PRACTICE PROBLEM SET 1

INTRODUCTORY MATHEMATICAL ECONOMICS

Ch. 2 – Ch. 3 Function and equilibrium analysis

Question 1 (Easy): Simple break-even analysis

1.1 Product A has a fixed cost at 5,000 Baht and variable cost for 7.5 baht per unit and price for 10 Baht per unit.

a) Construct the profit function of the producer of product A.

b) Determine break-even quantity and illustrate by the graph

1.2 Let TC = 2,000 + 20Q and price per unit is 40 Baht per unit. Determine the following:

a) Total revenue

b) Break-even quantity

c) If the company requires the minimum profit of 2,000 Baht, how many products company should produce?

Question 2 (Moderate): Individual vs market demand

Alex, Zander, and Clark are the only three consumers in the market for oatmeal. They have the following individual demand curves for oatmeal. The market supply curve for oatmeal is also provided to you.

Alex: $P = 20 - Q_d$

Zander: $Q_d = 40 - 2P$

Clark: $Q_d = 20 - 4P$

Supply: $Q_s = \frac{3}{2}P - 12$

Answer the following questions:

a. Derive equation for market demand. Go step-by-step slowly. You can use whichever methods that we discuss in the class.
b. What is the equilibrium price and quantity of oatmeal?
c. Under equilibrium, are there any consumers rationed or excluded from the market?

Question 3 (Easy)

In June, KFC lowers the price of fried chicken from 50 Baht per piece to 30 Baht per piece. Then, KFC can sell more fried chicken from 600 pieces to 1,800 pieces and the sale of the drinks increases from 300 to 1,500 cups.

From the above information, answer the following questions

a. Find "price elasticity" of demand for fried chicken with respect to price of fried chicken.
b. Find "cross-price elasticity" of demand for the drinks with respect to price of fried chicken.
c. Holding other things remain constant, if the fried chicken's price drops to be 25 Baht per piece, would the total revenue from selling fried chicken and drinks increase or not? Explain.

Question 4 (Tedious): Given the following information,

Price	**6**	**4**	**2**
Q_d	0	4	8
Q_s	120	80	40

Answer the following questions:

a. Demand equation for each individual consumer
b. Supply equation for each individual producer
c. Suppose there are 10,000 identical consumers in the market, find the market demand equation.

d. Suppose there are 10,000 identical producers in the market, find the market supply equation.
e. Equilibrium price and quantity (also illustrate by graph)
f. If government provides subsidy (to sellers) for each unit sold, 0.1 Baht per unit, find the total amount of money required for the subsidy program.

Question 5 (moderate): The IS-LM model

Consider the following IS-LM model

$C = 48 + 0.8Y^*$

$I = 98 - br^*$

$\frac{M_s}{P} = 250$

$L_d = 52 + 0.3Y^* - 150r^*$

where

C = consumption
Y = income
I = investment
r = interest rate
$\frac{M_s}{P}$ = real money supply
L_d = real money demand

a. Derive IS and LM equation. Graph the equations.
b. Suppose that b = 75, find the equilibrium income (Y*) and equilibrium interest rate (r*)
c. Now suppose instead that b = 0. Discuss the effectiveness of monetary policy.

Question 6 (moderate): Consider the ice cream market in Bangkok. In July, the ice cream market demand and supply curves are given by the following equations where Q is the quantity of ice cream units and P is the price in dollars per unit of ice cream:

Demand: Q = 14000 – 10P
Supply: Q = 2000 + 20P

a) Find the equilibrium price and quantity of ice cream in July.

b) Calculate the price elasticity of demand and supply at the equilibrium price in July. Use the point elasticity formula to compute the values of these two elasticity.

Suppose that the city of Bangkok imposes on producers an excise tax of B 15 per unit of ice cream.

c) Calculate the new equilibrium price and quantity in July for this ice cream market.

Question 7 (moderate)

The demand and supply functions of a two-commodity model are as follows:

$$\begin{aligned} Q_{d1} &= 18 - 3P_1 + P_2 \qquad & Q_{d2} &= 12 + P_1 - 2P_2 \\ Q_{s1} &= -2 + 4P_1 & Q_{s2} &= -2 + 3P_2 \end{aligned}$$

Find the equilibrium of the model.

Question 8 (easy)

Let the national-income model be:

$$\begin{aligned} Y &= C + I_0 + G \\ C &= a + b(Y - T_0) \qquad (a > 0, 0 < b < 1) \\ G &= gY \qquad (0 < g < 1) \end{aligned}$$

(a) Identify the endogenous variables.

(b) Give the economic meaning of the parameter g.

(c) Find the equilibrium national income.

(d) What restriction(s) on the parameters is needed for an economically reasonable solution to exist?

Question 9 (HARD)

A study has shown that there are three groups of iPhone users, namely, *crazy, love-it, and just-live-with-it*. Demand for iPhone of each group can be given by:

crazy: $Q_c = 100 - P$;

Love-it: $P = 50 - Q_L$;

Just-live-with-it: $Q_j = 20 - P$;

where Q_c is the quantity demanded by ***c***razy group, Q_L is quantity demanded by ***L***ove-it group, and Q_j is the quantity demanded by ***j***ust-love-with-it group.

a) Find the domain set of prices that justifies the demand equation for each group of iPhone user. And, rewrite each demand function in a more appropriate way.

b) At what domain set of prices, do all the three types of iPhone users stay active in the market?
c) Find the function for market demand for iPhone. Be precise about what is needed to make your equation justified.

Now, suppose that market supply equation is given by p=4+3w+3/8 Q.

d) Find the equilibrium when w = 1/3 where w is wage rate for each unit of labor hired.
e) How much does each type of consumer consume in the equilibrium?
f) What is the likely effect on market equilibrium when wage drops? State your prediction and develop intuition for your result. (Note: Answer to this question could be made in qualitative sense. You don't need to get into algebraic solution with numbers solved.)

Question 10: (Tricky) *Nonlinear* macroeconomics model

Find the equilibrium Y and C from the following:

$$Y = C + I_0 + G_0, \qquad C = 25 + 6Y^{1/2}, \qquad I_0 = 16, \qquad G_0 = 14.$$

Hint: let $\mathbf{Y^{\frac{1}{2}} = A}$, $\boldsymbol{Y = A^2}$

Question 11 (tedious)

In a 2-good market equilibrium model, the **inverse** demand functions are given by

$$P_1 = Q_1^{\frac{-2}{3}} Q_2^{\frac{1}{3}}, \quad P_2 = Q_1^{\frac{1}{3}} Q_2^{\frac{-2}{3}}.$$

(a) Find the demand functions $Q_1 = D^1(P_1, P_2)$ and $Q_2 = D^2(P_1, P_2)$.

(b) Suppose that the supply functions are

$$Q_1 = a^{-1} P_1, \qquad Q_2 = P_2.$$

Find the equilibrium prices (P_1^*, P_2^*) and quantities (Q_1^*, Q_2^*) as functions of a.

(Hint: Tricky. It might be useful to apply some logarithm rule)

Question 12 (Easy)

Find the equilibrium solution of the following model:

$$Q_d = 3 - P^2, \qquad Q_s = 6P - 4, \qquad Q_s = Q_d.$$

Question 13 (moderate)

Price support V.S. Price guarantee

a. Suppose that the supply and demand for rice are given by the following equations where Q is the quantity in units of rice and P is the price per unit of rice:

Supply of rice: Q = $\frac{1}{2}P - 3$

Demand for rice: Q = 27 - $\frac{1}{3}$P

What are the equilibrium price and quantity in the rice market without government intervention? Illustrate your answer with a well labeled graph.

b. The government tries to raise rice prices using a *price support program*. It sets the price at $54 per unit of rice, and *commits to buy any leftover rice*. Given this program, how much rice do consumers buy? How much rice does the government buy? What is the cost of the program to the government?

c. Now, suppose the government still wants to keep the price of rice at $54 per unit of rice, but instead of implementing a price support program the government decides to enact a price guarantee program that will *subsidize* the rice producers. Under this scenario, how much rice will consumers buy? What is the cost of the program to the government? Illustrate your answer with a well labeled graph.

d. Given the market and the programs described in this problem answer the following questions and provide a rationale for your answer to each question. Which program will the consumers prefer? Which program will the producers prefer? Which program will the government prefer?

e. Could you think of any reason for the government to prefer the price guarantee program?

Question 14 (Moderate)

*This question doesn't require your prior knowledge in calculus.

Consider the milk market in the U.S. There are a number of companies selling milk, so that the market is perfectly competitive. Let's look at Kemps, a firm that produces milk. Assume that Kemp's total cost is given by the following equation:

$$TC = q^2 + 8q + 5$$

where q denotes units of milk.

The market price is $50 per unit of milk.
a) Given the above information, how many gallons of milk will Kemps produce in the short run?

b) Given your answer in (a), find the short run profit for Kemps. Show your work.

Solution to PPS1

Question 1(Easy): Simple break-even analysis

1.1 Product A has a fixed cost at 5,000 Baht and variable cost for 7.5 baht per unit and price for 10 Baht per unit.

a) Construct the profit function of the producer of product A.

$$profit = 2.5Q - 5{,}000$$

b) Determine break-even quantity and illustrate by the graph [Q=2,000]

1.2 Let TC = 2,000 + 20Q and price per unit is 40 Baht per unit. Determine the following:

a) Total revenue [TR=40Q]

b) Break-even quantity [Q = 100]

c) If the company require the minimum profit of 2,000 Baht, how many products company should produce? [Q = 200]

Question 2 (Moderate): Individual vs market demand

Alex, Zander, and Clark are the only three consumers in the market for oatmeal. They have the following individual demand curves for oatmeal. The market supply curve for oatmeal is also provided to you.

Alex: P = 20 - Q_d

Zander: Q_d = 40 - 2P

Clark: Q_d = 20 - 4P

Supply: $Q_s = \frac{3}{2}P - 12$

Answer the following questions:

a. Derive equation for market demand. Go step-by-step slowly. You can use whichever methods that we discuss in the class.

$\boldsymbol{Q = 80 - 7P;} \quad \boldsymbol{0 \leq p < 5}$ **(Z + A + C)**

$= 60 - 3P; \quad 5 \leq p < 20$ (Z + A)

$= 0 \ , P \geq 20$ (all are out!)

b. What is the equilibrium price and quantity of oatmeal?
SOL: Q = 12; P = 16

c. Under equilibrium, are there any consumers rationed or excluded from the market?
SOL: Only Alex and Zander stay in the market, Clark is not.

Question 3 (Easy)

In June, KFC lowers the price of fried chicken from 50 Baht per piece to 30 Baht per piece. Then, KFC can sell more fried chicken from 600 pieces to 1,800 pieces and the sale of the drinks increases from 300 to 1,500 cups.

From the above information, answer the following questions

a. Find "price elasticity" of demand for fried chicken with respect to price of fried chicken. [-5]
b. Find "cross-price elasticity" of demand for the drinks with respect to price of fried chicken. [-10]
c. Holding other things remain constant, if the fried chicken's price drops to be 25 Baht per piece, would the total revenue from selling fried chicken and drinks increase or not? Explain. [Revenue should increase as elasticity should be greater than 5]

Question 4 (Tedious): Given the following information,

Price	**6**	**4**	**2**
Q_d	0	4	8
Q_s	120	80	40

Answer the following questions:

a. Demand equation for each individual consumer [Q = 12 - 2P]
b. Supply equation for each individual producer [Q=20P]
c. Suppose there are 10,000 identical consumers in the market, find the market demand equation. [Q = 10,000(12 - 2P)]
d. Suppose there are 10,000 identical producers in the market, find the market supply equation. [Q = 10,000(20P)]
e. Equilibrium price and quantity (also illustrate by graph)
   [P=12/22; Q = 200,000*(12/22)]
f. If government provides subsidy (to sellers) for each unit sold, 0.1 Baht per unit, find the total amount of money required for the subsidy program.
   [Pd=10/22; Ps=12.2/22; Resource = (20,000)*(12.2/22)]

Question 5 (moderate): The IS-LM model

Consider the following IS-LM model

$C = 48 + 0.8Y^*$

$I = 98 - br^*$

$\frac{M_s}{P} = 250$

$L_d = 52 + 0.3Y^* - 150r^*$

where

C = consumption
Y = income
I = investment
r = interest rate
$\frac{M_s}{P}$ = real money supply
L_d = real money demand

a. Derive IS and LM equation. Graph the equations.
   IS: $Y^* = 730 - 5br^*$

LM: r^* = (-198+0.3y)*(1/150)

b. Suppose that b = 75, find the equilibrium income (Y*) and equilibrium interest rate (r*) [Y* = 700] : $r^* = 0.08$

c. Now suppose instead that b = 0. Discuss the effectiveness of monetary policy. [Policy is NOT effective. In fact, monetary policy does not have any impacts on real GDP.]

Question 6 (moderate): Consider the ice cream market in Bangkok. In July, the ice cream market demand and supply curves are given by the following equations where Q is the quantity of ice cream units and P is the price in dollars per unit of ice cream:

Demand: Q = 14000 – 10P

Supply: Q = 2000 + 20P

a) Find the equilibrium price and quantity of ice cream in July. [p=400, Q = 10000]

b) Calculate the price elasticity of demand and supply at the equilibrium price in July. Use the point elasticity formula to compute the values of these two elasticity.

[demand = -0.4; supply = 0.8]

Suppose that the city of Bangkok imposes on producers an excise tax of B15 per unit of ice cream.

c) Calculate the new equilibrium price and quantity in **July** for this ice cream market. [P_s = 395; P_d =410; Q=9900]

Question 7 (moderate)

The demand and supply functions of a two-commodity model are as follows:

$$Q_{d1} = 18 - 3P_1 + P_2 \qquad Q_{d2} = 12 + P_1 - 2P_2$$
$$Q_{s1} = -2 + 4P_1 \qquad Q_{s2} = -2 + 3P_2$$

Find the equilibrium of the model.

[**P1=3.35 and P2= 3.45**] using precisely decimal P2=3.47

Question 8 (easy)

Let the national-income model be:

$$\begin{aligned} Y &= C + I_0 + G \\ C &= a + b(Y - T_0) \qquad (a > 0, 0 < b < 1) \\ G &= gY \qquad\qquad\qquad (0 < g < 1) \end{aligned}$$

(a) Identify the endogenous variables.

(b) Give the economic meaning of the parameter g.

(c) Find the equilibrium national income.

(d) What restriction(s) on the parameters is needed for an economically reasonable solution to exist?

(a) Exo: I_0 and T_0

Endo: Y, C, G

(b) g = marginal propensity to spend of government

(c) Y = $\frac{a-bT_0+I_0}{1-b-g}$;

(d) a-bT_0 > 0 and (1-b-g) are greater than 0

Question 9 (HARD)

A study has shown that there are three groups of iPhone users, namely, *crazy, love-it, and just-live-with-it.* Demand for iPhone of each group can be given by:

crazy: $Q_c = 100 - P$;

Love-it: $P = 50 - Q_L$;

Just-live-with-it: $Q_j = 20 - P$;

where Qc is the quantity demanded by crazy group, Q_L is quantity demanded by love-it group, and Q_j is the quantity demanded by just-love-with-it group.

a) Find the domain set of prices that justifies the demand equation for each group of iPhone user. And, rewrite each demand function in a more appropriate way.

Crazy: P < 100;
Love-it: P<50;
Just-live-with-it: P < 20;

b) At what domain set of prices, do all the three types of iPhone users stay active in the market?

[Sol: P < 20]

c) Find the function for market demand for iPhone. Be precise about what is needed to make your equation justified.

Q = 0 ; P ≥ 100
100 -P ; 50 ≤ P < 100
150 - 2P ; 20 ≤ P < 50
170 - 3P ; 0 ≤ P < 100

Suppose market supply equation is given by: p=4+3w+3/8 Q.

d) Find the equilibrium when w = 1/3 where w is wage rate for each unit of labor hired.

Q = 80 units P = $35.

e) How much does each type of consumer consume in the equilibrium?

Crazy: P < 100; Q = 65 units
Love-it: P<50; Q = 15 units
Just-live-with-it: P < 20; excluded

f) What is the likely effect on market equilibrium when wage drops? State your prediction and develop intuition for your result. (Note: Answer to this question could be made in qualitative sense. You don't need to get into algebraic solution with numbers solved.)

A drop in wage would increase the supply. Thus, it's likely that under the new equilibrium, more output will be produced.

Question 10: Nonlinear macroeconomics model

Find the equilibrium Y and C from the following:

$$Y = C + I_0 + G_0, \qquad C = 25 + 6Y^{1/2}, \qquad I_0 = 16, \qquad G_0 = 14.$$

[Y = 121; C = 91]. **Hint: let $Y^{\frac{1}{2}} = A$, $Y = A^2$**

Question 11

In a 2-good market equilibrium model, the **inverse** demand functions are given by

$$P_1 = Q_1^{\frac{-2}{3}} Q_2^{\frac{1}{3}}, \quad P_2 = Q_1^{\frac{1}{3}} Q_2^{\frac{-2}{3}}.$$

(a) Find the demand functions $Q_1 = D^1(P_1, P_2)$ and $Q_2 = D^2(P_1, P_2)$.

(b) Suppose that the supply functions are

$$Q_1 = a^{-1} P_1, \qquad Q_2 = P_2.$$

Find the equilibrium prices (P_1^*, P_2^*) and quantities (Q_1^*, Q_2^*) as functions of a.

a) $\boldsymbol{Q_1 = P_1^{-2} * P_2^{-1};\ Q_2 = P_1^{-1} P_2^{-2}}$

b) $\boldsymbol{P_1 = a^{3/8};\quad P_2 = a^{-1/8} \qquad Q_1^* = a^{\frac{-5}{8}} \quad Q_2^* = a^{\frac{-1}{8}}}$

Question 12

Find the equilibrium solution of the following model:

$$Q_d = 3 - P^2, \qquad Q_s = 6P - 4, \qquad Q_s = Q_d.$$

[Sol: Q = 2, P =1]

Question 13 (moderate)

Price support V.S. Price guarantee

a. Suppose that the supply and demand for rice are given by the following equations where Q is the quantity in units of rice and P is the price per unit of rice:

Supply of rice: $Q = \frac{1}{2}P - 3$

Demand for rice: $Q = 27 - \frac{1}{3}P$

What are the equilibrium price and quantity in the rice market without government intervention? Illustrate your answer with a well labeled graph.

Set the two equations equal to each other, and you get Q = 15 units of rice, P = $36 per unit of rice.

b. The government tries to raise rice prices using a *price support program*. It sets the price at $54 per unit of rice, and *commits to buy any leftover rice*. Given this program, how much rice do consumers buy? How much rice does the government buy? What is the cost of the program to the government?

Plug P = 54 into the demand curve and the supply curve respectively, you can get the quantity demanded = 9 units of rice and quantity supplied = 24 units of rice. The excess supply is thus 24 – 9 = 15 units of rice, which is purchased by the government under the price support program. Therefore, consumers buy 9 units of rice, and the government buys 15 units of rice, at the price of $54 per unit of rice. The cost to the government is 15*54 = $810.

c. Now, suppose the government still wants to keep the price of rice at $54 per unit of rice, but instead of implementing a price support program the government decides to enact a price guarantee program that will *subsidize* the rice producers. Under this scenario, how much rice will consumers buy? What is the cost of the program to the government? Illustrate your answer with a well labeled graph.

Under a price guarantee program, the producers receive a price of $54 per unit of rice and produce 24 units of rice. The consumers will buy at a discounted price, so that they are willing to buy all 24 units of rice. The price differential is the government subsidy per gallon of rice. For the consumers to be willing to buy 24 units of rice, the price they pay must be – 3*24 + 81 = $9 per unit of rice. Therefore, for each unit of rice the government must pay a subsidy of 54 – 9 = $45 per unit of rice. Since altogether 24 units of rice are produced and sold, the total cost to the government is 45*24 = $1,080.

d. Given the market and the programs described in this problem answer the following questions and provide a rationale for your answer to each question. Which program will the consumers prefer? Which program will the producers prefer? Which program will the government prefer?

Under the price support program, consumers buy 9 units of rice at the price of $54 per unit. Under the price guarantee program, they could buy a larger quantity (24 units) at a lower price ($9 per unit). So consumers will prefer the price guarantee program. The producers get to sell 24 units of rice at the price of $54 per unit under both programs, so they are indifferent between the programs. Finally, the government incurs a higher cost under the price guarantee program, so it prefers the price support program.

e. Could you think of any reason for the government to prefer the price guarantee program?

Under the price support program, the government buys a large quantity of rice and thus has to pay ***a storage cost***. So there is a "hidden" cost associated with the price support program.

Question 14 (Moderate)

*This questions doesn't require your prior knowledge in calculus.

Consider the milk market in the U.S. There are a number of companies selling milk, so that the market is perfectly competitive. Let's look at Kemps, a firm that produces milk. Assume that Kemp's total cost is given by the following equation:

$$TC = q^2 + 8q + 5$$

where q denotes units of milk.

The market price is $50 per unit of milk.

a) Given the above information, how many gallons of rice will Kemps produce in the short run?

$$\pi = \mathbf{50q - q^2 - 8q - 5 = -q^2 + 42q - 5} \Rightarrow \mathbf{q^* = -(42/2(-1))} = \mathbf{21\ units}$$

b) Given your answer in (a), find the short run profit for Kemps. Show your work.

$$profit = -(21)^2 + 42(21) - 5 = 436$$