EE312 Macroeconomic Theory

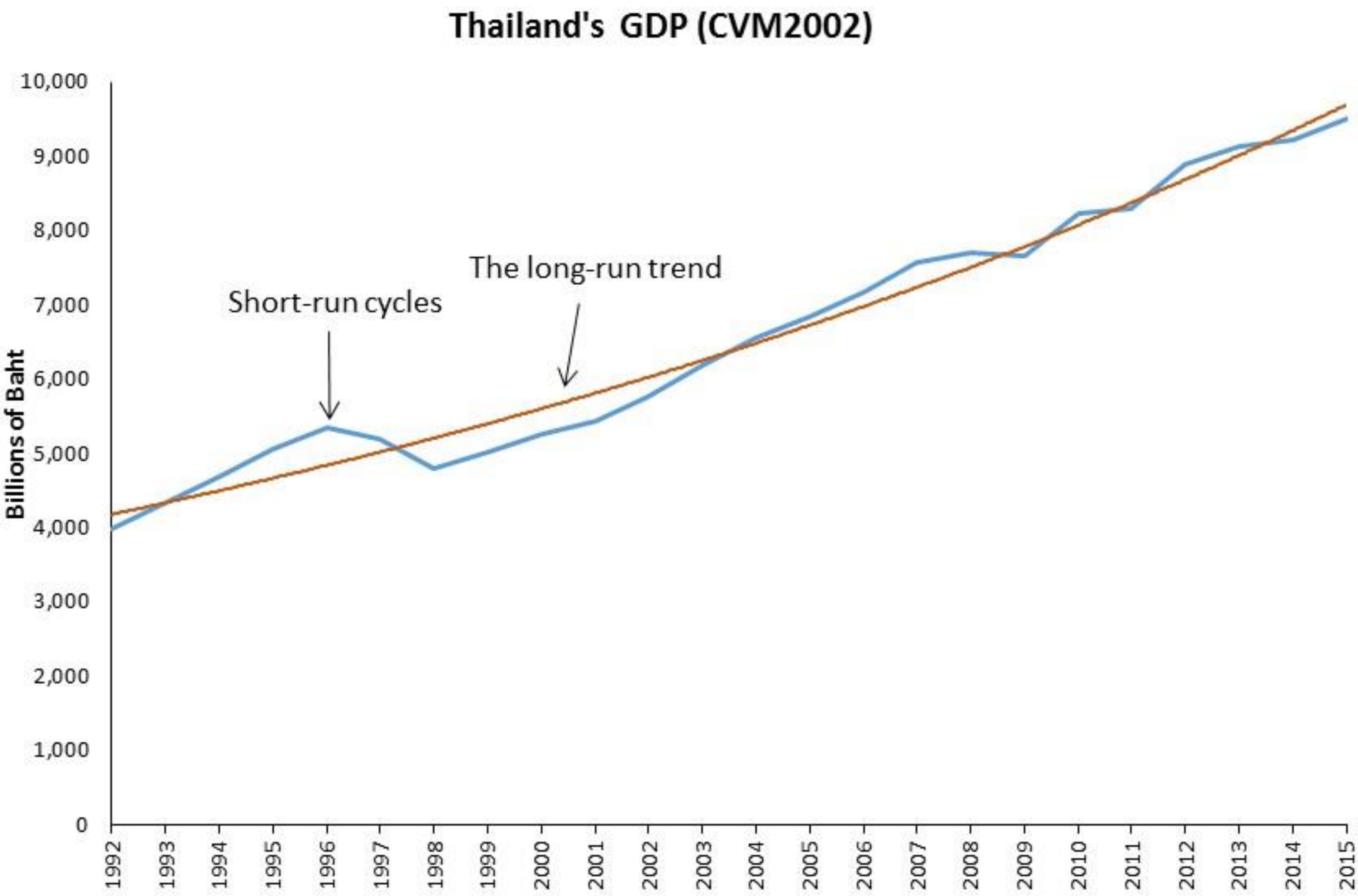
Chapter 5:

Economic Growth

The Solow growth model

Importance of growth

- The standards of living in the long term depend on **economic growth**.
  - Short-run fluctuations tend to cancel out in the long run.
- What determines economic growth?
  - Models of economic growth.
  - The Solow growth model.
  - Endogenous growth models.


Thailand's GDP (CVM2002)
Billions of Baht
10,000
9,000
8,000
7,000
6,000
5,000
4,000
3,000
2,000
1,000
0
1992
1993
1994
1995
1996
1997
1998
1999
2000
2001
2002
2003
2004
2005
2006
2007
2008
2009
2010
2011
2012
2013
2014
2015
Short-run cycles
The long-run trend

Facts about growth

1. Before the Industrial Revolution (1800), standards of living differed little over time and across regions.

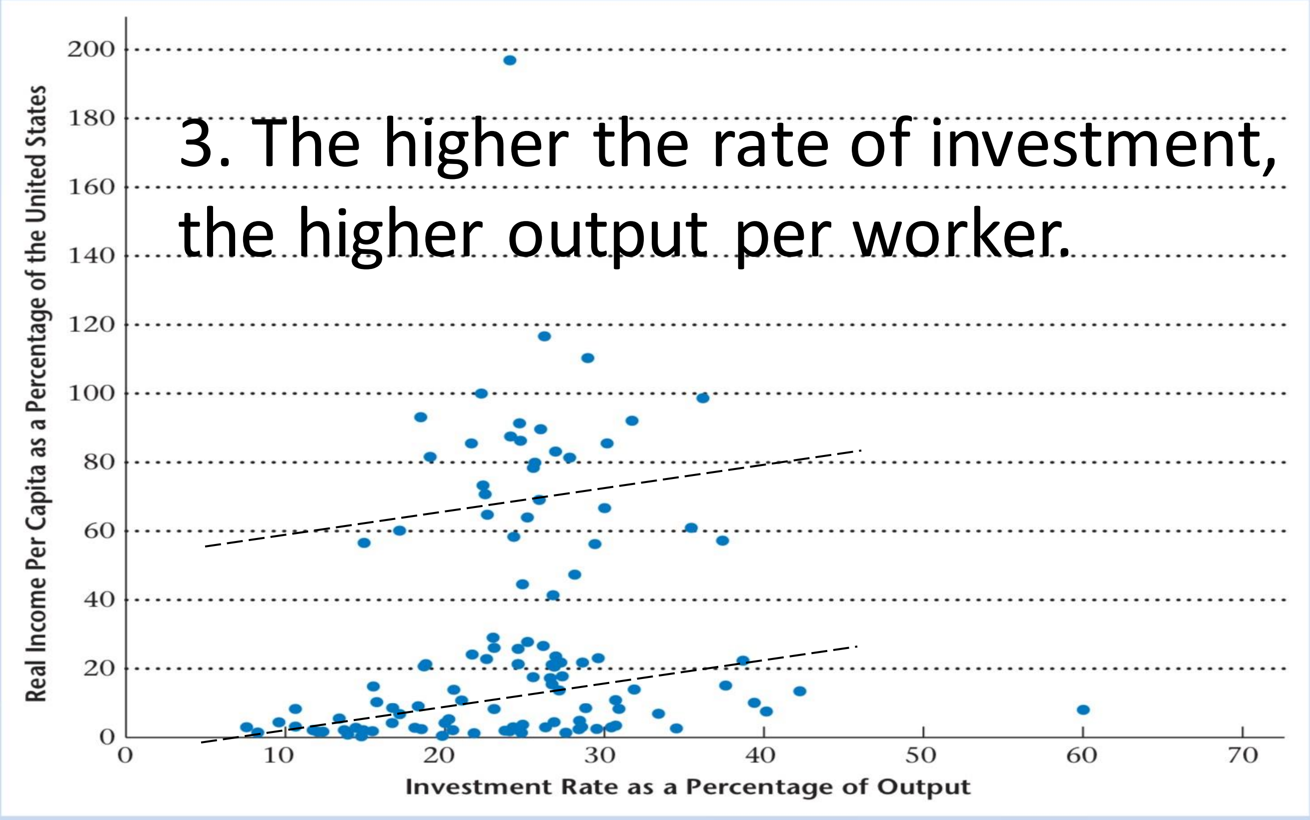
2. Since **the Industrial Revolution**, per capita income growth has been sustained in **developed countries**.

Facts about growth

Figure 7.2 Real Income Per Capita vs. Investment Rate

The figure shows a positive correlation across the countries of the world, between the output per capita and the investment rate.

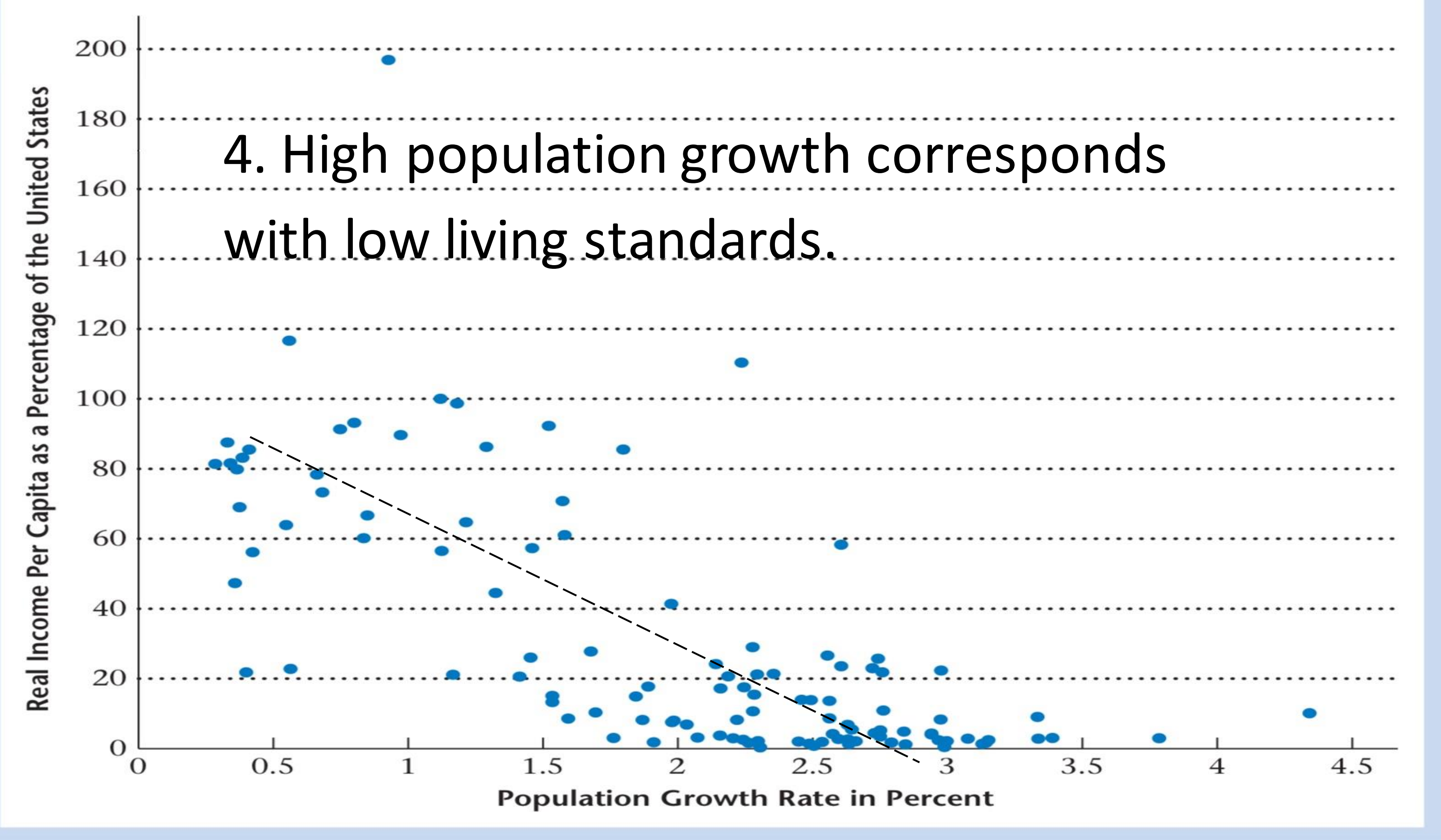
Source: A. Heston, R. Summers, and B. Aten, *Penn World Table Version 7.0*, Center for International Comparisons of Production, Income, and Prices at the University of Pennsylvania, May 2011.



3. The higher the rate of investment, the higher output per worker.

Facts about growth

Figure 7.3 Real Per Capita Income vs. the Population Growth Rate
Across the countries in the world, real per capita income and the population growth rate are negatively correlated.



Facts about growth

5. International differences in living standards **increasingly and persistently widen** between developed and developing countries (except East Asia).

Poverty in India

Affluent society in developed countries.

Facts about growth

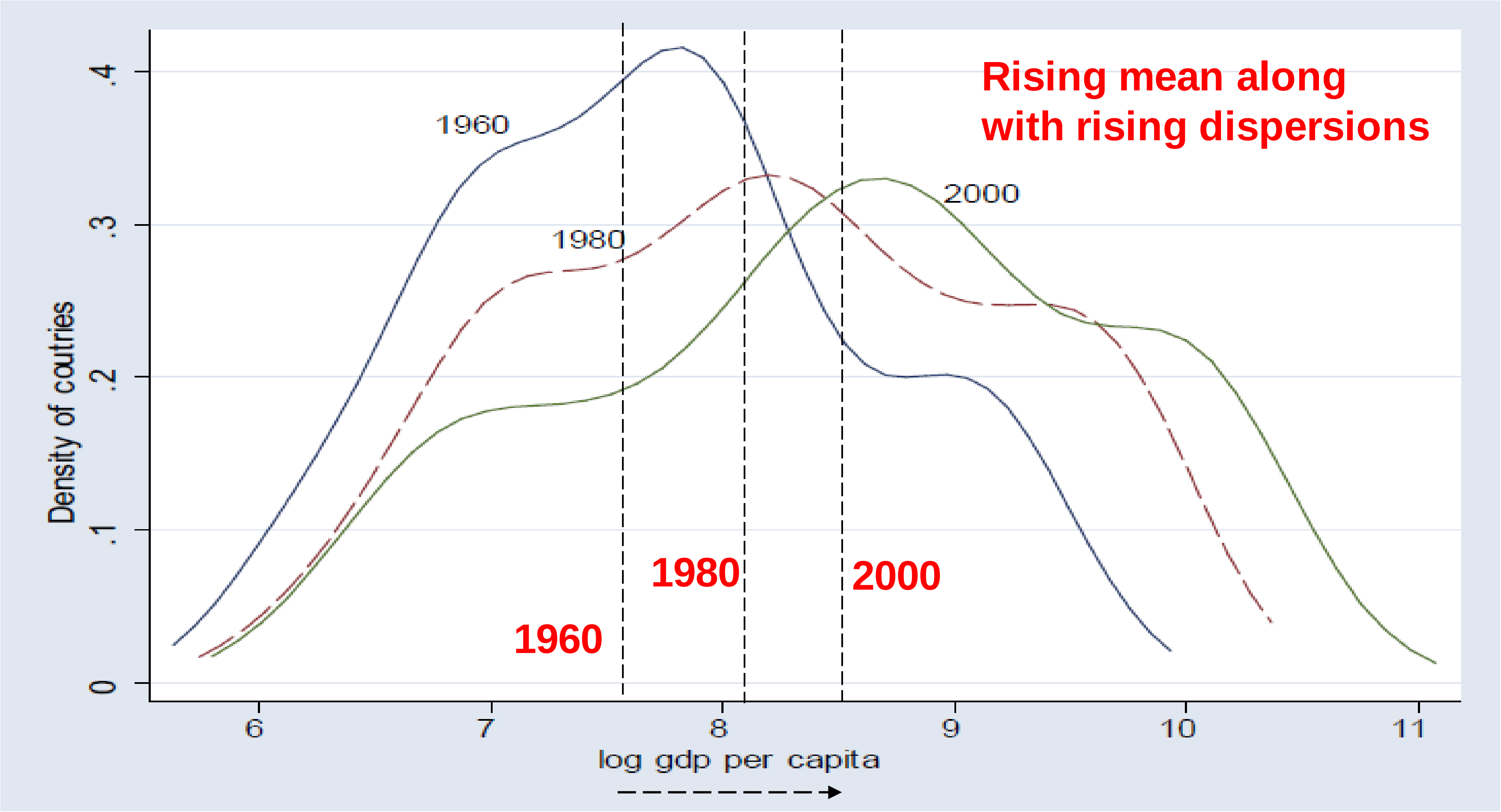


Figure: Estimates of the distribution of countries according to log GDP per capita (PPP-adjusted) in 1960, 1980 and 2000.

World Bank classification 2016

- **Gross National Income (GNI) per capita** (Atlas method).
  - High income (79) > $12,235
  - Upper middle income (55) $3,956 - $12,235
  - Lower middle income (52) $1,006 - $3,955
  - Low income (31) < $1,006
  - **World average** $10,302

- **High income**: North America, Western Europe, East Asia, oil-rich Middle East.
- **Upper middle income**: South America, Eastern Europe, Russia, South-East Asia.
- **Lower middle income**: South America, Central Asia, Africa.
- **Low income**: Central and East Africa

Rich and poor in Asia

Japan	38,000	**Singapore**	51,880
S. Korea	27,600	**Malaysia**	9,850
Hong Kong	43,420	**Thailand**	5,640
China	8,260	Philippines	3,580
Bhutan	2,510	Indonesia	3,400
India	1,680	Vietnam	2,050
Pakistan	1,510	Lao	2,150
Bangladesh	1,330	Myanmar	1,190
Nepal	730	Cambodia	1,140

Facts about growth

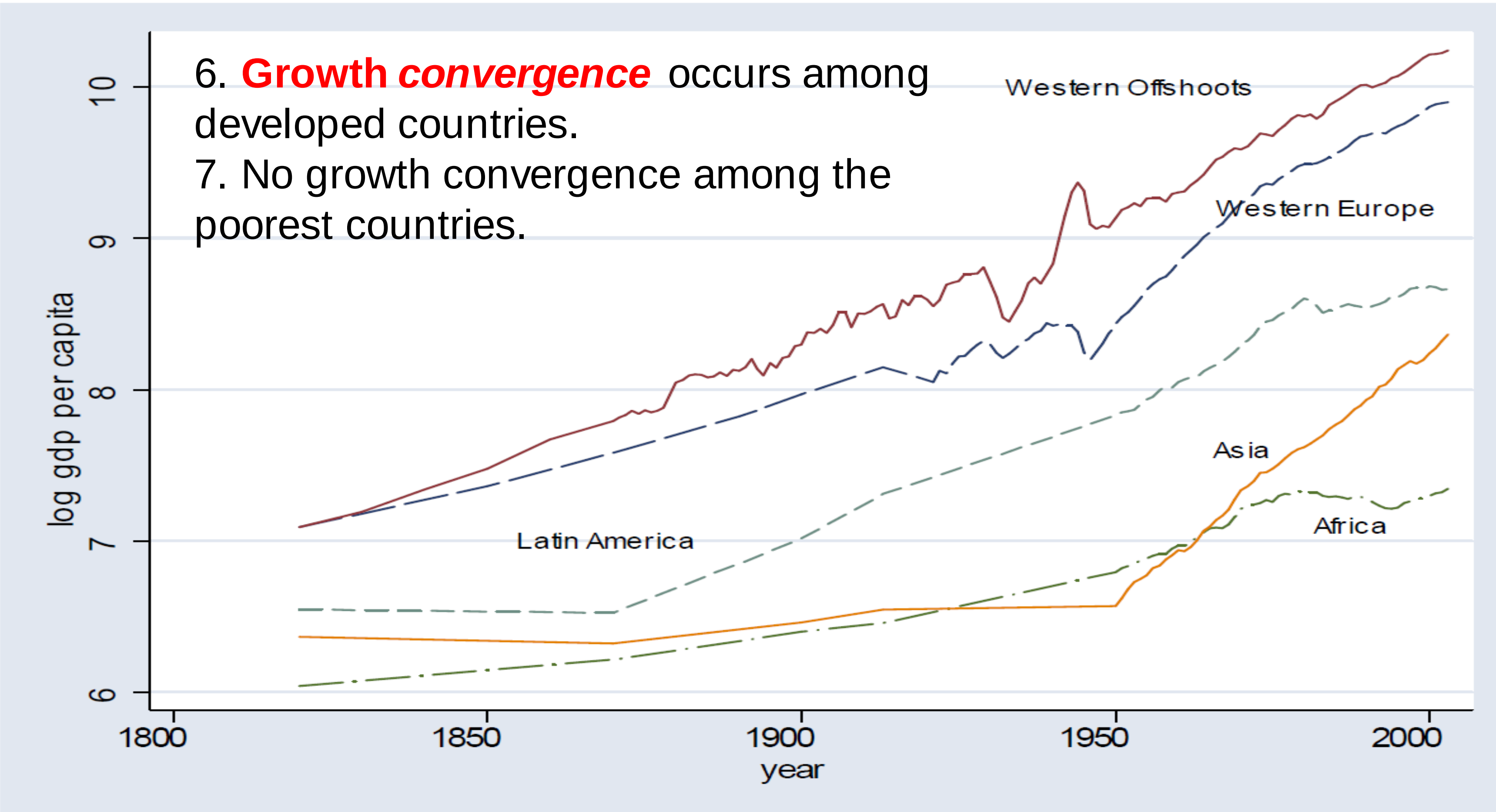
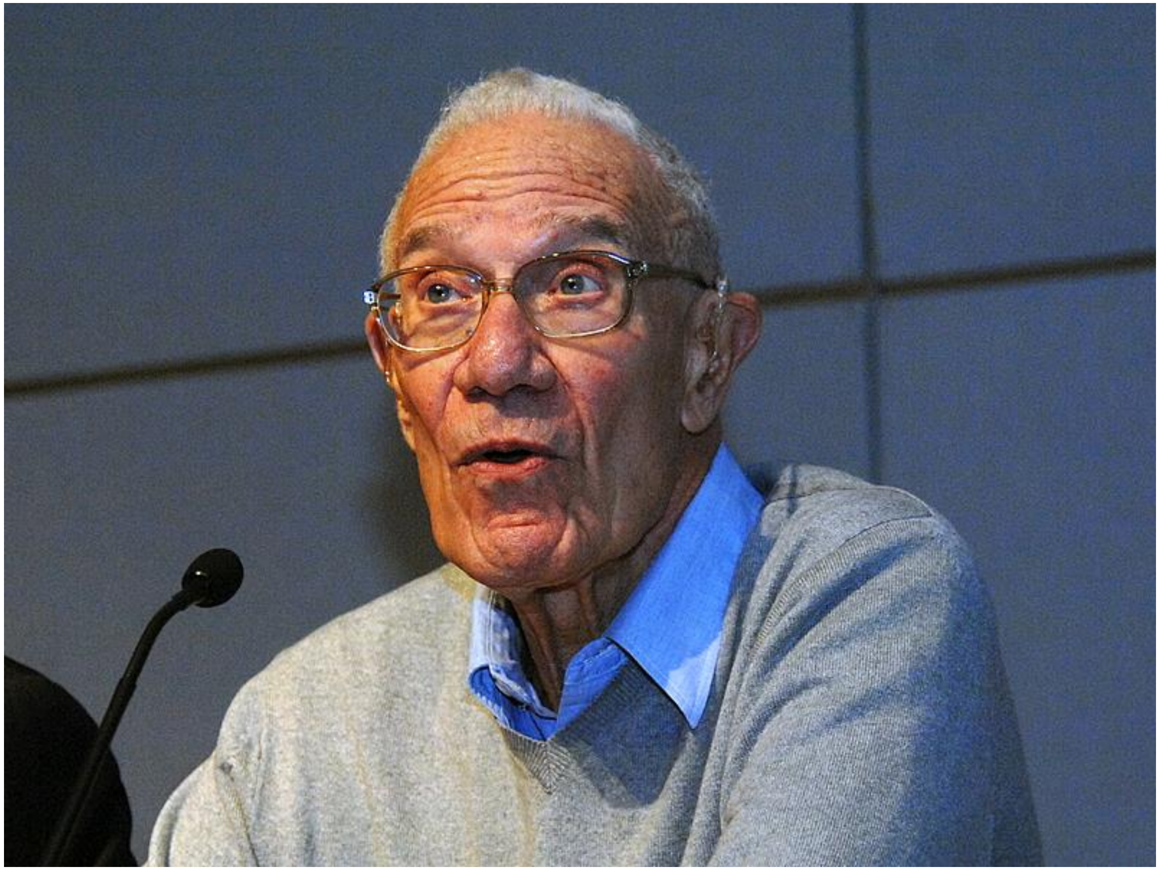


Figure: Evolution of GDP per capita 1820-2000.

Growth models

- **Solow growth**: sustainable growth based on technological progress.
  - **Exogenous growth**: technology is determined outside the model.
  - Growth convergence among countries.
- **Endogenous growth**: sustainable growth based on human capital.
  - Growth engine is **endogenous**.
  - No certainty in growth convergence.



Robert M. Solow (b.1924),
Massachusetts Institute of Technology,
Nobel Prize 1987

The Solow growth model

- The basis of all modern theories of growth.
- Long-term economic growth depends on one single factor --- **technological progress**.
  - Rising **total factor productivity (z)**.
  - Sustained improvement in living standards (real per capita income or output per worker).

Population growth

- Assume population grows exogenously at a constant rate.
- N = population in the current period.
- N' = population in the future period.
- n > -1; rate of population growth.

$$N' = (1+n)N$$

Consumers

- Consumers = population = workers.
- Consumers supply labor in production.
- Consumers receive real output (Y) as (wage and dividend) income.
  - Spend on consumption goods (C) and save a constant fraction (s) of Y as savings (S).

$$Y = C + S; \quad S = sY$$

$$C = (1 - s)Y$$

The representative firm

- The firm produces output using current capital stock (K) and current labor input (N).
- Assuming **constant returns to scale**.

$$Y = zF(K, N)$$

$$\frac{Y}{N} = zF(\frac{K}{N}, 1)$$

Per worker production function

$$Let\ y = \frac{Y}{N} = output\ \ per\ wor\,\text{ker}$$

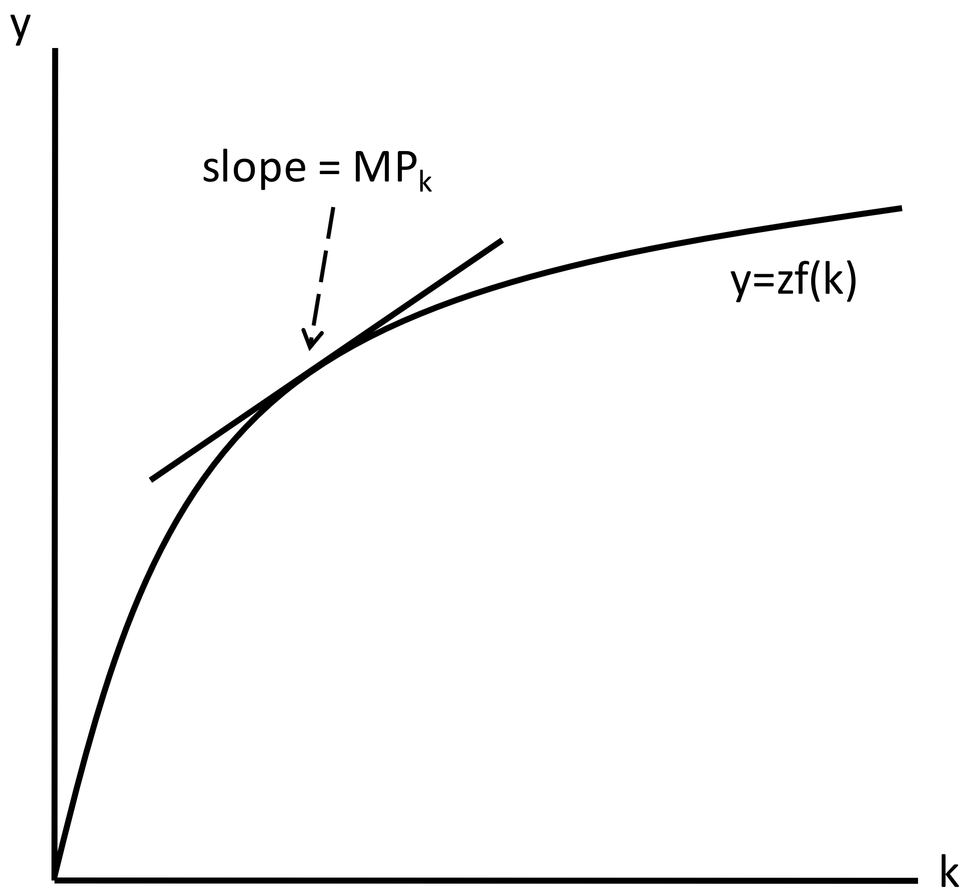
$$k = \frac{K}{N} = capital\ \ per\ wor\,\text{ker}$$

$$f(k) = zF\left(\frac{K}{N},1\right)$$

$$y = zf(k)$$

Marginal product of k

- Output per worker (y) **increases at a decreasing rate** as capital per worker (k) rises.
- Slope is the marginal product of k.

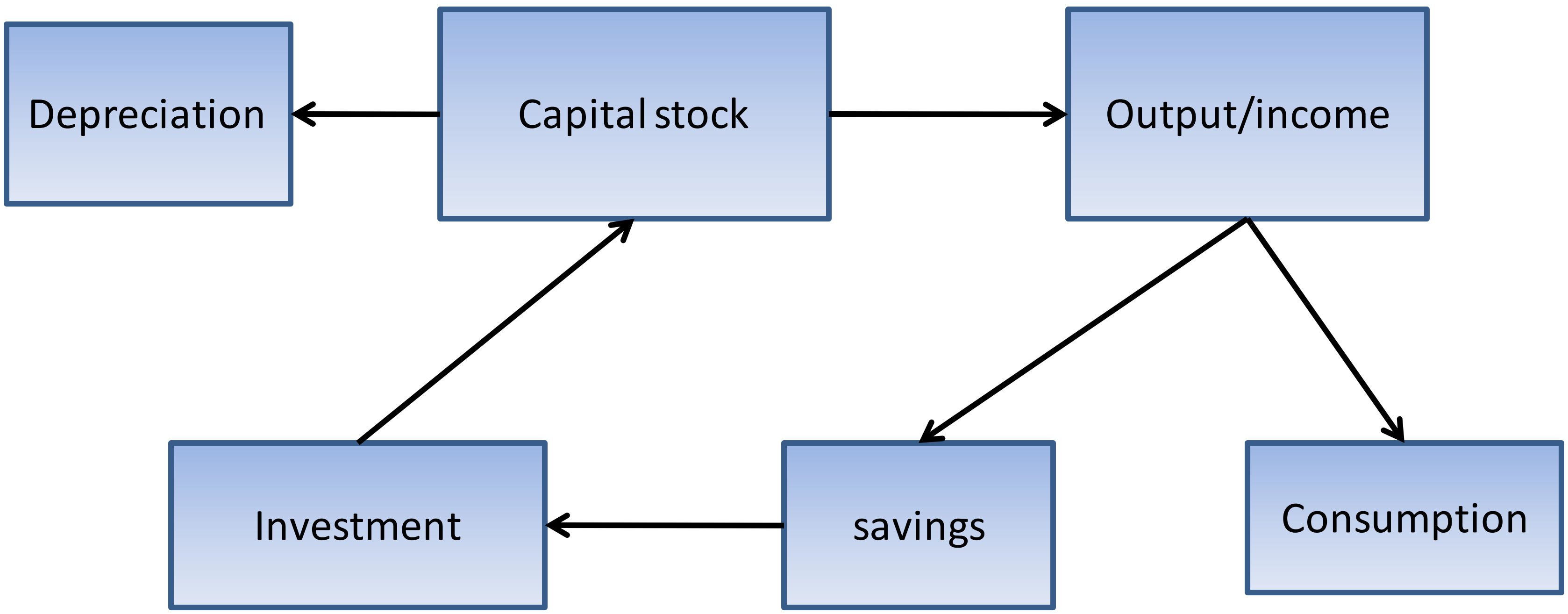


Growth of capital stock

- Assumer capital wears out over time at the rate of d (or depreciation).
  - where 0 > d > 1.
- I = investment = addition to capital stock.
- K’ = capital stock in the future period.

$$K' = (1-d)K + I$$

The working of growth



Equilibrium output

- At equilibrium, **savings equals investment** so that output consists of consumption and investment.

$$S = I$$

$$S = Y - C$$

$$Y = C + I$$

Equilibrium condition

- The future capital stock is current capital stock deducted by depreciation and added by investment (= savings).

$$Y = C + I$$
$$C = (1-s)Y$$
$$I = K' - (1-d)K$$

- Substitute C and I in the Y equation.

Per worker formulation

$$Y = (1-s)Y + K' - (1-d)K$$

rearrange the terms:

$$K' = sY + (1-d)K$$

but $\quad Y = zF(K,N)$

so $\quad K' = szF(K,N) + (1-d)K$

divide it by N *:*

$$\frac{K'}{N} = sz\frac{F(K,N)}{N} + (1-d)\frac{K}{N}$$

Future capital per worker function

$$\frac{K'}{N}\frac{N'}{N'} = szF(\frac{K}{N},1) + (1-d)\frac{K}{N}$$

$$where \quad k' = \frac{K'}{N'} \quad and \quad \frac{N'}{N} = (1+n)$$

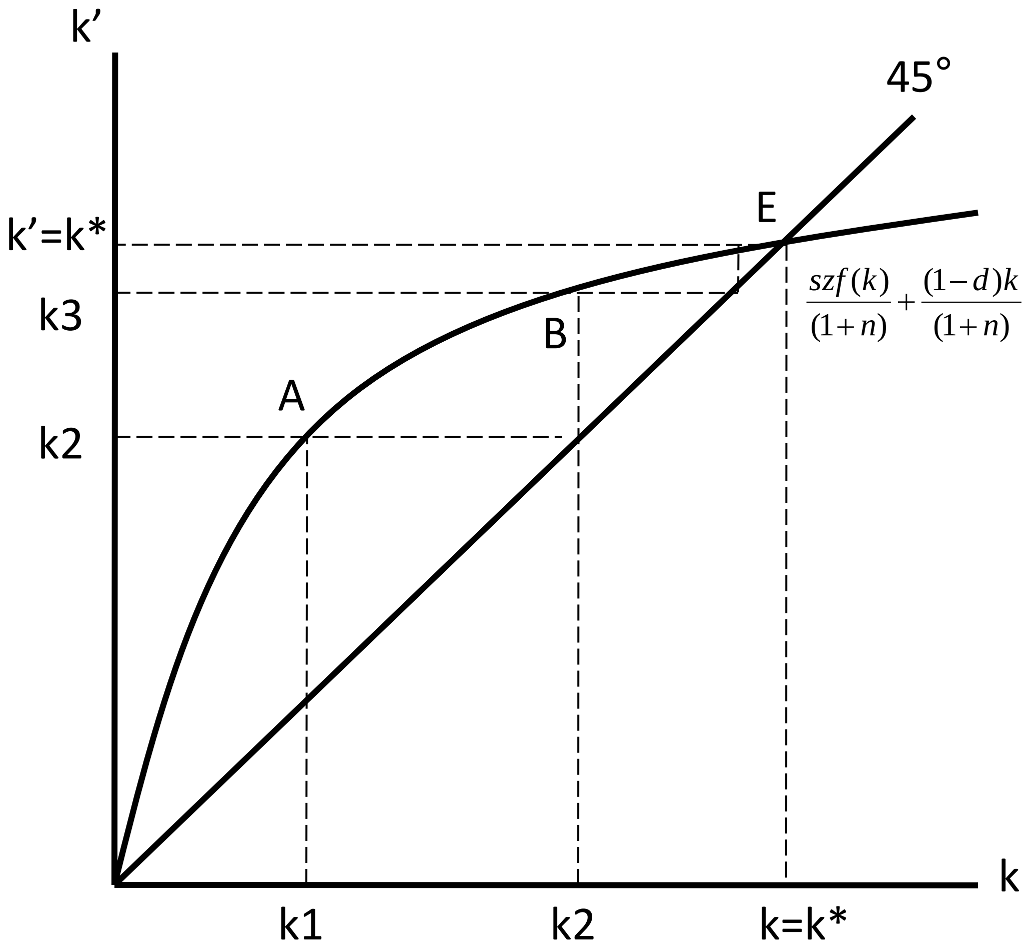
$$k'(1+n) = szf(k) + (1-d)k$$

$$k' = \frac{szf(k)}{(1+n)} + \frac{(1-d)k}{(1+n)}$$

- Future k’ as a function of current k.

The steady-state capital per worker

- At A, k2 > k1; k is growing.
- At B, k3 > k2; k is growing.
- k = k*; steady-state capital per worker.



Diminishing returns on k

- At E, k = k'= k* so that k* is steady.
  - To the left of k*, k'> k so that k is increasing.
  - To the right of k*, k'< k so that k is decreasing.
- As k is increasing, MP_k is falling so that **y is increasing at a decreasing rate**.
- Finally, **investment** (or new capital) is just sufficient to keep up with **population growth** and **depreciation**, so that k (and y) is stagnant.

Steady-state aggregates

- With k* at the steady state, y*, c* and szf(k*) are all at **the steady-state**.
  - No further improvement in output per worker (y).
- Given population growth (n), total factor productivity (z) and the savings rate (s), the steady-state growth rate is ‘n’ for aggregate quantities:
  - Capital stock (K) and output (Y);
  - Consumption (C), savings (S) and investment (I).

Analysis of the steady-state

$$k^{*} = \frac{szf(k^{*})}{(1+n)} + \frac{(1-d)k^{*}}{(1+n)}$$

Multiplying k* by (1+n)

$$szf(k^{*}) = (n+d)k^{*}$$

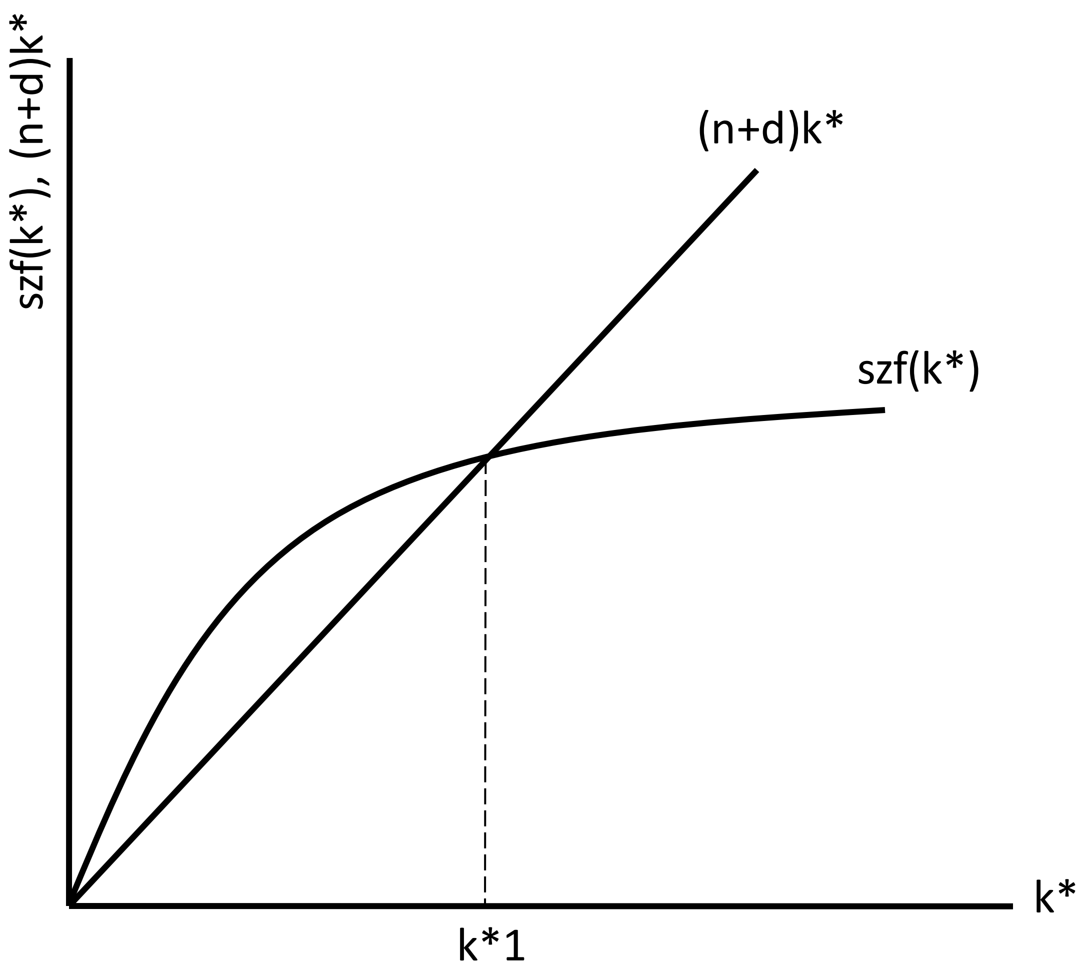
- Or steady-state savings = steady-state investment.

$$szf(k^*) = (n+d)k^*$$

- szf(k*) = savings per worker;
- (n+d)k* = investment per worker needed to keep up with population growth and depreciation.
- At k*, the capital stock is still growing, but just sufficient to equip each worker with the same k and depreciation (so k* is steady).
  - ‘**Capital widening**’: growing K just to keep the steady k and y.

Determination of steady-state k*

- szf(k*) is concave due to zf(k*).
- (n+d)k* has the slope = (n+d).

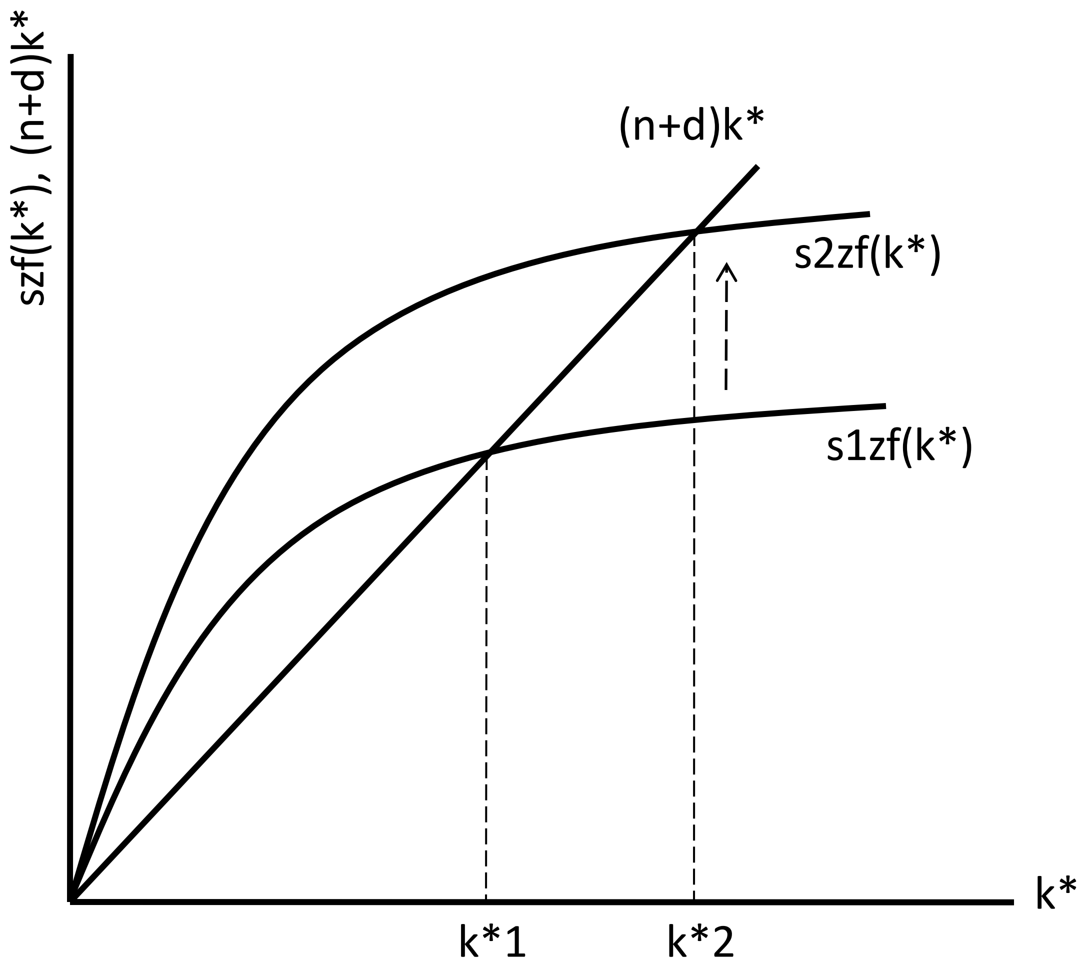


Effect of an increase in s

- Savings rate may increase due to changes in consumers' propensity or government policy.
- Assume a permanent increase in s:
  - szf(k*) rotates upwards.
  - Higher steady-state k* and y* (on a different 'growth path').
  - Higher growth of K and Y is transitional.
  - Convergence to the same steady-state growth rate of 'n'.

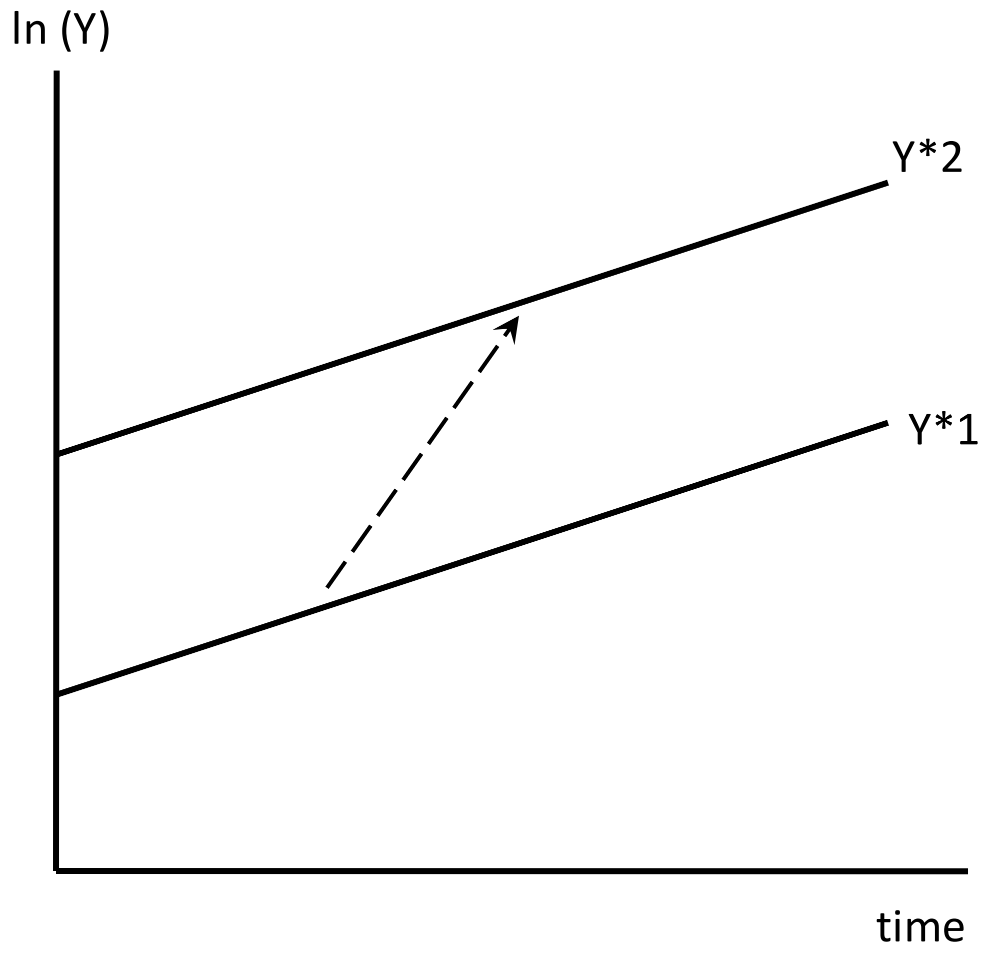
A rise in s raises k*.

- Higher savings rate results in a higher k* and y*.



Temporary gain in growth rate

- K and Y move to new ‘growth paths’.
- Higher growth rates of K and Y are transitional, converging to n.



Steady-state consumption per worker

- A broad measure of aggregate welfare.

$$y^* = zf(k^*)$$

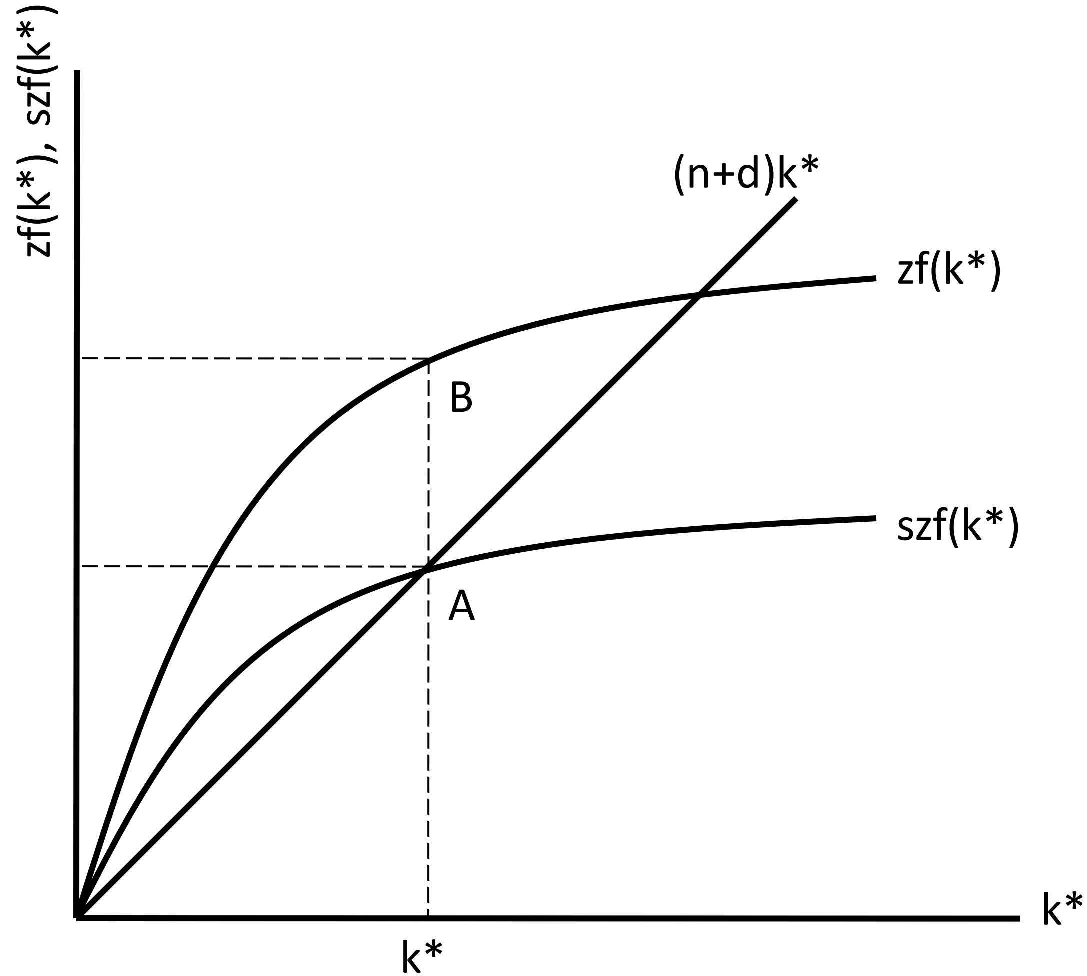
$$\frac{S}{N} = szf(k^*)$$

$$c^* = zf(k^*) - szf(k^*)$$

$$c^* = zf(k^*) - (n + d)k^*$$

$$c^* = (1 - s)zf(k^*)$$

- AB = c*.
- Each savings rate is associated with a value of steady-state consumption per worker (c**).



Maximized c**

$$c^* = zf(k^*) - (n+d)k^*$$

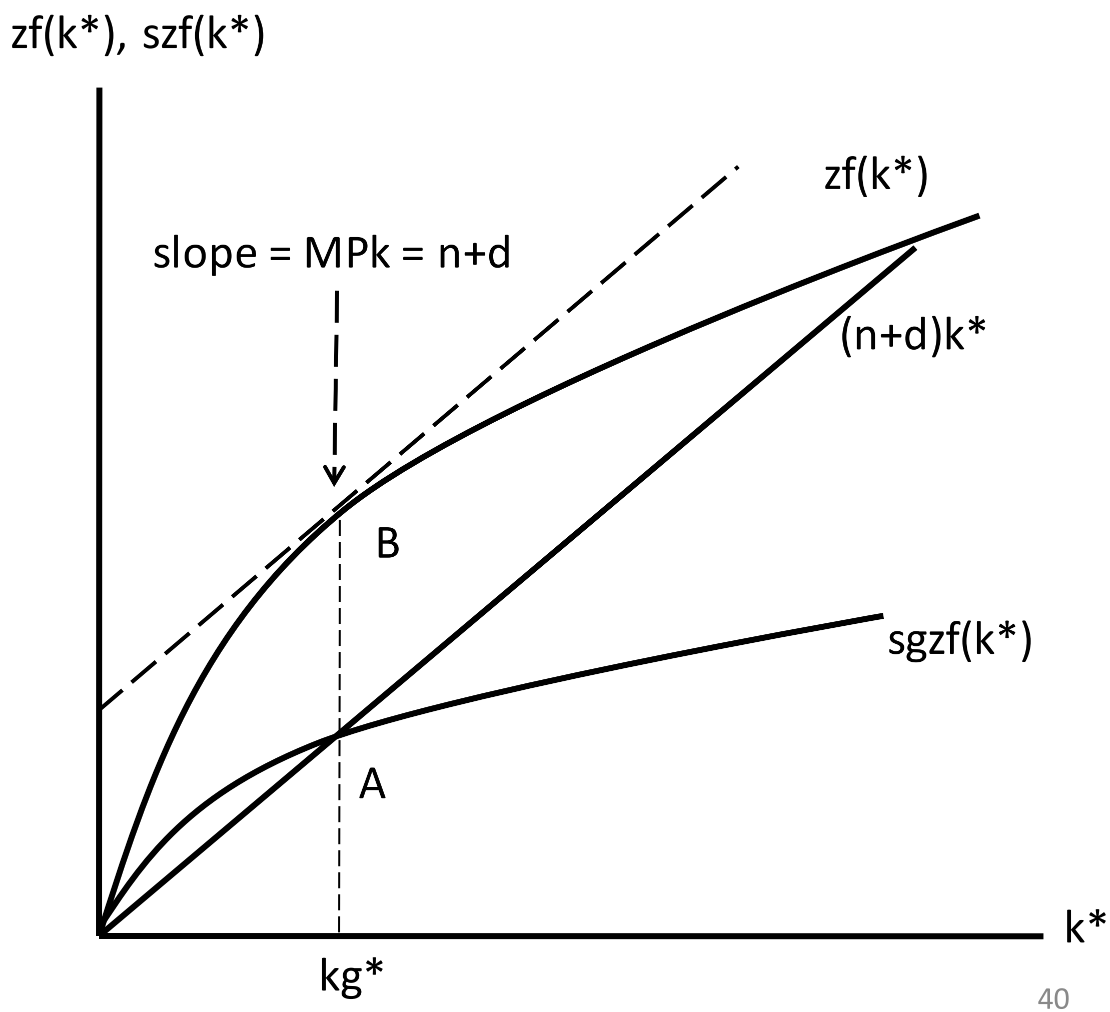
$$Set\ \frac{dc^*}{dk^*} = \frac{d(zf(k^*))}{dk^*} - (n+d) = 0$$

$$\frac{d(zf(k^*))}{dk^*} = n+d$$

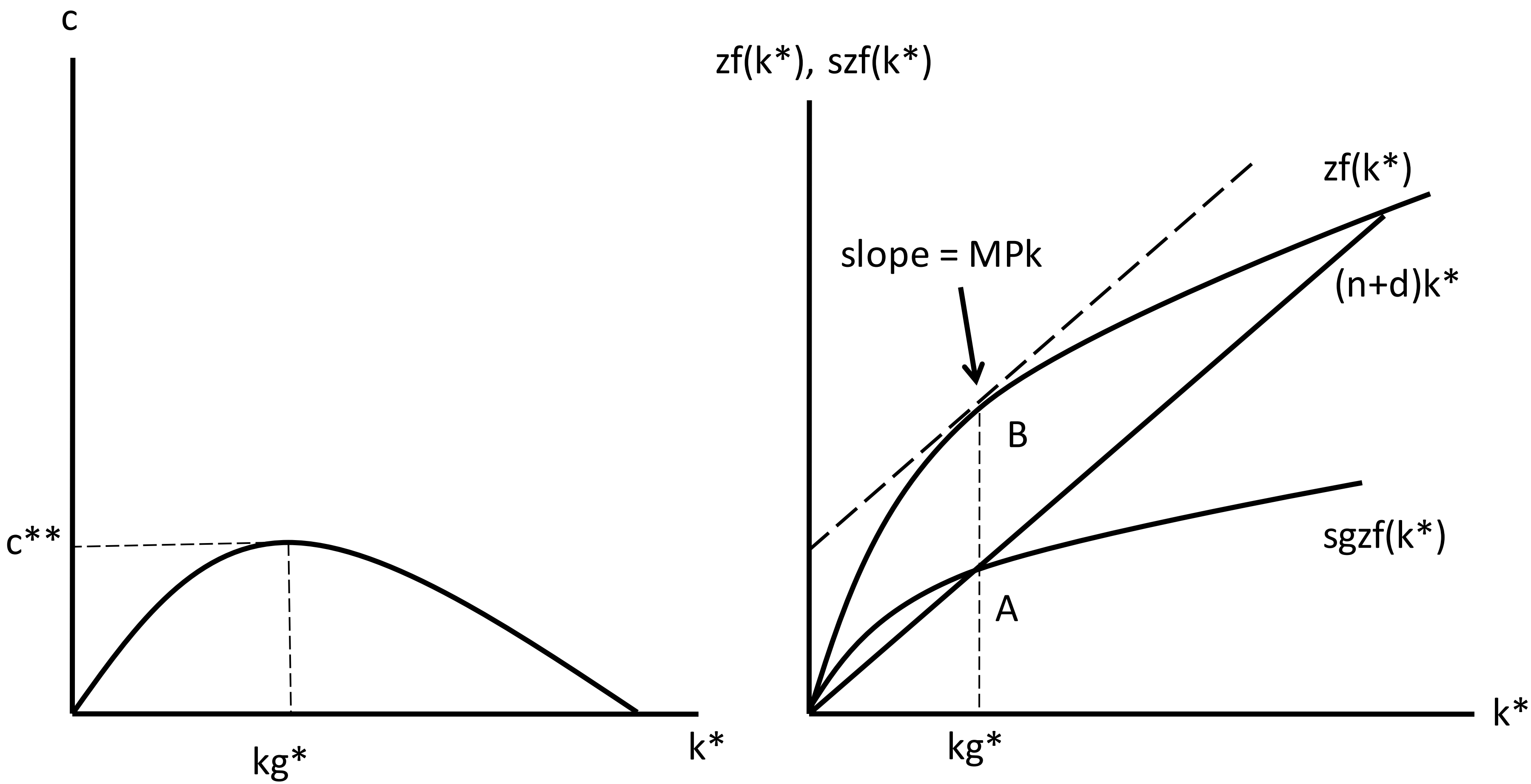
$$MP_k = n+d$$

Golden-rule s_g

- The rate with max. c** is the **'golden-rule' savings rate** (s_g).



Maximized c** and golden-rule s_g



Golden-rule s_g and policy?

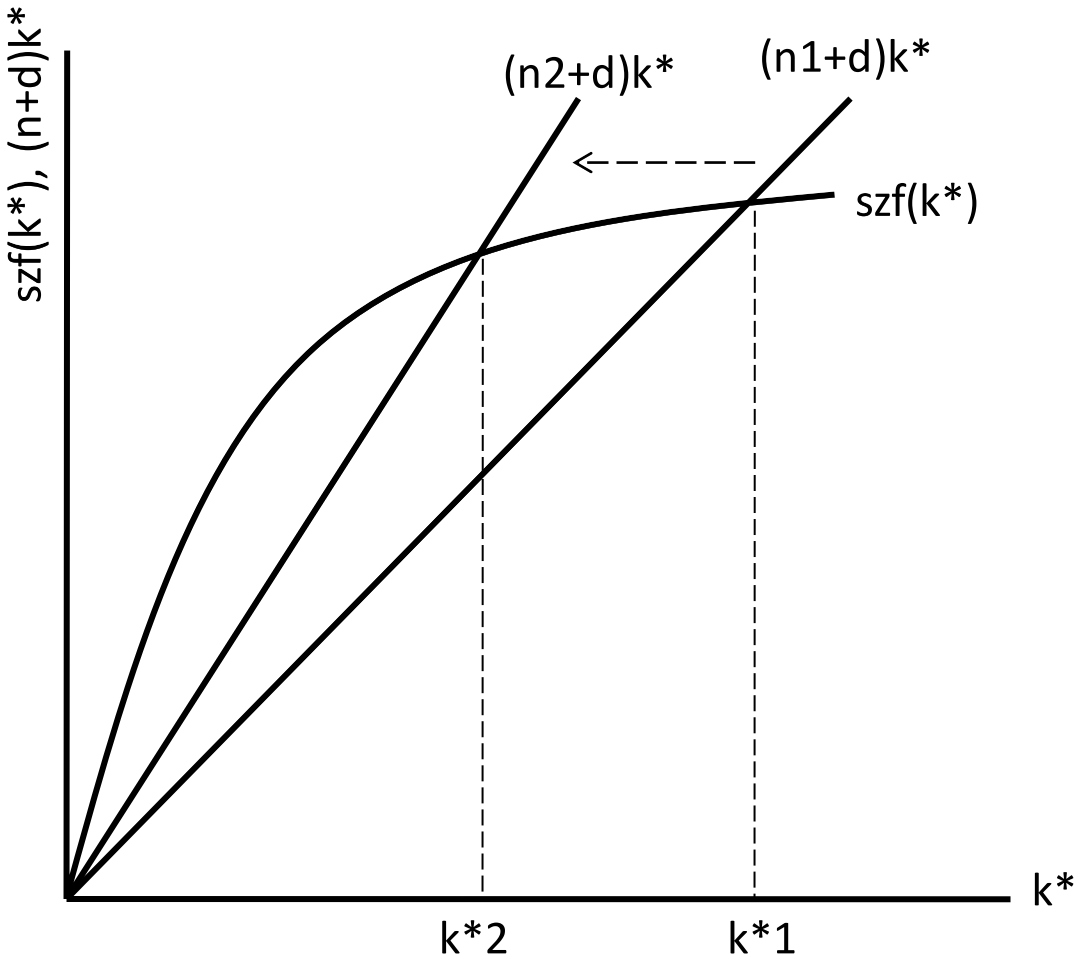
- With the golden-rule s_g, c** is maximized and steady in all periods.
  - Other s results in different c* which is not max.
- Should we achieve s_g if the current $s \neq s_g$?
  - A sacrifice of current c to build up a larger capital stock in the future is needed; is it worth?
  - s depends on individuals' preference and the market for investment.

Effect of an increase in n

- The increase in population growth (n1 to n2) rotates (n+d)k* upwards.
- Decreased steady-state capital (k*) and output per worker (y*).
  - More workers (N*) produce larger output (Y*).
  - But falling productivity of labor results in lower output per worker (y*).
- The steady-state growth rate is higher at n2 for the capital stock (K) and total output (Y).

A higher n with lower k*

- Higher population growth (n) results in lower k* and y*.

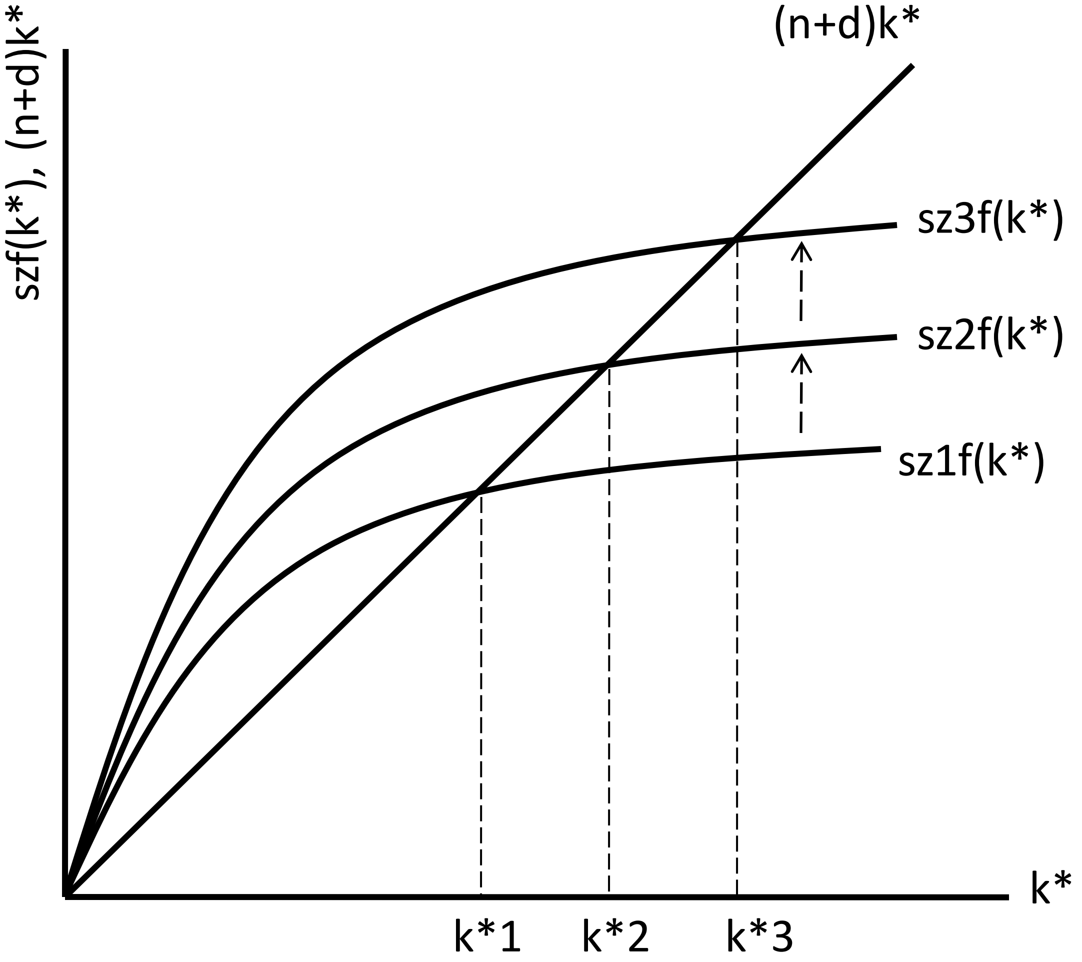


Effect of an increase in z

- A rising s or falling n raises steady-state output per worker (living standards).
  - But the improvement will cease at some point (s cannot exceed 1; n cannot fall indefinitely).
- An increase in total factor productivity (z) raises steady-state capital (k*) and output per worker (y*).
  - Sustained increases in z cause sustained increases in output per worker (y).

Sustained increases in z

- Sustained increases in z cause sustained improvement in y*.



Sources of sustained growth

- Growth from increases in **productive inputs**:
  - Physical capital accumulation, F(K, N).
  - Human capital accumulation, F(K, H).
- Growth from **total factor productivity** (z):
  - Technical progress, inventions, better management and organization.
  - Weather, improved government regulations, falling input prices.

Solow model predictions

- In the long run, higher savings rate results in higher income per worker.
  - **Fact**: positive correlation between GDP per capita and the ratio of investment to GDP.
- An increase in population growth causes a decrease in income per worker.
  - **Fact**: negative correlation between population growth and GDP per capita

Growth accounting

- Growth since the Industrial Revolution has come mainly from rising z.
  - Will this continue indefinitely into the future?
- **Growth accounting**: identify sources of growth.
  - Increases in productive inputs (K, N) or in total factor productivity (z).
  - Calculation based on the production function and the Solow residual.

Thailand’s production function

$$Y = zF(K^{\alpha} N^{1-\alpha})$$

$$where\ 0 < \alpha < 1\ and\ a + (1 - \alpha) = 1$$

- Assume constant returns to scale (CRS).
- α = share of the capital input in GDP.
- 1 - α = share of the labor input in GDP.

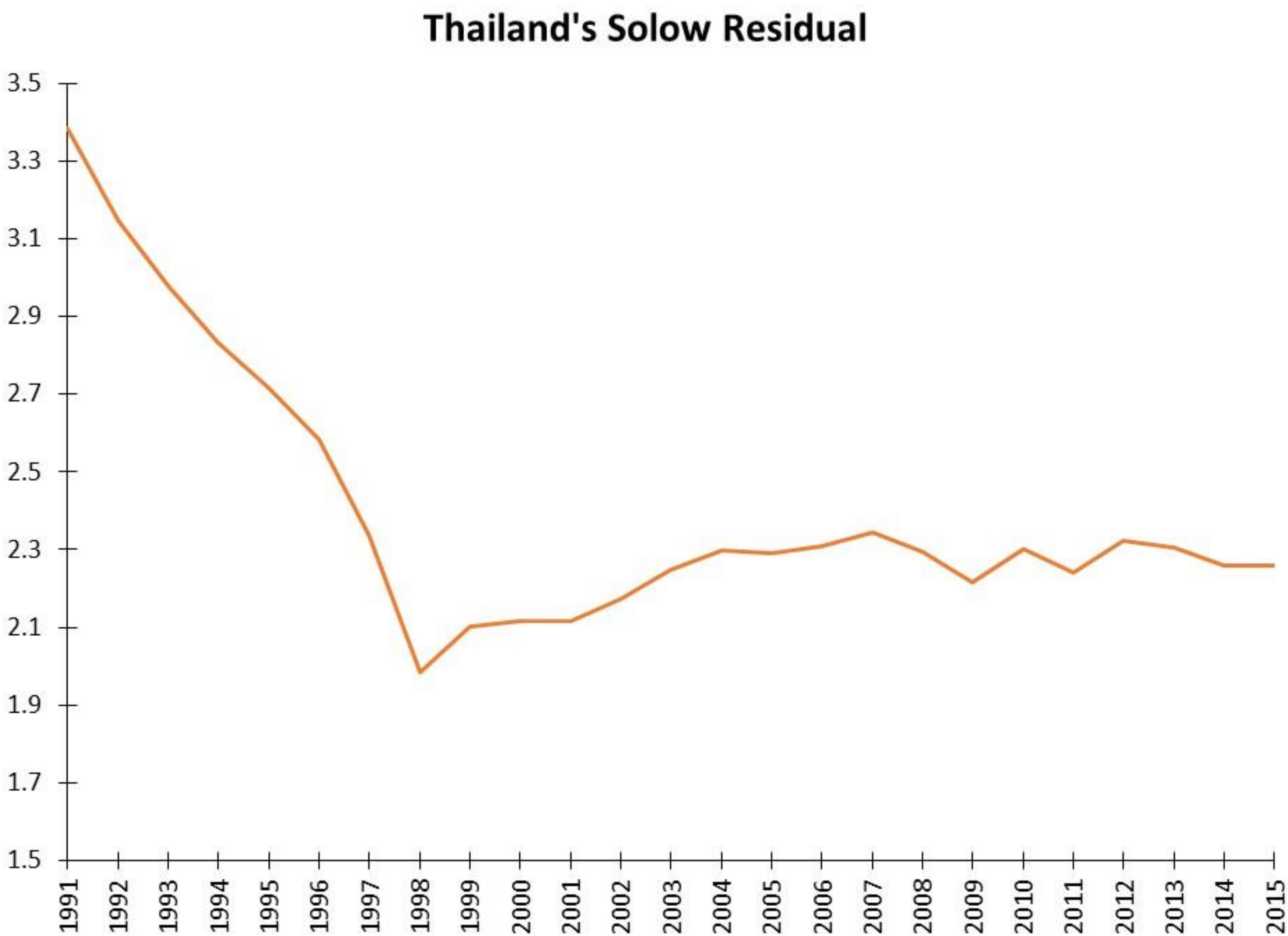
$$Y = zK^{0.6} N^{0.4}$$

The Solow residual for Thailand

$$Y = zK^{0.6}N^{0.4}$$

$$z = \frac{Y}{K^{0.6}N^{0.4}}$$

- Estimated z = Solow residual.
- It measures the level of total factor productivity for Thailand.



Thailand's Solow Residual
3.5
3.3
3.1
2.9
2.7
2.5
2.3
2.1
1.9
1.7
1.5
1991
1992
1993
1994
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2014
2015

Thailand's growth accounting

Year	Output	Capital	Labor	Solow residual
1990-1997	5.4%	18.0%	0.9%	-6.1%
1998-2015	3.8%	5.0%	0.2%	0.7%