A Review of Some Statistical Concepts

Summation and Product Operators

The Greek capital letter $\sum$ (sigma) is used to indicate summation. Thus,

$$\sum_{i=1}^{n} x_i = x_1 + x_2 + \dots + x_n$$

Some of the important properties of the summation operator $\sum$ are

1. $\sum_{i=1}^{n} k = nk$, where k is constant.

$$\sum_{i=1}^{n} k = \underbrace{k + k + \dots + k}_{n \text{ term}}$$

$$= nk$$

e.g. $\sum_{i=1}^{4} 3 = (4)(3) = 12$

$= 3+3+3+3 = 12$

2. $\sum_{i=1}^{n} kx_i = k\sum_{i=1}^{n} x_i$, where k is constant.

$$\sum_{i=1}^{n} kx_i = kx_1 + kx_2 + \dots + kx_n$$

$$= k\underbrace{(x_1 + x_2 + \dots + x_n)}_{\sum_{i=1}^{n} x_i}$$

$$= k\sum_{i=1}^{n} x_i$$

3. $\sum_{i=1}^{n}(a + bx_i) = na + b\sum_{i=1}^{n} x_i$, where a and b are constants and where use is made of properties 1 and 2 above.

$$\sum_{i=1}^{n}(a+bx_i) = \underbrace{na}_{1^{st}\text{ property}} + \underbrace{b\sum_{i=1}^{n} x_i}_{2^{nd}\text{ property}}$$

$$= na + b(\underbrace{x_1 + x_2 + x_3 + \dots x_n}_{\sum x_i})$$

4. $\sum_{i=1}^{n}(x_i + y_i) = \sum_{i=1}^{n} x_i + \sum_{i=1}^{n} y_i$

Additivity

$$= \underset{1^{st}}{(x_1+y_1)} + \underset{2^{nd}}{(x_2+y_2)} + \dots + \underset{n \text{ term}}{(x_n+y_n)}$$

1st, 2nd, ..., n term, nth.

$$= (x_1+x_2+\dots+x_n) + (y_1+y_2+\dots+y_n)$$

$$= \sum_{i=1}^{n} x_i + \sum_{i=1}^{n} y_i$$

x_i	5	8	6	9
y_i	4	6	8	9

1 $\sum_{i=1}^{4} x_i$

2 $\sum_{i=1}^{4} x_i^2$

3 $\sum_{i=1}^{4} x_i y_i$

4 $\sum_{i=1}^{4} (x_i + y_i)$

5 $\sum_{i=1}^{4} (3x_i + 5y_i)$

x_i	5	8	6	9
y_i	4	6	8	9

1 $\sum_{i=1}^{4} x_i$

2 $\sum_{i=1}^{4} x_i^2$

$5^2+8^2+6^2+9^2$

$\sum x_i^2 \neq \left(\sum x_i\right)^2$

3 $\sum_{i=1}^{4} x_i y_i$

$(5+4)+(8+6)+(6+8)+(9+9)$

4 $\sum_{i=1}^{4} (x_i+y_i) = \sum_{i=1}^{n=4} x_i + \sum_{i=1}^{n=4} y_i$

5 $\sum_{i=1}^{4} (3x_i+5y_i) = 3\sum_{i=1}^{n} x_i + 5\sum_{i=1}^{n} y_i$

x_i	5	8	6	9
y_i	4	6	8	9

1 $\sum_{i=1}^{4} x_i = 5+8+6+9$

2 $\sum_{i=1}^{4} x_i^2 = 5^2+8^2+6^2+9^2$

3 $\sum_{i=1}^{4} x_i y_i = (5)(4)+(8)(6)+(6)(8)+(9)(9)$

4 $\sum_{i=1}^{4} (x_i+y_i) = (5+4)+(8+6)+(6+8)+(9+9)$

5 $\sum_{i=1}^{4} (3x_i+5y_i) = (3(5)+5(4))+(3(8)+5(6))+\ldots$

$$\left(\sum x_i\right)^2 \neq \sum x_i^2$$

Example

x_i	5	8	6	9
y_i	4	6	8	9

6. $\sum_{i=1}^{10} 5$

7. $\sum_{i=1}^{4} (x_i + y_i)^2 =$

Example

x_i	5	8	6	9
y_i	4	6	8	9

6. $\sum_{i=1}^{10} 5 =$ $nk = 10(5)$

7. $\sum_{i=1}^{4} (x_i + y_i)^2 =$

$(x_1+y_1)^2 + (x_2+y_2)^2 + \ldots\ldots$

Example

x_i	5	8	6	9
y_i	4	6	8	9

1 $\sum_{i=1}^{4} x_i = x_1 + x_2 + x_3 + x_4 = 28$

2 $\sum_{i=1}^{4} x_i^2 = x_1^2 + x_2^2 + x_3^2 + x_4^2 = 206$

3 $\sum_{i=1}^{4} x_i y_i = x_1 y_1 + x_2 y_2 + x_3 y_3 + x_4 y_4$

$= 197$

4 $\sum_{i=1}^{4} (x_i + y_i) = (x_1 + y_1) + \dots + (x_4 + y_4)$

$= \sum_{i=1}^{4} x_i + \sum_{i=1}^{4} y_i = 55$

5 $\sum_{i=1}^{4} (3x_i + 5y_i) = n\sum_{i=1}^{n} x_i + n\sum_{i=1}^{n} y_i$

$= 219$

Example

x_i	5	8	6	9
y_i	4	6	8	9

6. $\sum_{i=1}^{10} 5 = 5 \times 10 = 50$

7. $\sum_{i=1}^{4} (x_i + y_i)^2 = \sum_{i=1}^{4} (x_i^2 + 2x_iy_i + y_i^2$

$(a+b)^2 = a^2 + 2ab + b^2$

$= \sum_{i=1}^{n} x_i^2 + 2\sum_{i=1}^{4} x_iy_i + \sum y_i^2$

$= 797$

The summation operator can also be extended to multiple sums. This, $\sum\sum$, The double summation operator, is defined as

m term

$$\sum_{i=1}^{n}\sum_{j=1}^{m} x_{ij} = \sum_{i=1}^{n}(x_{i1} + x_{i2} + \cdots + x_{im})$$

$$=(x_{11} + x_{21} + \cdots + x_{n1}) + (x_{12} + x_{22} + \cdots + x_{n2}) + \cdots + (x_{1m} + x_{2m} + \cdots + x_{nm})$$

$j=1$ $j=2$

Some of the properties of $\sum\sum$ are

1. $\sum_{i=1}^{n}\sum_{j=1}^{m} x_{ij} = \sum_{j=1}^{m}\sum_{i=1}^{n} x_{ij}$; that is, the order in which the double summation is performed is interchangeable.

2. $\sum_{i=1}^{n}\sum_{j=1}^{m} x_i y_j = \sum_{i=1}^{n} x_i \sum_{j=1}^{m} y_j$.

The finite double series can be written as a product of series

$x_n y_1$ ... $+ x_n y_2 + \ldots\ldots$

$= x_1 y_1 + x_2 y_1 + \ldots + x_1 y_2 + \ldots\ldots + x_n y_m$ ①

$= (x_1 + x_2 + \ldots x_n) y_1 + (x_1 + x_2 + \ldots x_n) y_2 + \ldots$ ②

$= \sum_{i=1}^{n} x_i (y_1 + y_2 + \ldots y_m) = \sum_{i=1}^{n} x_i \sum_{j=1}^{m} y_j$ ③

3. $\sum_{i=1}^{n}\sum_{j=1}^{m}(x_{ij}+y_{ij}) = \sum_{i=1}^{n}\sum_{j=1}^{m}x_{ij} + \sum_{i=1}^{n}\sum_{j=1}^{m}y_{ij}$

nterm

$$= \sum_{j=1}^{m}(x_{1j}+y_{1j}) + \sum_{j=1}^{m}(x_{2j}+y_{2j}) + \dots + \sum_{j=1}^{m}(x_{nj}+y_{nj}) \quad ①$$

$$= \sum_{j=1}^{m}\underbrace{(x_{1j}+x_{2j}+x_{3j}+\dots+x_{nj})}_{\sum_{i=1}^{n} x_{ij}} + \sum_{j=1}^{m}\underbrace{(y_{1j}+y_{2j}+\dots y_{nj})}_{\sum_{i=1}^{n} y_{ij}} \quad ②$$

$$= \sum_{j=1}^{m}\sum_{i=1}^{n} x_{ij} + \sum_{j=1}^{m}\sum_{i=1}^{n} y_{ij} \quad ③$$

$$\sum_{j=1}^{m}(x_{1j}+y_{1j}) + \sum_{j=1}^{m}(x_{2j}+y_{2j})$$

$$\sum_{j=1}^{m}\underbrace{(x_{1j}+x_{2j})}_{\sum_{i=1}^{2} x_{ij}} + \sum_{j=1}^{m}\underbrace{(y_{1j}+y_{2j})}_{\sum_{i=1}^{2} y_{ij}}$$