Chapter 6
Inputs and Production Functions in the Short Run: A Quick Review

Key Concepts

The production function tells us the *maximum* possible output that can be attained by the firm for any given quantity of inputs.

Production Function:

$$Q = f(L, K)$$

- Q = output
- K = Capital
- L = Labor

The production set is a set of technically feasible combinations of inputs and outputs.

The Production Function & Technical Efficiency

Total Product (TP), Average Product (AP), and Marginal Product (MP)

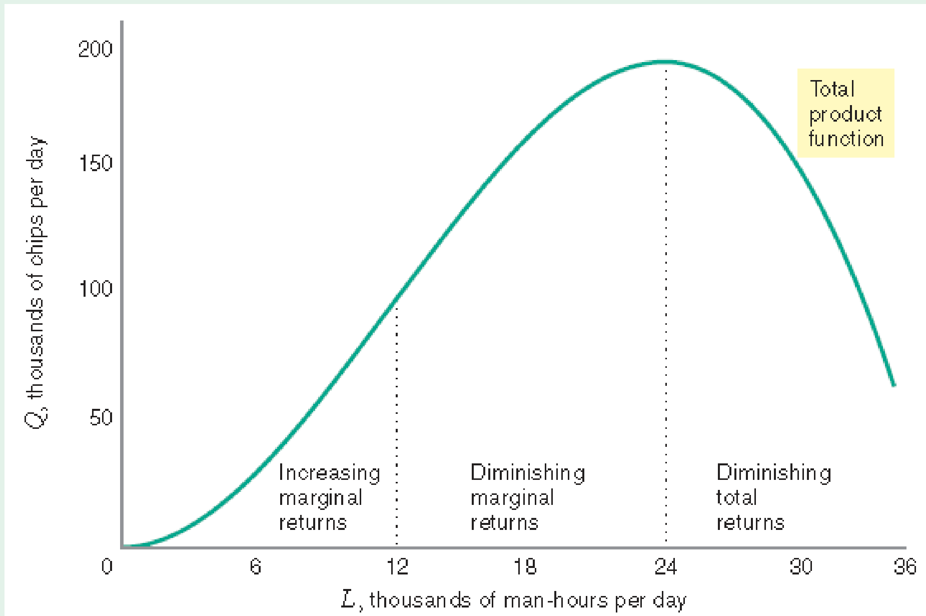
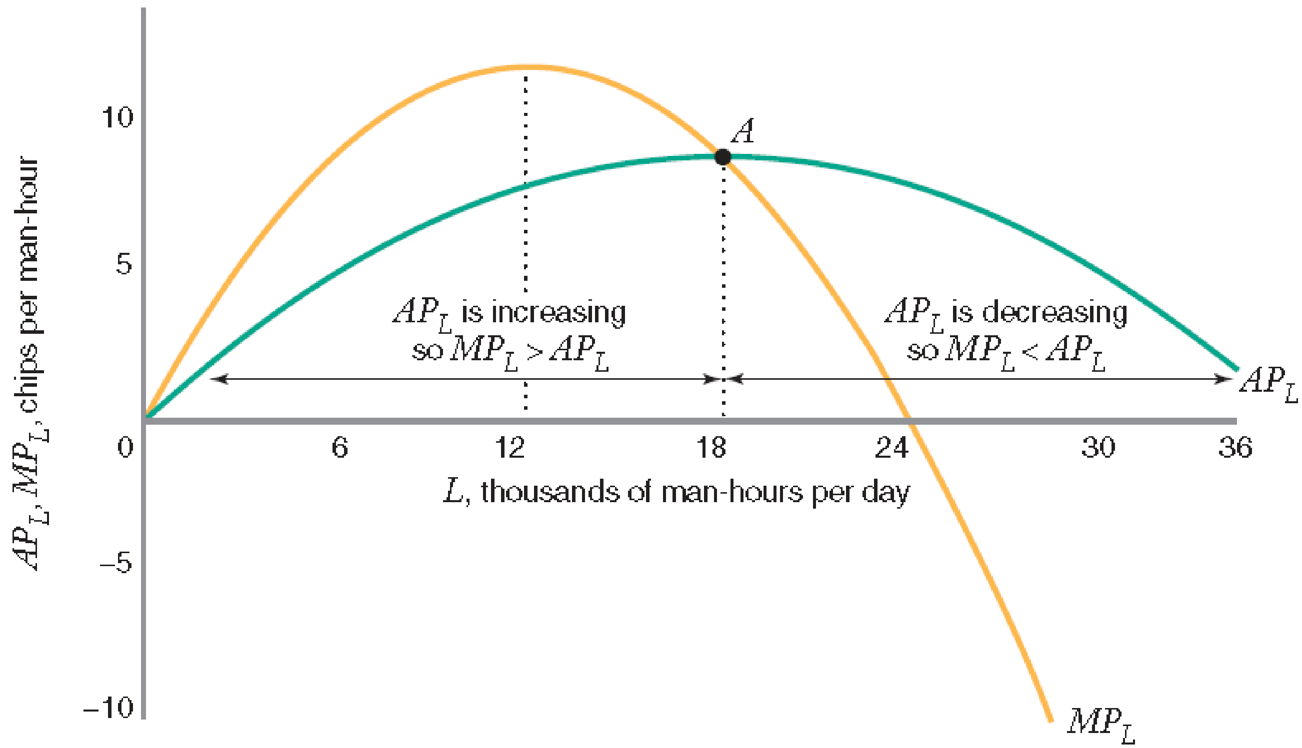


FIGURE 6.2 Total Product Function

TABLE 6.1 Total Product Function

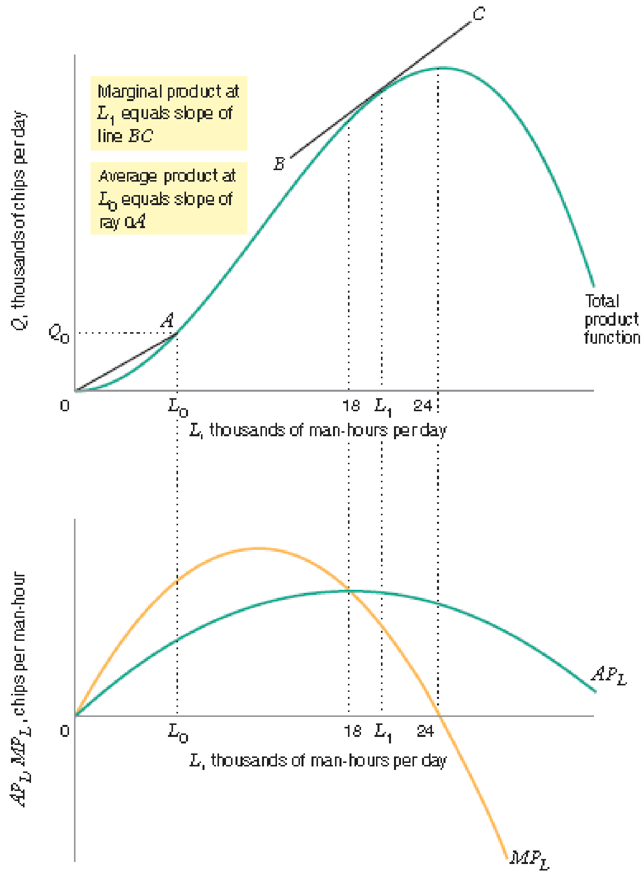
*L**	*Q*
0	0
6	30
12	96
18	162
24	192
30	150

**L* is expressed in thousands of man-hours per day, and *Q* is expressed in thousands of semiconductor chips per day.



Average Product of Labor

L	Q	$AP_L = \frac{Q}{L}$
6	30	5
12	96	8
18	162	9
24	192	8
30	150	5

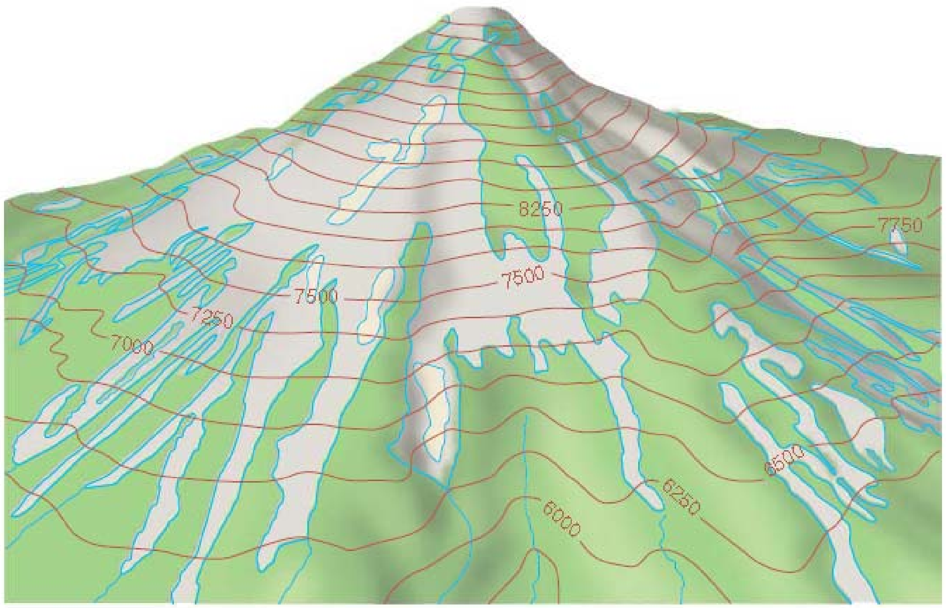

Marginal product at L_1 equals slope of line BC
Average product at L_0 equals slope of ray $0A$
C
B
A
Q_0
Q, thousands of chips per day
Total product function
0
L_0
18
L_1
24
L, thousands of man-hours per day
AP_L, MP_L, chips per man-hour
AP_L
MP_L
0
L_0
18
L_1
24
L, thousands of man-hours per day

Relationship between TP, AP, MP

Law of Diminishing Marginal Product of Labor

Chapter 6
Inputs and Production Functions
in the Long Run

Product Mountain



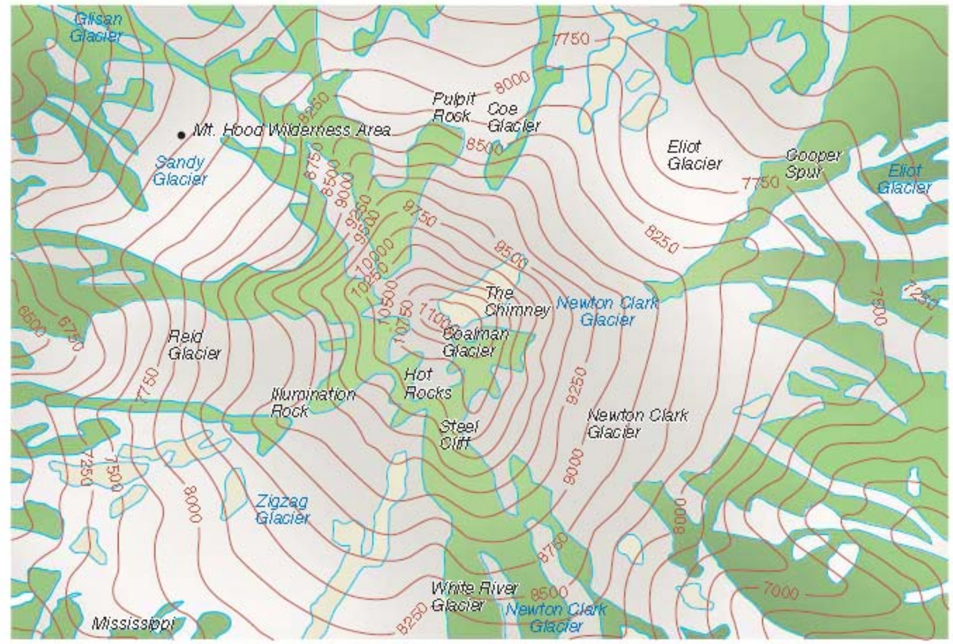


TABLE 6.3 Production Function for Semiconductors*

		K**					
		0	**6**	**12**	**18**	**24**	**30**
L**	**0**	0	0	0	0	0	0
	6	0	5	15	25	30	23
	12	0	15	48	81	96	75
	18	0	25	81	137	162	127
	24	0	30	96	162	192	150
	30	0	23	75	127	150	117

*Numbers in table equal the output that can be produced with various combinations of labor and capital.

***L* is expressed in thousands of man-hours per day; *K* is expressed in thousands of machine-hours per day; and *Q* is expressed in thousands of semiconductor chips per day.

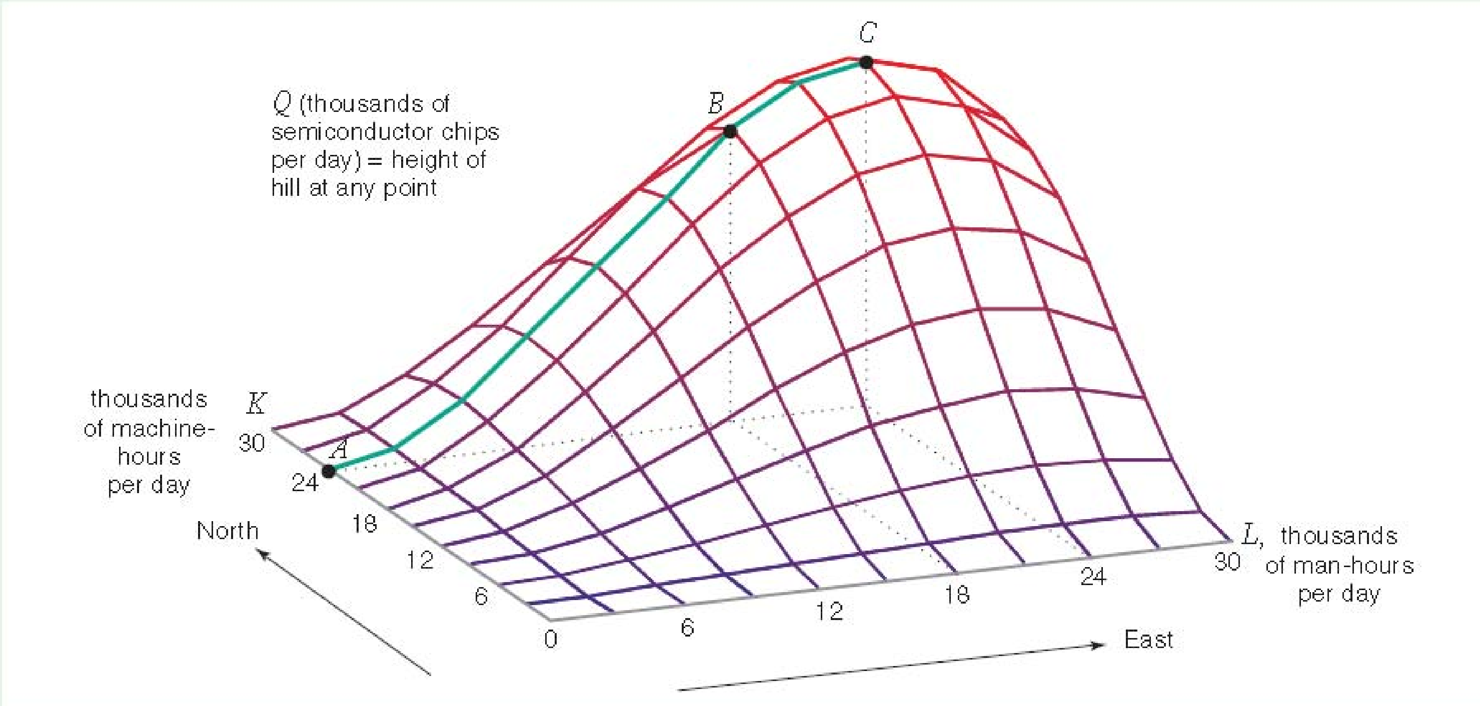
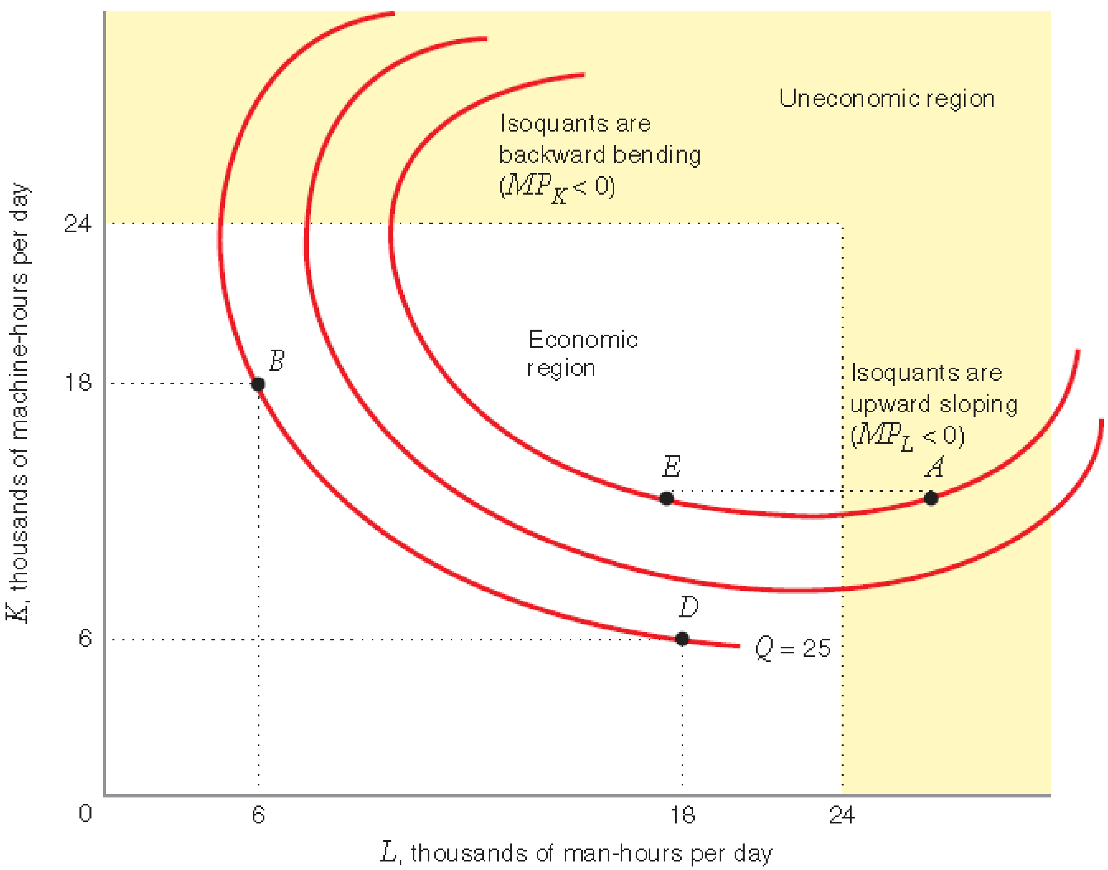
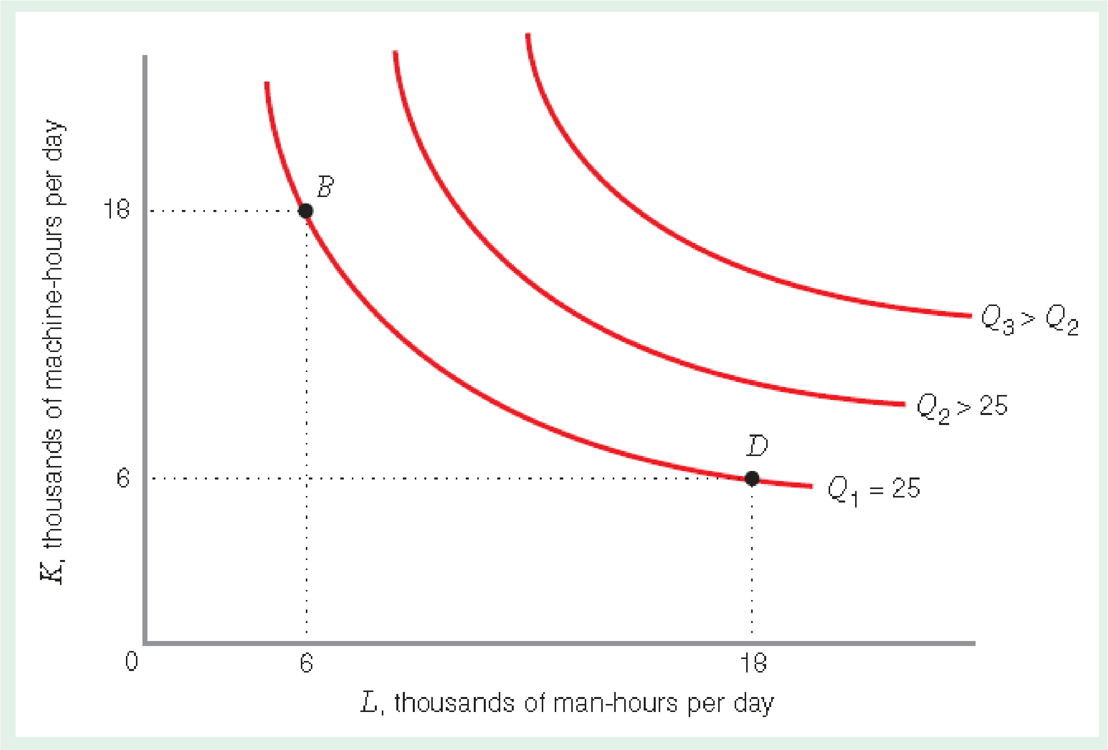


FIGURE 6.5 **Total Product Hill**

The height of the hill at any point is equal to the quantity of output Q attainable from the quantities of labor L and capital K corresponding to that point.

Economic and Uneconomic Regions of Production





K, thousands of machine-hours per day
18
6
0
6
18
L, thousands of man-hours per day
B
D
Q3 > Q2
Q2 > 25
Q1 = 25

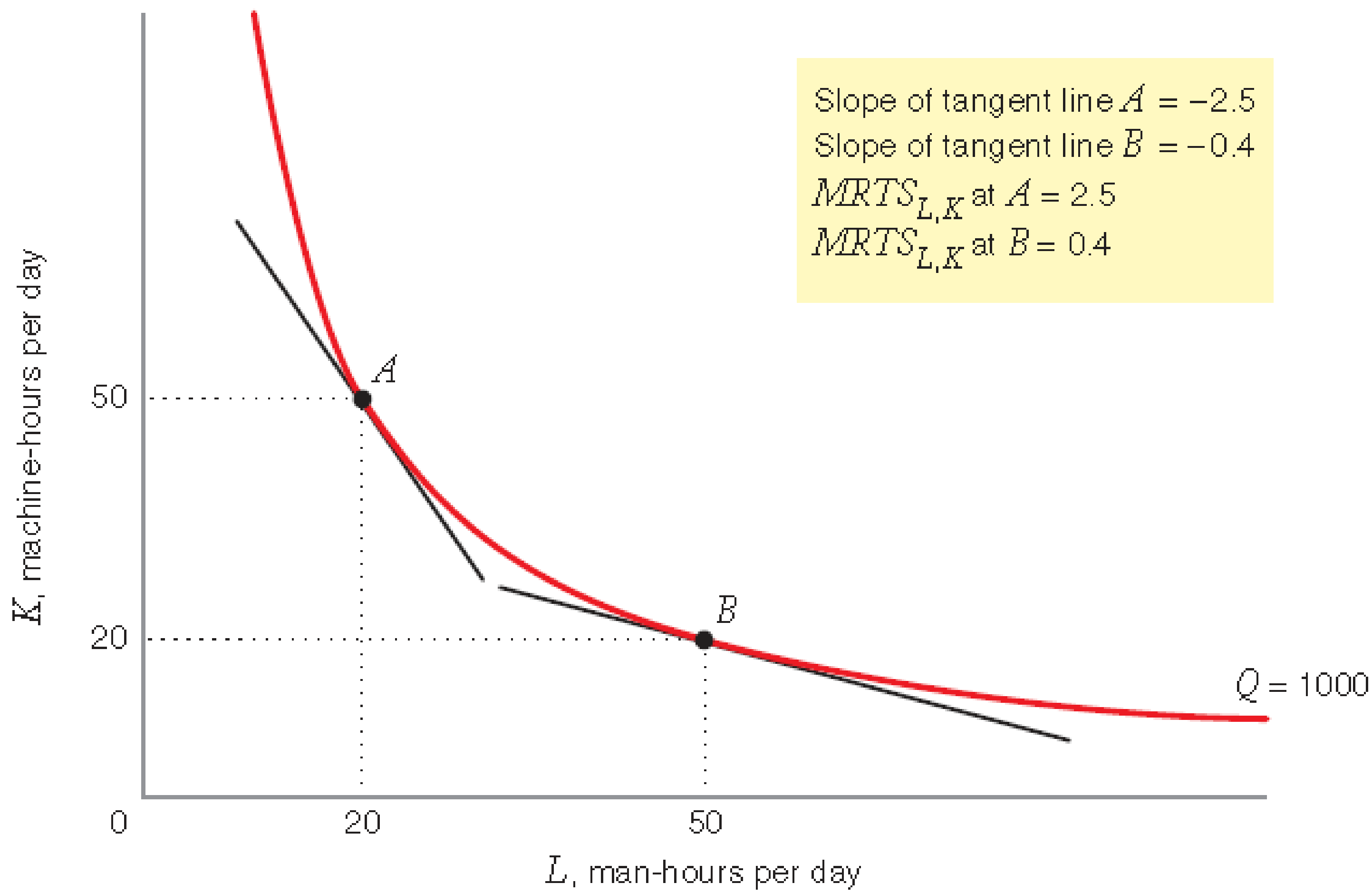
MARGINAL RATE OF TECHNICAL SUBSTITUTION

marginal rate of technical substitution of labor for capital: The rate at which the quantity of capital can be reduced for every one-unit increase in the quantity of labor, holding the quantity of output constant.

- The rate at which the quantity of capital can be *decreased* for every one-unit *increase* in the quantity of labor, holding the quantity of output constant, *or*

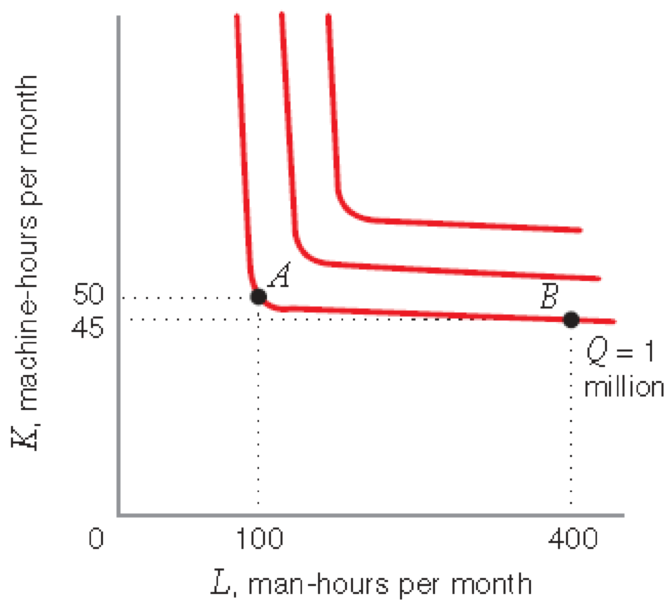
- The rate at which the quantity of capital must be *increased* for every one-unit *decrease* in the quantity of labor, holding the quantity of output constant.

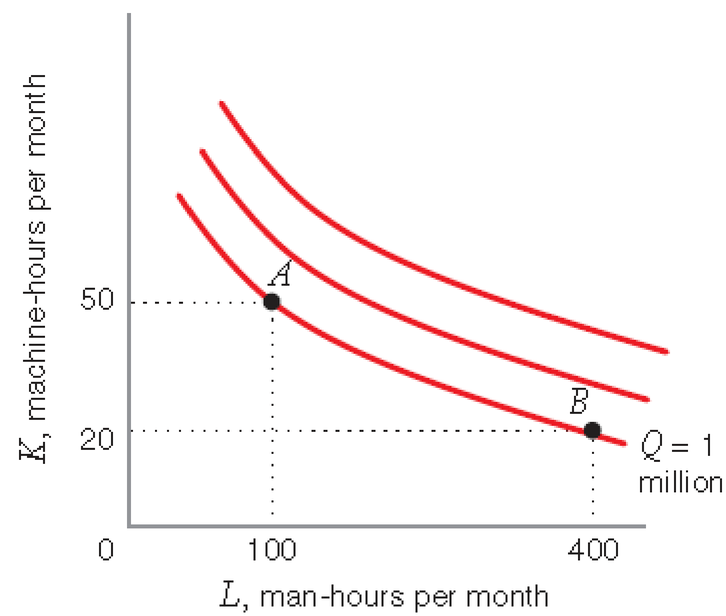
MARGINAL RATE OF TECHNICAL SUBSTITUTION IS DIMINISHING.



MARGINAL RATE OF TECHNICAL SUBSTITUTION

we explore how to describe the ease or difficulty with which a firm can substitute between different inputs





Input Substitution Opportunities and the Shape of Isoquants

The ease or difficulty with which a firm can substitute among inputs depends on the curvature of its isoquants

- When the production function offers limited input substitution opportunities, the *$MRTS_{L,K}$* changes substantially as we move along an isoquant. In this case, the isoquants are nearly L-shaped, as in Figure a

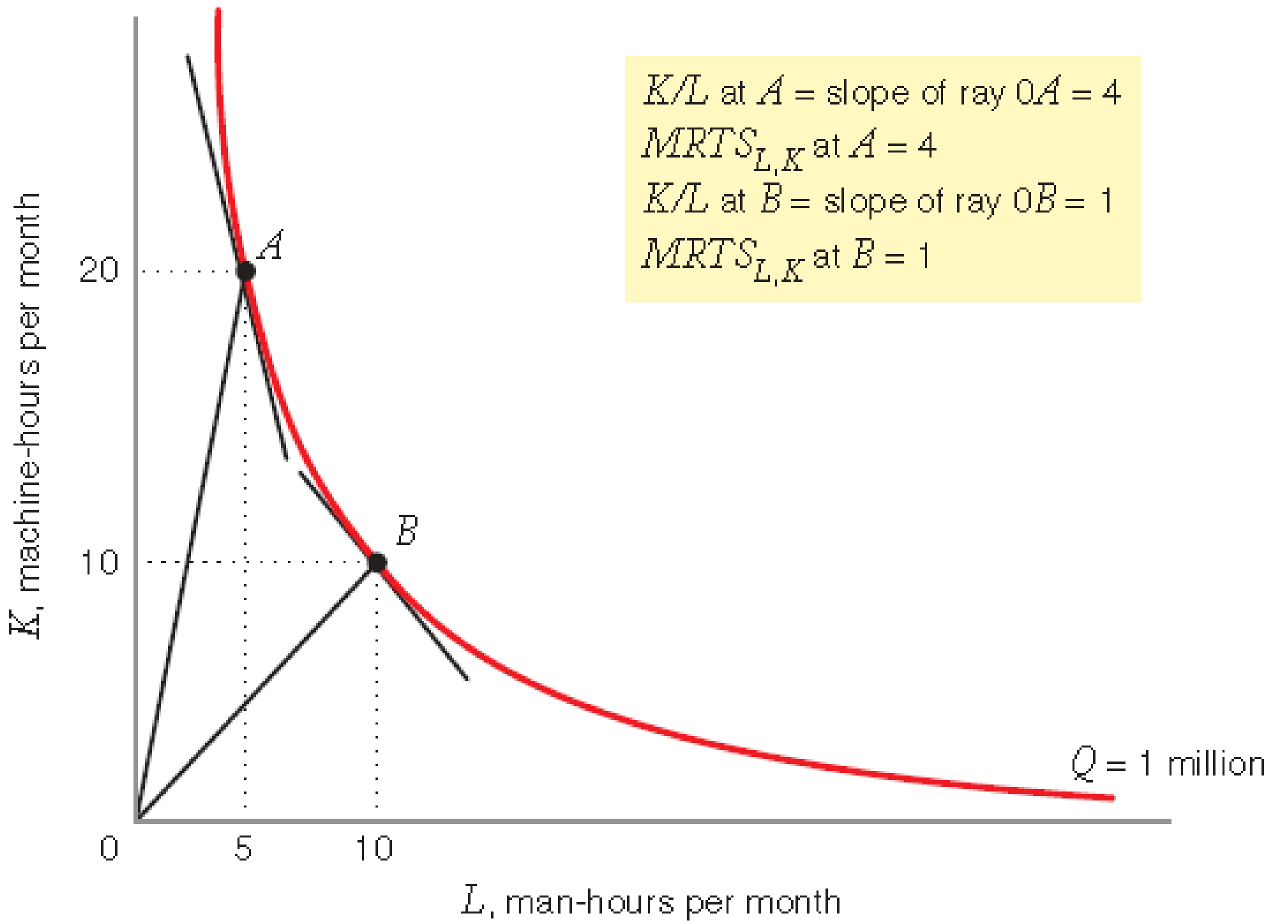
- When the production function offers abundant input substitution opportunities, the *$MRTS_{L,K}$* changes gradually as we move along an isoquant. In this case, the isoquants are nearly straight lines, as in Figure b

Elasticity of Substitution can help us describe the firm's input substitution opportunities

Elasticity of substitution: A measure of how easy it is for a firm to substitute labor for capital. It is equal to the percentage change in the capital–labor ratio for every 1 percent change in the marginal rate of technical substitution of labor for capital as we move along an isoquant

$$\sigma = \frac{\textit{percentage change} \text{ in capital–labor ratio}}{\textit{percentage change} \text{ in } MRTS_{L,K}}$$

$$= \frac{\%\Delta\left(\frac{K}{L}\right)}{\%\Delta MRTS_{L,K}}$$



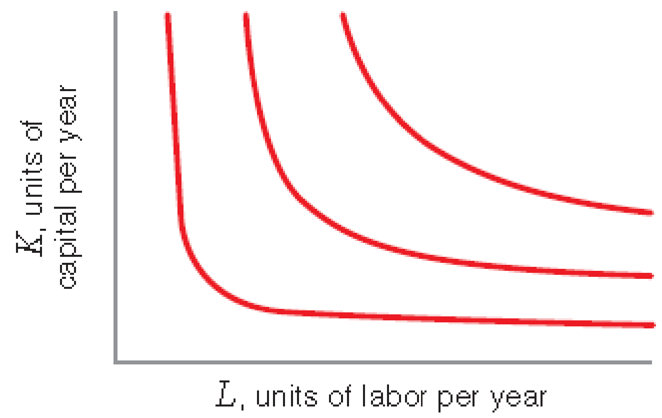
Elasticity of Substitution of Labor for Capital

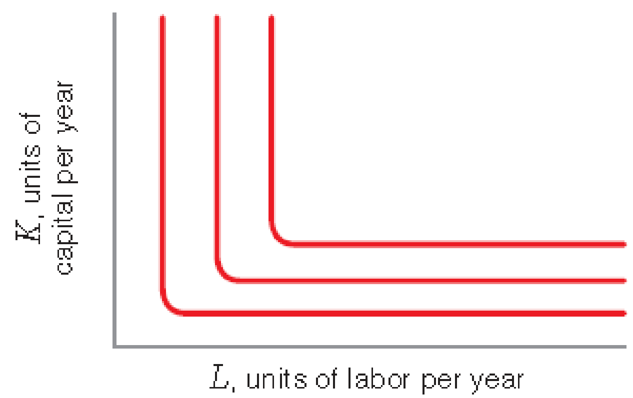
Elasticities of Substitution in German Industries[11]

TABLE 6.5 Elasticities of Substitution in German Manufacturing Industries, 1970–1988

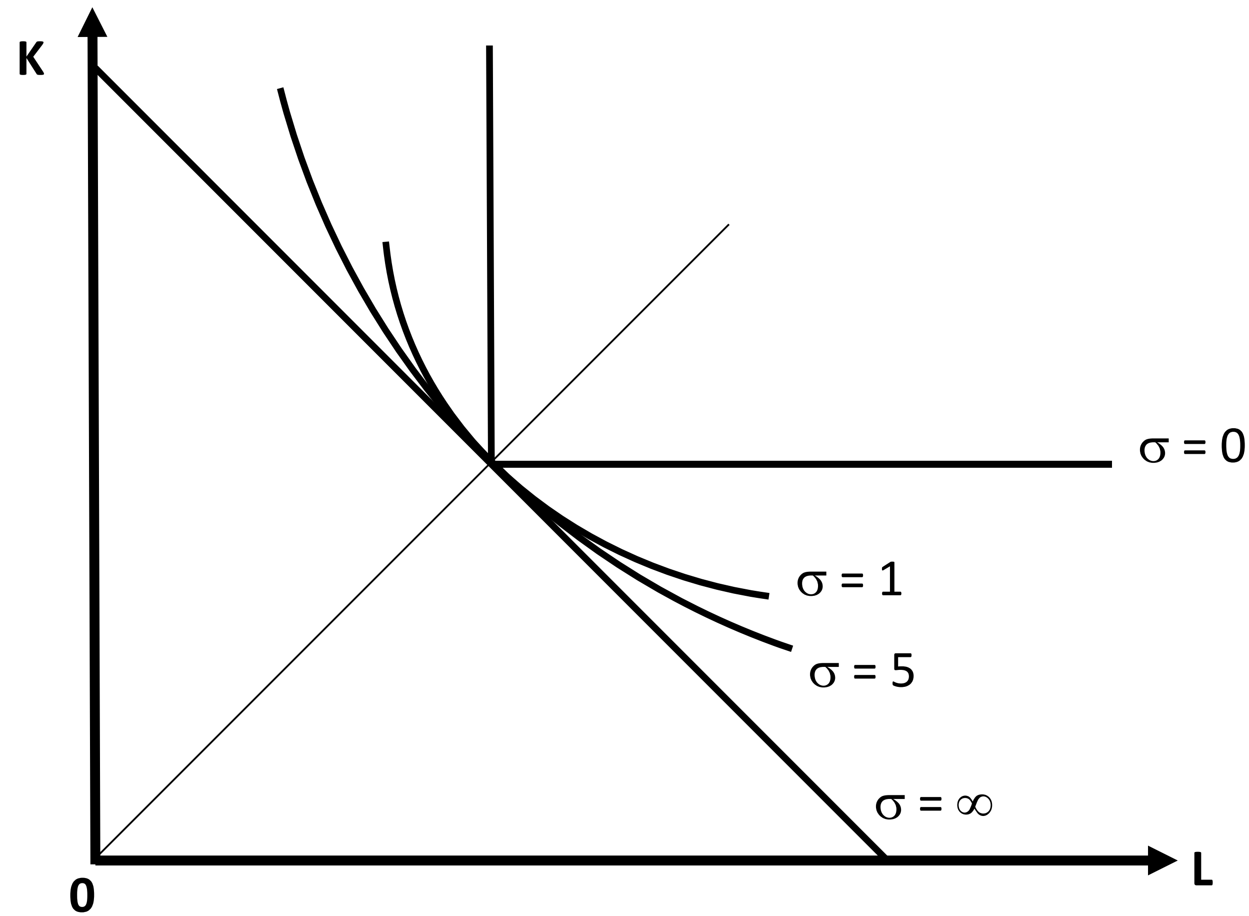
Industry	Elasticity of Substitution
Chemicals	0.37
Stone and earth	0.21
Iron	0.50
Motor vehicles	0.10
Paper	0.35
Food	0.66

[11]This example is based on "Estimated Substitution Elasticities of a Nested CES Production Function Approach for Germany," *Energy Economics* 20 (1998): 249–264.





The shape of the isoquant indicates the degree of substitutability of the input.



4 Special Production Functions

- The linear production function
- The fixed-proportions production function
- The Cobb–Douglas production function
- The constant elasticity of substitution production function

4 Special Production Functions

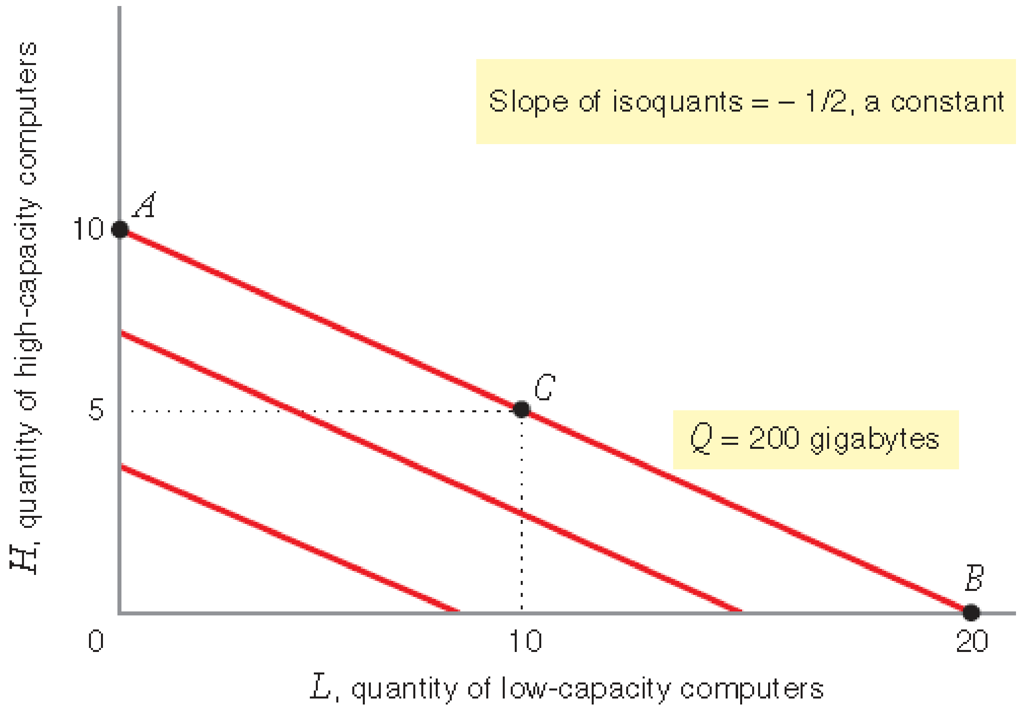
The Linear Production Function

The Fixed-Proportion Production Function

The Cobb-Doughlas Production Function

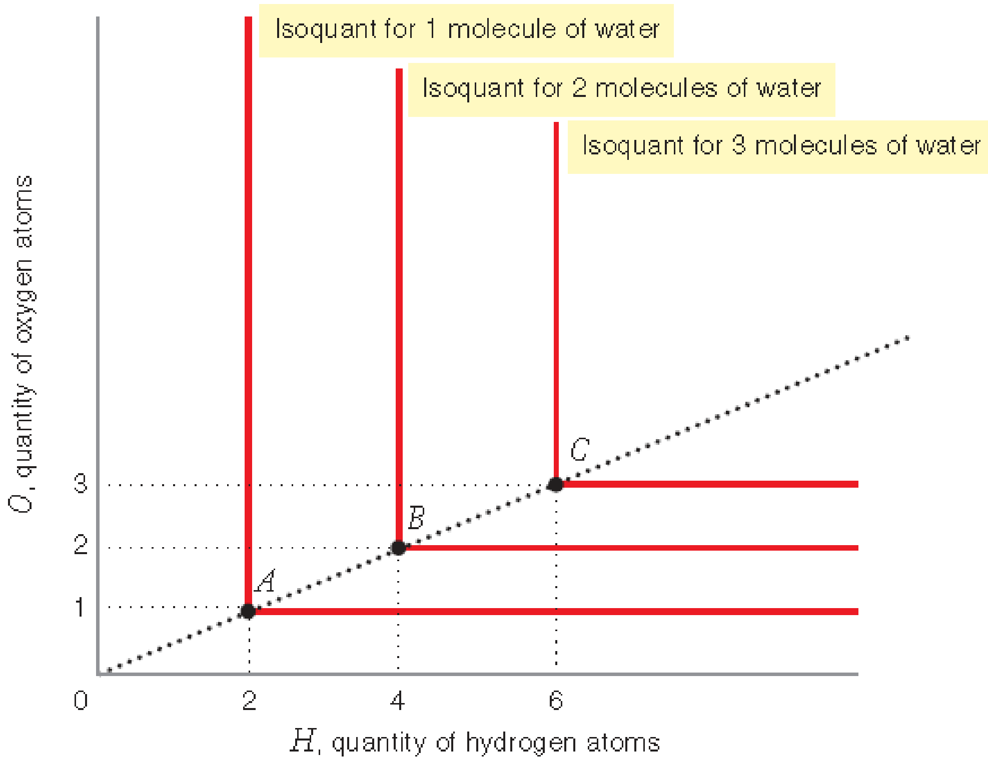
The Constant Elasticity Production Function

Linear Production Function (Perfect Substitutes)



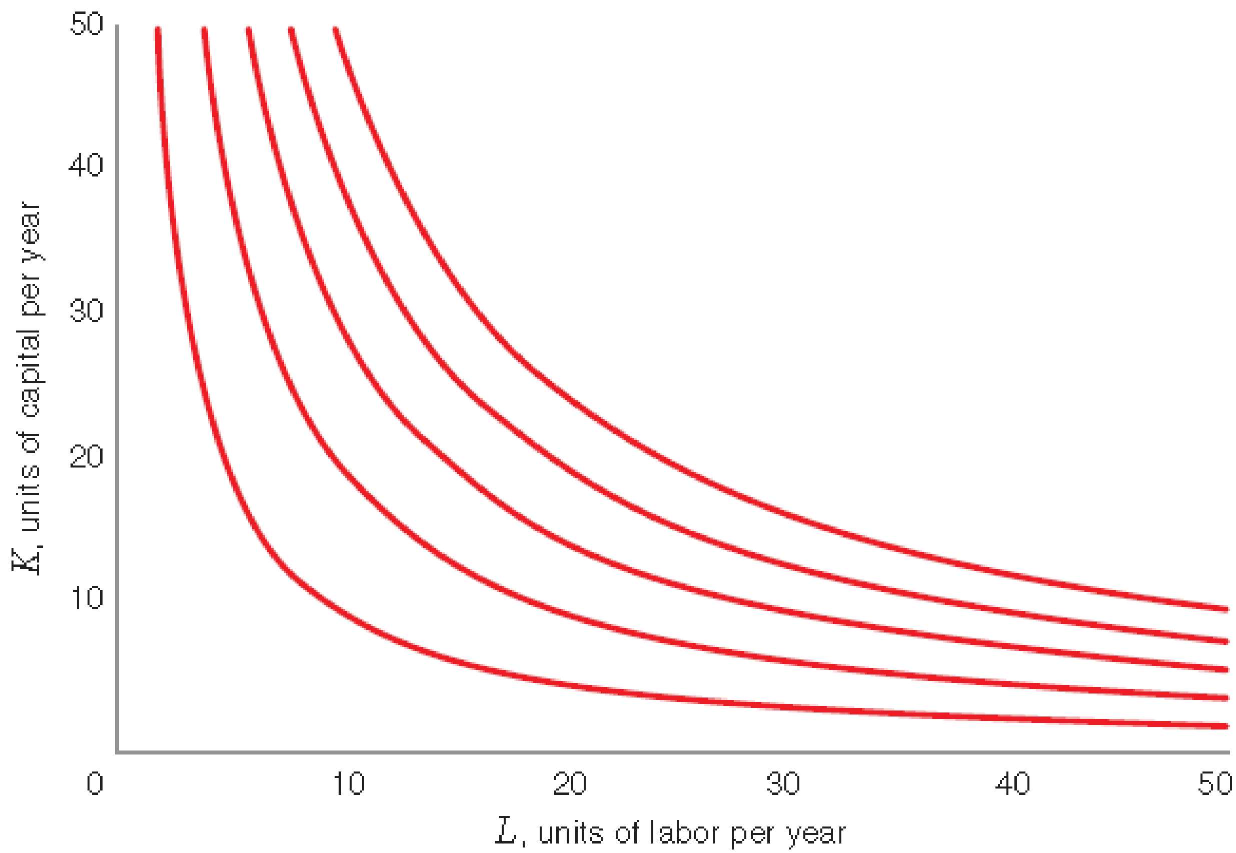
Isoquants for a Linear Production Function

Fixed-Proportions Production Function (Perfect Complements)



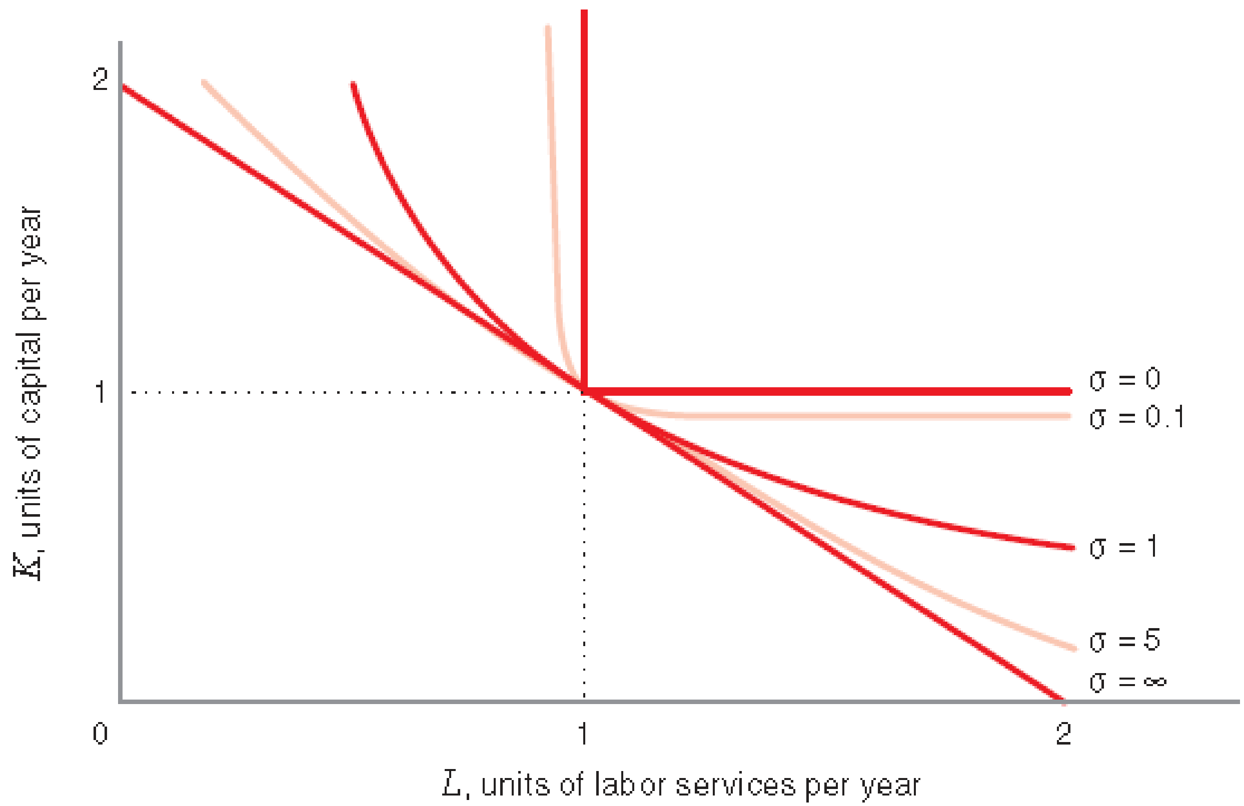
Isoquants for a Fixed-Proportions Production Function

Cobb–Douglas Production Function



Isoquants for a Cobb–Douglas Production Function

Constant Elasticity of Substitution Production Function



Isoquants for the CES Production Function

$$Q = \left[aL^{\frac{\sigma-1}{\sigma}} + bK^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$$

Characteristics of Production Functions: A Summary

Production Function	Elasticity of Substitution (σ)	Other Characteristics
Linear production function	$\sigma = \infty$	Inputs are perfect substitutes Isoquants are straight lines
Fixed-proportions production function	$\sigma = 0$	Inputs are perfect complements Isoquants are L-shaped
Cobb—Douglas production function	$\sigma = 1$	Isoquants are curves
CES production function	$0 \leq \sigma \leq \infty$	Indudes other three production functions as special cases Shape of isoquants varie

APPLICATION 6.6

Estimating a CES Production Function for U.S. Industries

Using data from the Bureau of Economic Analysis for 1947–1998, economists Edward Balistreri, Christine McDaniel, and Eina Vivian Wong (BMW) estimated the constant σ in a CES production function relating the quantity of output to the quantities of labor and capital in each of 28 U.S. industries.[14] Because, as discussed in the text, σ represents the elasticity of substitution, BMW's estimates provide insight into the opportunities for substituting between labor and capital in these industries.

Table 6.7 shows the estimates of σ for a subset of the 28 industries BMW studied. The table shows two types of estimates for each industry: a long-run elasticity of substitution and a short-run elasticity of

TABLE 6.7 Estimates of σ for Selected U.S. Industries

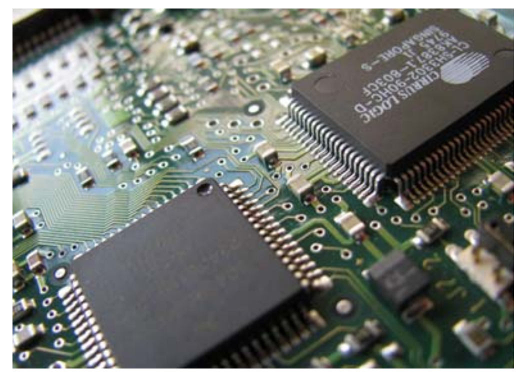
	Estimated Value of σ	
Industry	Short Run	Long Run
Agricultural services, forestry, and fishing	0.23	0.36
Coal mining	0.10	1.27
Furniture and fixtures	0.10	1.01
Fabricated metal products	0.11	1.39
Industrial machinery and equipment	0.23	0.82
Motor vehicles and equipment	0.05	0.40
Textile mill products	0.05	1.14
Apparel and other textile products	0.13	2.05

[14] Edward Balistreri, Christine McDaniel, and Eina Vivian Wong, "An Estimation of US Industry-Level Capital–Labor Substitution Elasticities: Support for Cobb-Douglas," *North American Journal of Economics and Finance* 14 (2003): 343–356.

Returns to Scale

Question: By how much would a semiconductor firm be able to increase its output *if it doubled its man-hours of labor and its machine-hours of robots*?

Returns to scale The concept that tells us the percentage by which output will increase when all inputs are increased by a given percentage.



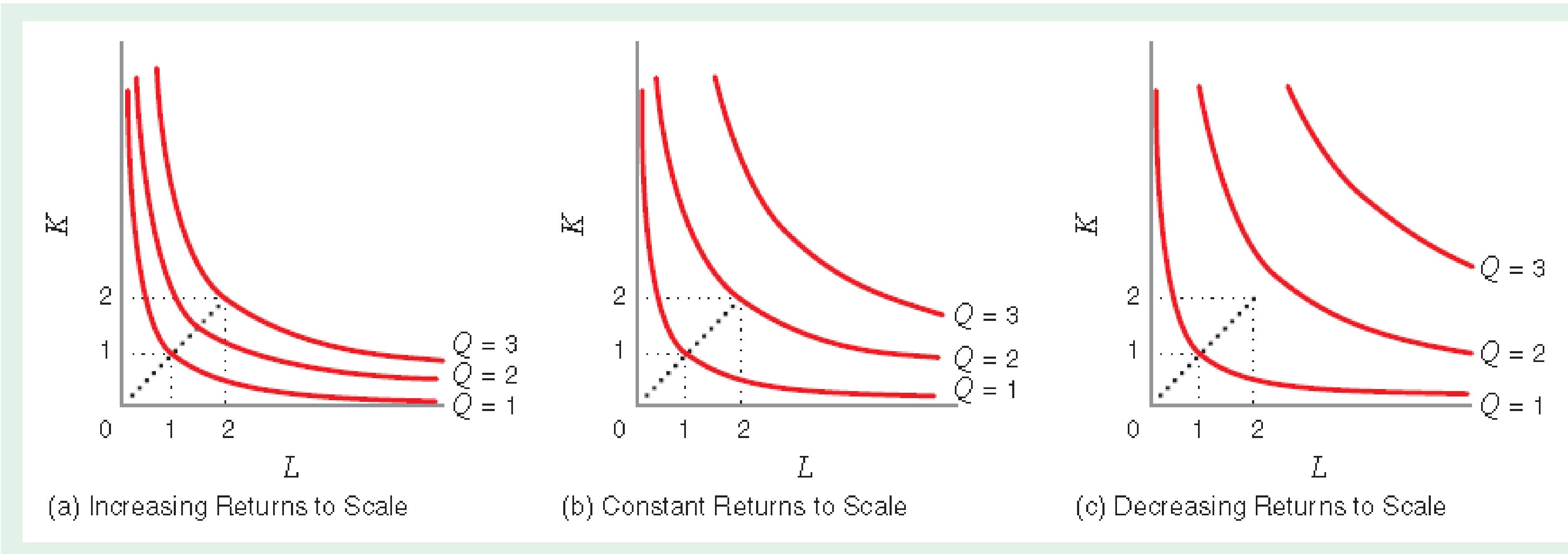
3 Types of Returns to Scale

- **increasing returns to scale:** proportionate increase in all input quantities resulting in a greater than proportionate increase in output.

- **constant returns to scale:** A proportionate increase in all input quantities simultaneously that results in the same percentage increase in output.

- **decreasing returns to scale** A proportionate increase in all input quantities resulting in a less than proportionate increase in output.

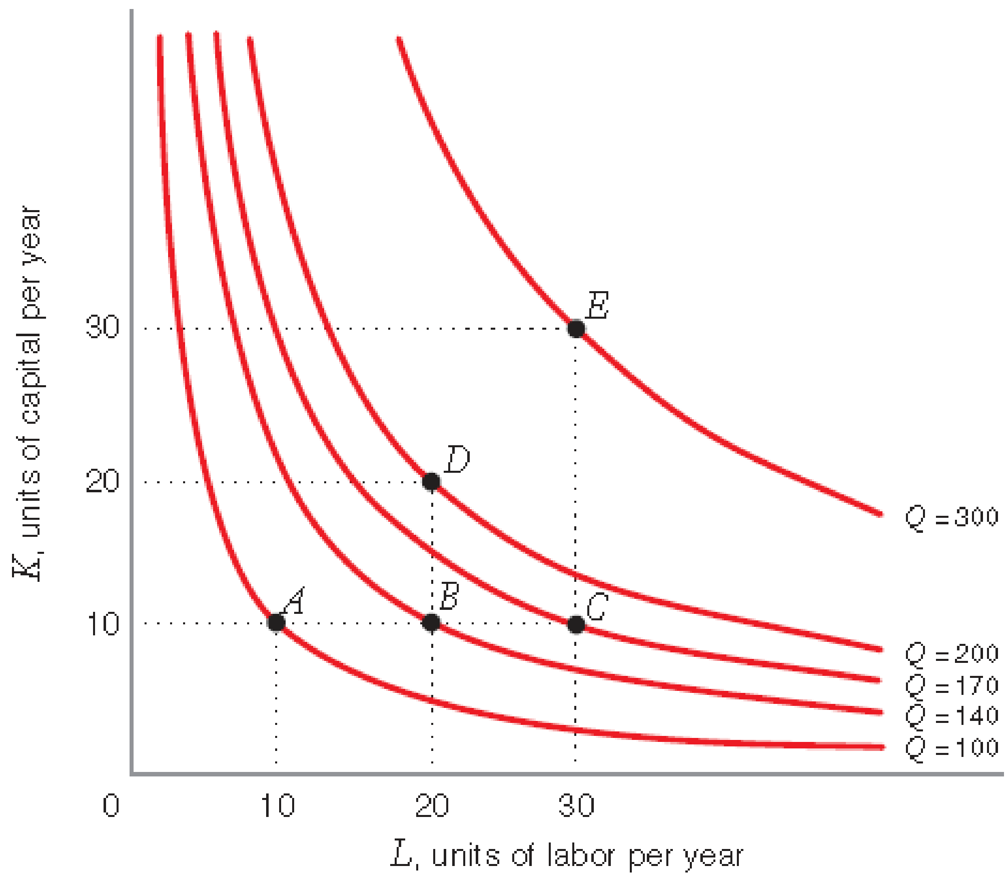
Increasing, Constant, and Decreasing Returns to Scale



Learning-by-Doing: Returns to Scale for a Cobb–Douglas Production Function

- Does a Cobb–Douglas production function, $Q = AL^{\alpha}K^{\beta}$ exhibit increasing, decreasing, or constant returns to scale?

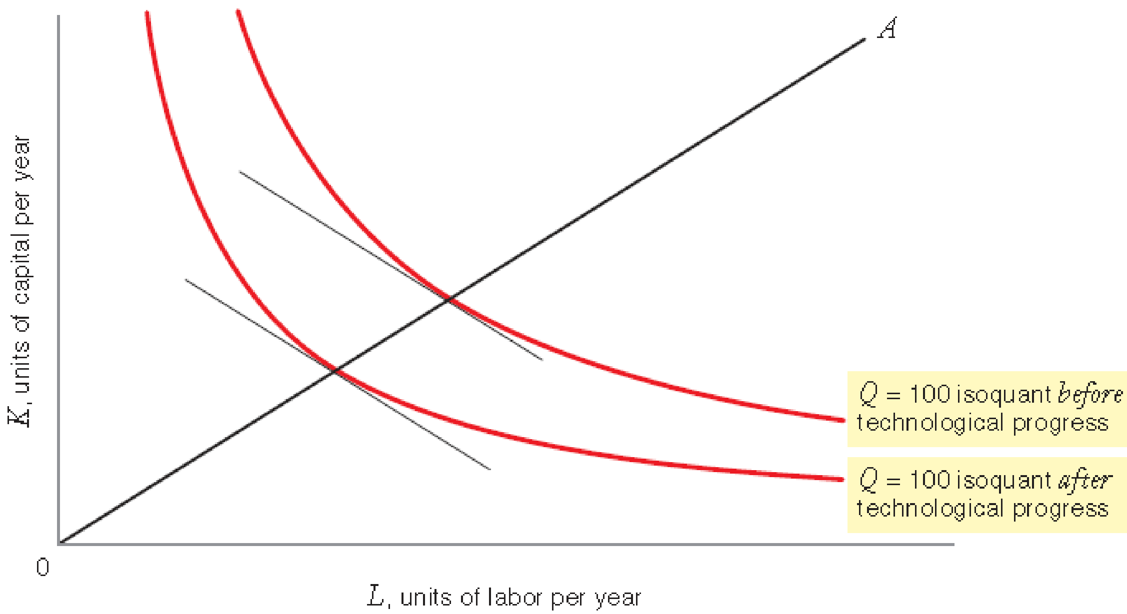
Returns to Scale Versus Diminishing Marginal Returns



Technological progress

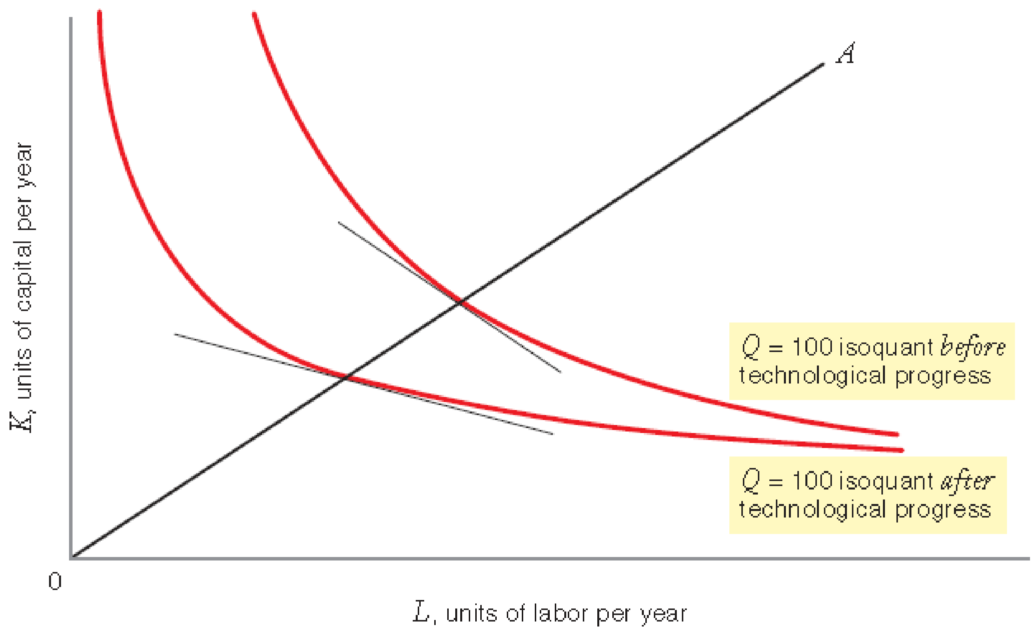
technological progress : A change in a production process that enables a firm to achieve more output from a given combination of inputs or, equivalently, the same amount of output from less inputs.

Neutral Technological Progress (*MRTSL,K* Remains the Same)



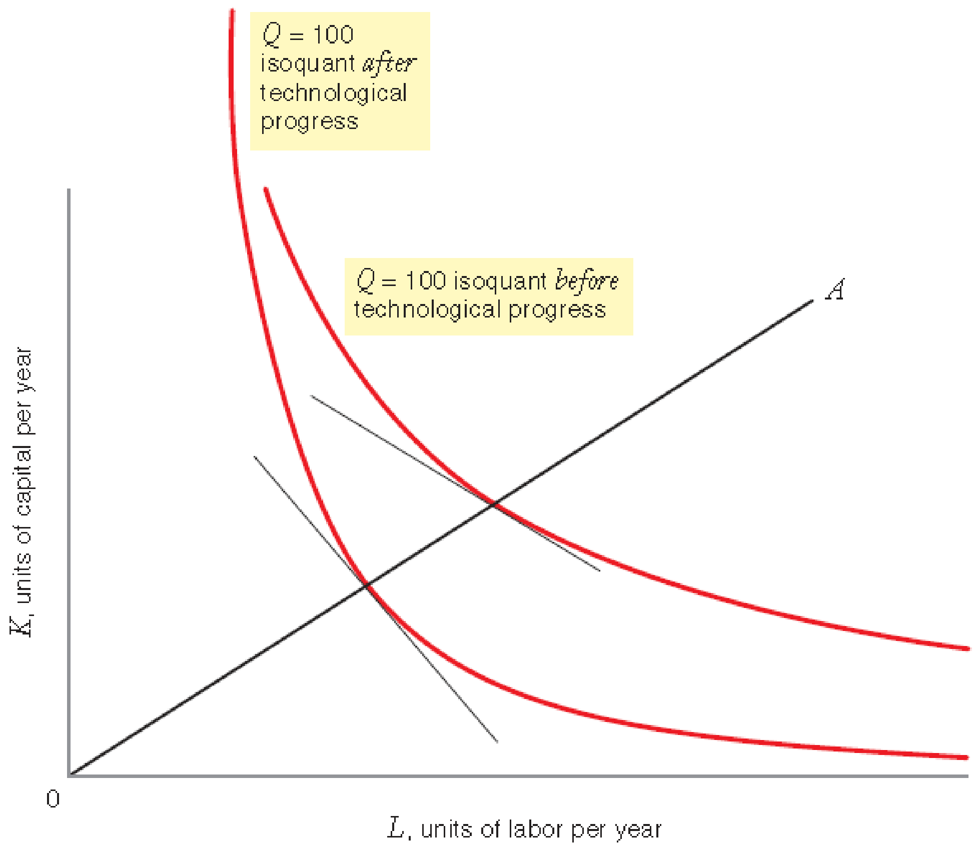
neutral technological progress Technological progress that decreases the amounts of labor and capital needed to produce a given output, without affecting the marginal rate of technical substitution of labor for capital.

Labor-Saving Technological Progress (*MRTSL,K* Decreases)



labor-saving technological progress
Technological progress that causes the marginal product of capital to increase relative to the marginal product of labor.

Capital- Saving Technological Progress (*MRTSL,K* Increases)



capital-saving technological progress

Technological progress that causes the marginal product of labor to increase relative to the marginal product of capital.

Learning-by-Doing: Technological Progress