + const
##
##              Estimate Std. Error t value Pr(>|t|)
## UKrealgdp.l1   1.42694    0.07494  19.040  < 2e-16 ***
## USArealgdp.l1   0.08187    0.02684   3.050  0.00282 **
## UKrealgdp.l2  -0.52537    0.06902  -7.611 6.82e-12 ***
## const          0.48200    0.14500   3.324  0.00118 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 0.005604 on 120 degrees of freedom
## Multiple R-Squared:      1, Adjusted R-squared:      1
## F-statistic: 1.568e+08 on 4 and 120 DF, p-value: < 2.2e-16
##
##
## Estimation results for equation Canadarealgdp:
## =====
## Canadarealgdp = UKrealgdp.l1 + Canadarealgdp.l1 + USArealgdp.l1 + UKrealgdp.l2 + Canadarealgdp.l2 + USArealgdp.l2
##
##              Estimate Std. Error t value Pr(>|t|)
## UKrealgdp.l1     0.29148    0.08482   3.437 0.000814 ***
## Canadarealgdp.l1  1.22221    0.08543  14.306  < 2e-16 ***
## USArealgdp.l1     0.37334    0.09622   3.880 0.000173 ***
## UKrealgdp.l2     -0.28088    0.08397  -3.345 0.001104 **
## Canadarealgdp.l2 -0.22529    0.08689  -2.593 0.010722 *

```

```

## USArealgdp.l2    -0.37887    0.09326   -4.062 8.78e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 0.00588 on 118 degrees of freedom
## Multiple R-Squared:    1,    Adjusted R-squared:    1
## F-statistic: 4.186e+08 on 6 and 118 DF,  p-value: < 2.2e-16
##
##
## Estimation results for equation USArealgdp:
## =====
## USArealgdp = UKrealgdp.l1 + Canadarealgdp.l1 + USArealgdp.l1 + UKrealgdp.l
2 + Canadarealgdp.l2 + const
##
##              Estimate Std. Error t value Pr(>|t|)
## UKrealgdp.l1    0.38363    0.08076   4.750 5.77e-06 ***
## Canadarealgdp.l1 0.19941    0.07597   2.625 0.009817 **
## USArealgdp.l1    0.98379    0.02958  33.261 < 2e-16 ***
## UKrealgdp.l2    -0.28365    0.08069  -3.515 0.000624 ***
## Canadarealgdp.l2 -0.28569    0.07340  -3.892 0.000165 ***
## const           1.17762    0.43779   2.690 0.008183 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 0.005826 on 118 degrees of freedom
## Multiple R-Squared:    1,    Adjusted R-squared:    1
## F-statistic: 5.264e+07 on 6 and 118 DF,  p-value: < 2.2e-16
##
##
##
## Covariance matrix of residuals:
##              UKrealgdp Canadarealgdp USArealgdp
## UKrealgdp    3.221e-05    1.088e-06  5.881e-06
## Canadarealgdp 1.088e-06    3.487e-05  1.432e-05
## USArealgdp    5.881e-06    1.432e-05  3.423e-05
##
## Correlation matrix of residuals:
##              UKrealgdp Canadarealgdp USArealgdp
## UKrealgdp    1.00000    0.03246    0.1771
## Canadarealgdp 0.03246    1.00000    0.4145
## USArealgdp    0.17712    0.41450    1.0000

```

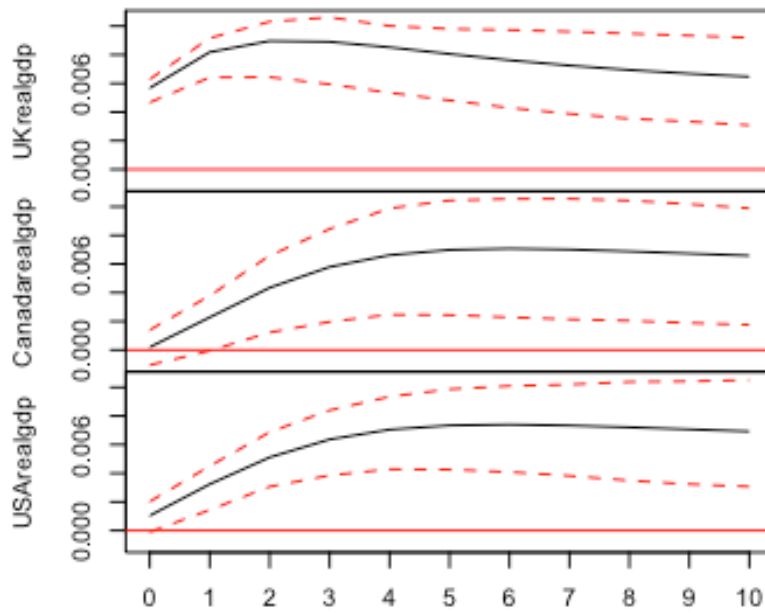
#3.2

```

impresp=irf(varfit1)
plot(impresp)

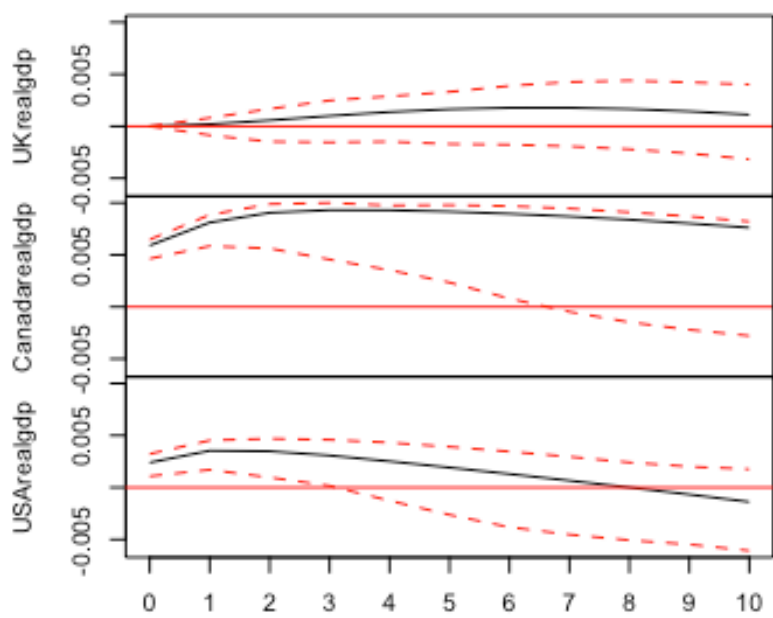
```

Orthogonal Impulse Response from UKrealgdp



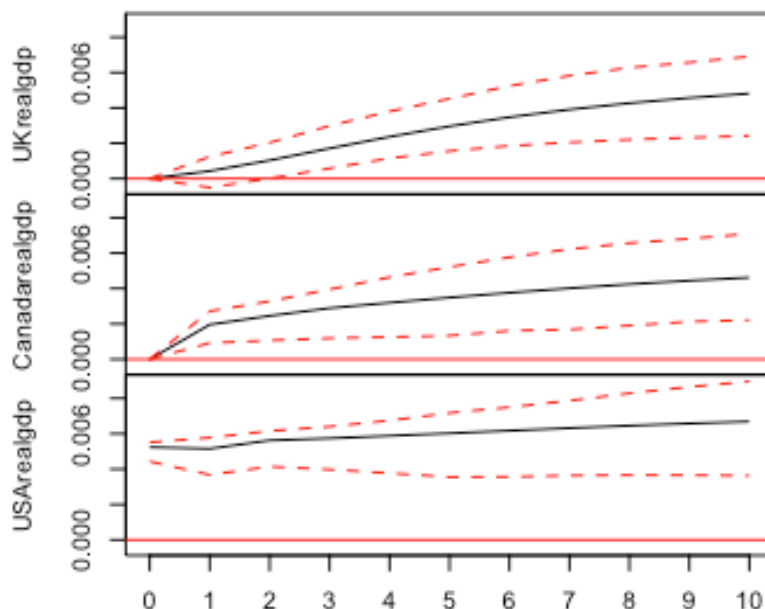
95 % Bootstrap CI, 100 runs

Orthogonal Impulse Response from Canadarealgdp



95 % Bootstrap CI, 100 runs

Orthogonal Impulse Response from USArealgdp



95 % Bootstrap CI, 100 runs

#3.3

```
fevd(varfit1, n.ahead=6)
```

```
## $UKrealgdp
##      UKrealgdp Canadarealgdp USArealgdp
## [1,] 1.0256410  0.000000000 0.000000000
## [2,] 1.0231875  0.000395965 0.001897277
## [3,] 1.0161004  0.002048088 0.007092450
## [4,] 1.0034766  0.005229373 0.016201814
## [5,] 0.9857968  0.009598402 0.029100759
## [6,] 0.9644918  0.014393977 0.045148122
##
## $Canadarealgdp
##      UKrealgdp Canadarealgdp USArealgdp
## [1,] 0.001062768  1.0074842 0.000000000
## [2,] 0.048016806  0.9258963 0.03521163
## [3,] 0.112459302  0.8516191 0.04597886
## [4,] 0.169008504  0.7885718 0.05331976
## [5,] 0.211524052  0.7407981 0.05921652
## [6,] 0.241406398  0.7054123 0.06516548
##
## $USArealgdp
##      UKrealgdp Canadarealgdp USArealgdp
```

```
## [1,] 0.03163904      0.1686788  0.8082292
## [2,] 0.13920528      0.2194761  0.6512818
## [3,] 0.24806953      0.1995651  0.5639124
## [4,] 0.33469862      0.1702819  0.5078846
## [5,] 0.39615015      0.1428221  0.4748435
## [6,] 0.43791211      0.1198690  0.4566839
```

#3.4

```
logUKrealgdp=log(CLVMNACSCAB1GQUK[21:146,])
m1 <- adfTest(logUKrealgdp, lags=3, type=c("ct"), title=NULL, description = NULL)
```

```
## Warning in if (class(x) == "timeSeries") x = series(x): the condition has
length
## > 1 and only the first element will be used
```

```
m1@test$p.value
```

```
##
## 0.7535667
```

```
logCanadarealgdp=log(NAEXKP01CAQ189S[77:202,])
m2 <- adfTest(logCanadarealgdp, lags=3, type=c("ct"), title=NULL, description = NULL)
```

```
## Warning in if (class(x) == "timeSeries") x = series(x): the condition has
length
## > 1 and only the first element will be used
```

```
m2@test$p.value
```

```
##
## 0.3721507
```

```
logUSArealgdp=log(GDPC1[133:258,])
m3 <- adfTest(logUSArealgdp, lags=3, type=c("ct"), title=NULL, description = NULL)
```

```
## Warning in if (class(x) == "timeSeries") x = series(x): the condition has
length
## > 1 and only the first element will be used
```

```
m3@test$p.value
```

```
##
## 0.804053
```

```
m4 <- adfTest(diff(logUKrealgdp), lags=3, type=c("ct"), title=NULL, description = NULL)
```

```
## Warning in if (class(x) == "timeSeries") x = series(x): the condition has
length
## > 1 and only the first element will be used
```

```

## Warning in adfTest(diff(logUKrealgdp), lags = 3, type = c("ct"), title = N
ULL, :
## p-value smaller than printed p-value

m4@test$p.value

##
## 0.01

m5 <- adfTest(diff(logCanadarealgdp), lags=3, type=c("ct"), title=NULL, descr
iption = NULL)

## Warning in if (class(x) == "timeSeries") x = series(x): the condition has
length
## > 1 and only the first element will be used

## Warning in adfTest(diff(logCanadarealgdp), lags = 3, type = c("ct"), title
=
## NULL, : p-value smaller than printed p-value

m5@test$p.value

##
## 0.01

m6 <- adfTest(diff(logUSArealgdp), lags=3, type=c("ct"), title=NULL, descript
ion = NULL)

## Warning in if (class(x) == "timeSeries") x = series(x): the condition has
length
## > 1 and only the first element will be used

## Warning in adfTest(diff(logUSArealgdp), lags = 3, type = c("ct"), title =
## NULL, : p-value smaller than printed p-value

m6@test$p.value

##
## 0.01

fit1 <- lm(logCanadarealgdp~logUKrealgdp+logUSArealgdp)
summary(fit1)

##
## Call:
## lm(formula = logCanadarealgdp ~ logUKrealgdp + logUSArealgdp)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.031869 -0.016180 -0.000016  0.010834  0.060533
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)

```

```

## (Intercept)    15.3400    0.5305   28.919 < 2e-16 ***
## logUKrealgdp   0.6720    0.1138    5.906 3.19e-08 ***
## logUSArealgdp  0.2855    0.0978    2.919 0.00417 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.02058 on 123 degrees of freedom
## Multiple R-squared:  0.9928, Adjusted R-squared:  0.9927
## F-statistic: 8493 on 2 and 123 DF,  p-value: < 2.2e-16

error1=residuals(fit1)
m7 <- adfTest(error1, lags=3, type=c("nc"), title=NULL, description = NULL)

## Warning in if (class(x) == "timeSeries") x = series(x): the condition has
length
## > 1 and only the first element will be used

m7@test$p.value

##
## 0.02300044

fit2 <- lm(logUKrealgdp~logUSArealgdp+logCanadarealgdp)
summary(fit2)

##
## Call:
## lm(formula = logUKrealgdp ~ logUSArealgdp + logCanadarealgdp)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.038761 -0.008042 -0.000668  0.010062  0.029801
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)   -1.43595    1.02802   -1.397    0.165
## logUSArealgdp  0.57420    0.04819   11.914 < 2e-16 ***
## logCanadarealgdp 0.32877    0.05566    5.906 3.19e-08 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.0144 on 123 degrees of freedom
## Multiple R-squared:  0.9964, Adjusted R-squared:  0.9964
## F-statistic: 1.717e+04 on 2 and 123 DF,  p-value: < 2.2e-16

error2=residuals(fit2)
m8 <- adfTest(error2, lags=3, type=c("nc"), title=NULL, description = NULL)

## Warning in if (class(x) == "timeSeries") x = series(x): the condition has
length
## > 1 and only the first element will be used

```



```

## Warning in adfTest(error2, lags = 3, type = c("nc"), title = NULL, descrip
tion =
## NULL): p-value smaller than printed p-value

m8@test$p.value

##
## 0.01

fit3 <- lm(logUSArealgdp~logCanadarealgdp+logUKrealgdp)
summary(fit3)

##
## Call:
## lm(formula = logUSArealgdp ~ logCanadarealgdp + logUKrealgdp)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.039741 -0.014332  0.001084  0.016284  0.033875
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)   -8.46852    1.07774   -7.858 1.67e-12 ***
## logCanadarealgdp  0.22695    0.07775    2.919  0.00417 **
## logUKrealgdp     0.93307    0.07831   11.914 < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.01835 on 123 degrees of freedom
## Multiple R-squared:  0.9957, Adjusted R-squared:  0.9956
## F-statistic: 1.429e+04 on 2 and 123 DF,  p-value: < 2.2e-16

error3=residuals(fit3)
m9 <- adfTest(error3, lags=3, type=c("nc"), title=NULL, description = NULL)

## Warning in if (class(x) == "timeSeries") x = series(x): the condition has
length
## > 1 and only the first element will be used

## Warning in adfTest(error3, lags = 3, type = c("nc"), title = NULL, descrip
tion =
## NULL): p-value smaller than printed p-value

m9@test$p.value

##
## 0.01

#3.5
diff.logUKrealgdp=diff(logUKrealgdp)
diff.logUSArealgdp=diff(logUSArealgdp)
diff.logCanadarealgdp=diff(logCanadarealgdp)

```

```

diff.logUKrealgdp.L.1=Lag(diff(logUKrealgdp),k=1)
diff.logUSArealgdp.L.1=Lag(diff(logUSArealgdp),k=1)
diff.logCanadarealgdp.L.1=Lag(diff(logCanadarealgdp),k=1)
error1.L.1=Lag(error1,k=1)
error2.L.1=Lag(error2,k=1)
error3.L.1=Lag(error3,k=1)
fit11 <- lm(formula=diff.logUKrealgdp~diff.logUKrealgdp.L.1+diff.logUSArealgdp.L.1+diff.logCanadarealgdp.L.1+error2.L.1)
summary(fit11)

##
## Call:
## lm(formula = diff.logUKrealgdp ~ diff.logUKrealgdp.L.1 + diff.logUSArealgdp.L.1 +
##     diff.logCanadarealgdp.L.1 + error2.L.1)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.0167184 -0.0028007 -0.0002558  0.0032301  0.0147306
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)      0.0022858   0.0007188    3.180  0.00188 **
## diff.logUKrealgdp.L.1  0.5079756   0.0811516    6.260 6.33e-09 ***
## diff.logUSArealgdp.L.1  0.0854195   0.0893827    0.956  0.34118
## diff.logCanadarealgdp.L.1 0.0104570   0.0837193    0.125  0.90081
## error2.L.1        -0.0989680   0.0376738   -2.627  0.00975 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.005671 on 119 degrees of freedom
## (2 observations deleted due to missingness)
## Multiple R-squared:  0.3601, Adjusted R-squared:  0.3385
## F-statistic: 16.74 on 4 and 119 DF, p-value: 6.55e-11

fit22 <- lm(formula=diff.logUSArealgdp~diff.logUKrealgdp.L.1+diff.logUSArealgdp.L.1+diff.logCanadarealgdp.L.1+error3.L.1)
summary(fit22)

##
## Call:
## lm(formula = diff.logUSArealgdp ~ diff.logUKrealgdp.L.1 + diff.logUSArealgdp.L.1 +
##     diff.logCanadarealgdp.L.1 + error3.L.1)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.0204776 -0.0032885  0.0000605  0.0035886  0.0157192
##
## Coefficients:

```

```
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)      0.0029882  0.0007858   3.803 0.000227 ***
## diff.logUKrealgdp.L.1  0.3285073  0.0861542   3.813 0.000219 ***
## diff.logUSArealgdp.L.1  0.1598648  0.0983524   1.625 0.106717
## diff.logCanadarealgdp.L.1 0.1725787  0.0885850   1.948 0.053749 .
## error3.L.1        -0.0215164  0.0316593  -0.680 0.498064
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.006131 on 119 degrees of freedom
## (2 observations deleted due to missingness)
## Multiple R-squared:  0.3058, Adjusted R-squared:  0.2825
## F-statistic: 13.11 on 4 and 119 DF, p-value: 7.094e-09

fit33 <- lm(formula=diff.logCanadarealgdp~diff.logUKrealgdp.L.1+diff.logUSAre
algrp.L.1+diff.logCanadarealgdp.L.1+error1.L.1)
summary(fit33)

##
## Call:
## lm(formula = diff.logCanadarealgdp ~ diff.logUKrealgdp.L.1 +
##     diff.logUSArealgdp.L.1 + diff.logCanadarealgdp.L.1 + error1.L.1)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.0129234 -0.0035559 -0.0001957  0.0033786  0.0171672
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)      0.0008393  0.0007464   1.124 0.263079
## diff.logUKrealgdp.L.1  0.2601149  0.0822372   3.163 0.001982 **
## diff.logUSArealgdp.L.1  0.3309112  0.0947854   3.491 0.000676 ***
## diff.logCanadarealgdp.L.1 0.2540305  0.0852632   2.979 0.003502 **
## error1.L.1        -0.0464788  0.0278360  -1.670 0.097600 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.005795 on 119 degrees of freedom
## (2 observations deleted due to missingness)
## Multiple R-squared:  0.488, Adjusted R-squared:  0.4708
## F-statistic: 28.36 on 4 and 119 DF, p-value: < 2.2e-16
```

Question 3

3.1

$$\text{UK real gdp}_t = 0.482 + 1.42694 \text{UK real gdp}_{t-1} + 0.08187 \text{USA real gdp}_t - 0.52537 \text{UK real gdp}_{t-2}$$

(0.145) (0.07494) (0.02684) (0.06902)

$$\begin{aligned} \text{Canada real gdp}_t &= 0.27148 \text{UK real gdp}_{t-1} + 1.2221 \text{Canada real gdp}_{t-1} + 0.37734 \text{USA real gdp}_{t-1} \\ &\quad - 0.28088 \text{UK real gdp}_{t-2} - 0.22529 \text{Canada real gdp}_{t-2} - 0.37887 \text{USA real gdp}_{t-2} \end{aligned}$$

(0.08492) (0.08543) (0.09622) (0.08397) (0.08689) (0.09326)

$$\begin{aligned} \text{USA real gdp}_t &= 1.17762 + 0.38363 \text{UK real gdp}_{t-1} + 0.19941 \text{Canada real gdp}_{t-1} + 0.98379 \text{USA real gdp}_{t-1} - 0.28365 \text{UK real gdp}_{t-2} \\ &\quad - 0.28569 \text{Canada real gdp}_{t-2} \end{aligned}$$

(0.43779) (0.08076) (0.07597) (0.02958) (0.08069) (0.07340)

3.2 Shock from UK real gdp

- The shock from UK real gdp have positive impact to itself for the whole period.
- The shock from UK real gdp have positive impact to Canada real gdp and USA real gdp from period 1 to the end of period.

Shock from Canada real gdp

- it have slightly positive impact to UK real gdp from period 1 to the end of period.
- it have significantly positive impact to Canada real gdp for the whole period.
- it have moderately positive impact to USA real gdp from initial to period 7 before generate negative impact.

Shock from USA real gdp

- it have significantly positive impact to UK real gdp, Canada real gdp and USA real gdp.

3.3 For UK real gdp

Majority impact come from $\varepsilon_{\text{UK real gdp}}$ itself with 96-100% that make us forecast error.

For Canada real gdp

Majority impact come from $\varepsilon_{\text{Canada real gdp}}$ itself with 70-92% and $\varepsilon_{\text{UK real gdp}}$ with around 5-24%. The rest of impact come from $\varepsilon_{\text{USA real gdp}}$ (3-5%) that make us forecast error.

For USA real gdp

Majority impact come from $\varepsilon_{\text{USA real gdp}}$ itself with 45-80% and $\varepsilon_{\text{UK real gdp}}$ with around 3-43%. The rest of impact come from $\varepsilon_{\text{Canada real gdp}}$ with around 11-21% that make us forecast error.

3.4 First we check each variable by performed Dickey-Fuller test

For $\log \text{UK real gdp}$

H_0 : there is unit-root in this series

H_1 : no unit root

$\alpha = 0.05$

p-value = 0.7535667

Since p-value is higher than $\alpha = 0.05$.

We can not reject H_0 at 95% CI.

Then, there is unit-root in this series.

For $\log \text{Canada real gdp}$

H_0 : there is unit-root in this series

H_1 : no unit root

$\alpha = 0.05$

p-value = 0.3721507

Since p-value is higher than $\alpha = 0.05$.

We can not reject H_0 at 95% CI.

Then, there is unit-root in this series.

For $\log \text{USA real gdp}$

H_0 : there is unit-root in this series

H_1 : no unit root

$\alpha = 0.05$

p-value = 0.804053

Since p-value is higher than $\alpha = 0.05$.

We can not reject H_0 at 95% CI.

Then, there is unit-root in this series.

After we use first difference to each series and test ADF again, we get p-value < 0.01 for all series. Then, we can conclude that the integrated order $I(p)$ of each series is $I(1)$.

Now we perform the model and find residual term. Next, we test unit-root in residual term by ADF. As a result, we get p-value = 0.0000004, 0.01, 0.01 which all reject H_0 and no unit-root in each residual term; $I(0)$. This mean that $\log \text{UK real gdp}$, $\log \text{Canada real gdp}$ and $\log \text{USA real gdp}$ is cointegrate with CI(1,1).

3.5

$$\Delta \log UK \text{ real gdp} = 0.0022858 + 0.5079756 \Delta \log UK \text{ real gdp}_{t-1} + 0.0854195 \Delta \log USA \text{ real gdp}_{t-1} + 0.010457 \Delta \log Canada \text{ real gdp}_{t-1} - 0.098968 e_{t-1}$$

(0.0007188) (0.0811516) (0.0897327) (0.0837193)

(0.0376738)

- If there is no change in other country real gdp and mismatch between actual $\log UK \text{ real gdp}$ and estimate model Change in $\log UK \text{ real gdp}$ is 0.0022858
- All of the change in other countries real gdp in last period have positive impact to change in $UK \text{ real gdp}$ in this period. Moreover, the actual real gdp of UK in last period exceed the expectation, so it will adjust to the normal by decrease in this period. This may, because cash flow in to UK and make return go down, while return in other country go up, so people move out from UK in this period.

$$\Delta \log USA \text{ real gdp} = 0.0029882 + 0.3285077 \Delta \log UK \text{ real gdp}_{t-1} + 0.1598648 \Delta \log USA \text{ real gdp}_{t-1} + 0.1725787 \Delta \log Canada \text{ real gdp}_{t-1} - 0.0215164 e_{t-1}$$

(0.0007859) (0.086542) (0.0983524) (0.0885850)

(0.0316593)

- If there is no change in other country real gdp and mismatch between actual $\log USA \text{ real gdp}$ and estimate model Change in $\log USA \text{ real gdp}$ is 0.0029882
- All of the change in other countries real gdp in last period have positive impact to change in $USA \text{ real gdp}$ in this period. Moreover, the actual real gdp of USA in last period exceed the expectation, so it will adjust to the normal by decrease in this period. This may, because cash flow in to USA and make return go down, while return in other country go up, so people move out from USA in this period.

$$\Delta \log Canada \text{ real gdp} = 0.0008393 + 0.2601179 \Delta \log UK \text{ real gdp}_{t-1} + 0.3309112 \Delta \log USA \text{ real gdp}_{t-1} + 0.2590385 \Delta \log Canada \text{ real gdp}_{t-1} - 0.0464788 e_{t-1}$$

(0.0007464) (0.0822372) (0.0947854) (0.0852632)

(0.027876)

- If there is no change in other country real gdp and mismatch between actual $\log Canada \text{ real gdp}$ and estimate model Change in $\log Canada \text{ real gdp}$ is 0.0008393
- All of the change in other countries real gdp in last period have positive impact to change in $Canada \text{ real gdp}$ in this period. Moreover, the actual real gdp of Canada in last period exceed the expectation, so it will adjust to the normal by decrease in this period. This may, because cash flow in to Canada and make return go down, while return in other country go up, so people move out from Canada in this period.