Chapter 4

Classical Normal Linear Regression Model (CNLRM)

Classical theory of statistical inference

- Estimation
- Hypothesis testing

The Probability Distribution of Disturbances

$$\hat{\beta}_2 = \sum k_i Y_i$$

$$\text{where} \quad k_i = \frac{x_i}{\sum x_i^2} = \frac{(X_i - \bar{X})}{\sum (X_i - \bar{X})^2}$$

$$Y_i = \beta_1 + \beta_2 X_i + u_i$$

$$\hat{\beta}_2 = \sum k_i(\beta_1 + \beta_2 X_i + u_i)$$

The Normality Assumption for

The classical normal linear regression model assumes that each u_i is distributed normally with

$$E(u_i) = 0$$

$$E[u_i - E(u_i)]^2 = E(u_i^2) = \sigma^2$$

$$E\{[(u_i - E(u_i)][(u_j - E(u_j)]\} = E(u_i u_j) = 0 \quad i \neq j$$

$$u_i \sim N(0, \sigma^2)$$

Where **N** stands for normal distribution

$$u_i \sim NID(0, \sigma^2)$$

Where **NID** stands for normally and independent distributed

Properties of OLS Estimators under the Normality Assumption

- Unbiased
- Minimum variance unbiased or efficient estimators
- Consistency

  Sample size increases $\longrightarrow$ the estimators converge to their true population values

Properties of OLS Estimators under the Normality Assumption

- $\hat{\beta}_1$ is normally distributed with

$$Mean: \; E(\hat{\beta}_1) = \beta_1$$

$$\text{var}(\hat{\beta}_1): \; \sigma^2_{\hat{\beta}_1} = \frac{\sum X_i^2}{n \sum x_i^2} \sigma^2$$

$$\hat{\beta}_1 \sim N(\beta_1, \sigma^2_{\hat{\beta}_1})$$

Properties of OLS Estimators under the Normality Assumption

By the properties of the normal distribution, the variable Z,

$$Z = \frac{\hat{\beta}_1 - \beta_1}{\sigma_{\hat{\beta}_1}}$$

follows the standard normal distribution, that is, a normal distribution with zero mean and unit variance

$$Z \sim N(0,1)$$

Properties of OLS Estimators under the Normality Assumption

- $\hat{\beta}_2$ is normally distributed with

$$Mean: \; E(\hat{\beta}_2) = \beta_2$$

$$\text{var}(\hat{\beta}_2): \; \sigma^2_{\hat{\beta}_2} = \frac{\sigma^2}{\sum x_i^2}$$

$$\hat{\beta}_2 \sim N(\beta_2, \sigma^2_{\hat{\beta}_2})$$

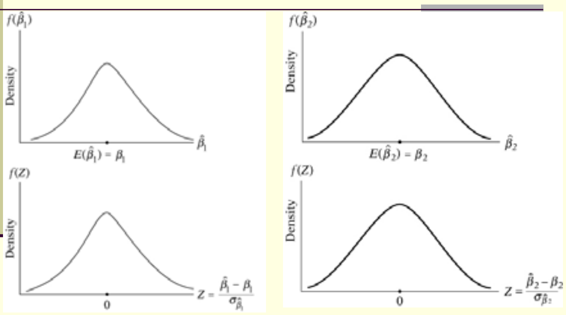
Properties of OLS Estimators under the Normality Assumption

By the properties of the normal distribution, the variable Z,

$$Z = \frac{\hat{\beta}_2 - \beta_2}{\sigma_{\hat{\beta}_2}}$$

follows the standard normal distribution

The probability distributions of $\hat{\beta}_1$ and $\hat{\beta}_2$



Properties of OLS Estimators under the Normality Assumption

- $(n-2)(\hat{\sigma}^2/\sigma^2)$ is distributed as the χ^2 (chi square) distribution with (n-2) degree of freedom
- $(\hat{\beta}_1, \hat{\beta}_2)$ are distributed independently of $\hat{\sigma}^2$
- $\hat{\beta}_1$ *and* $\hat{\beta}_2$ have minimum variance in the entire class of unbiased estimators, whether linear or not