homework 6304640771

1) $P = a - bQ$; $Q = q_1 + q_2 + q_3$

$C_1 = C_2 = C_3 = C$

What is equilibrium price ? : P^*

What are firms' profit ? $\pi_1 = \pi_2 = \pi_3 = ?$

$$\pi_i = (P - C)q_i = [a - b(q_1 + q_2 + q_3) - C]q_i$$

max π_i : $\frac{\partial \pi_i}{\partial q_i} = 0 \rightarrow \frac{\partial \pi_i}{\partial q_1} = (a - b(q_1 + q_2 + q_3) - c) + q_1(-b) = 0$

$$\Rightarrow q_1 = a - c - bq_1 - bq_2 - bq_3 - bq_1$$

$$0 = a - c - bq_2 - bq_3 - 2bq_1$$

$$2bq_1 = a - c - bq_2 - bq_3$$

$\boxed{q_1^* = \frac{a - c - bq_2 - bq_3}{2b}}$ —(1) BR of firm 1

$\boxed{q_2^* = \frac{a - c - bq_1 - bq_3}{2b}}$ —(2) $\boxed{q_3^* = \frac{a - c - bq_1 - bq_2}{2b}}$ —(3)

sub (2) in (1) ; $q_1^* = \frac{a - c - bq_2 - b\left(\frac{a - c - bq_1 - bq_3}{2b}\right)}{2b}$

$q_1^* = \frac{a - bq_3}{3b}$

sub (2) in (1) ; $q_2^* = \frac{a - bq_3}{3b}$

sub q_1^* and q_2^* in (3) ; $q_3^* = \frac{a - c - b\left(\frac{a - bq_3}{3b}\right) - b\left(\frac{a - bq_3}{3b}\right)}{2b}$

$= \frac{a + 2bq_3}{6b}$

$6bq_3 = a + 2bq_3$

$4bq_3 = a$

$\boxed{q_3^* = \frac{a}{4b}}$

sub q_3^* ; $q_1^* = \frac{a - b\left(\frac{a}{4b}\right)}{3b}$

$\boxed{q_1^* = \frac{a}{4b}}$

$\boxed{q_2^* = \frac{a}{4b}}$

$P = a - bQ = a - 3\cancel{b}\left(\frac{a}{4\cancel{b}}\right)$

$= a - \frac{3}{4}a$ $\therefore P = 0.25a$

$\pi_1 = (P - C)q_i$

$\pi_1 = (0.25a - c)\frac{a}{4b}$ $\therefore \pi_1 = \frac{a^2}{16b} - c$ $\therefore \pi_2 = \frac{a^2}{16b} - c$ $\therefore \pi_3 = \frac{a^2}{16b} - c$

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2) If there are N firms

$q_i^* = f(N)$, $P = f(N)$, $\pi_i = f(N)$

assume $q_1 + q_2 + .. + q_n = A$

$P = a - b(q_1 + q_2 + .. + q_n)$

$P = a - bq_1 - bq_2 - .. - bq_n$

$\boxed{\pi_i = (P-C)q_i}$ $\quad \pi_1 = (a - bq_1 - bq_2 - .. - bq_n)q_1 - C_1$

$$\frac{\partial \pi_1}{\partial q_1} = 0 \Rightarrow q_1 = \frac{a}{2b} - 0.5(q_2 + q_3 + ... + q_n)$$

$$\vdots$$

$$q_n = \frac{a}{2b} - 0.5(q_2 + q_3 + ... + q_{n-1})$$

$$\boxed{\therefore q_1^* = q_2^* = ... = q_n^* = \frac{a}{b} - A} \quad -(1)$$

$A = q_1 + q_2 + .. + q_n$

$A = n\left(\frac{a}{b} - A\right)$

$A = \frac{na}{b} - nA$

$A + nA = n\left(\frac{a}{b}\right)$

$A(n+1) = n\left(\frac{a}{b}\right) \rightarrow A = \frac{na}{(n+1)b}$

sub A into (1) ; $q_1 = \frac{a}{(n+1)b}$

$\therefore q_i = \frac{a}{(n+1)b}$

$P = a - bA$

$= a - b\left(\frac{na}{(n+1)b}\right)$

$= a - \left(\frac{n}{n+1}\right)a$

$= \frac{na + a - na}{n+1}$

$\therefore p = \frac{a}{n+1}$

$\pi_i = P \cdot q_i + C_i$

$= \frac{a}{n+1} \cdot \frac{a}{(n+1)b} - C_i$

$\therefore \pi_i = \frac{a^2}{(n+1)^2 b} - C_i$

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3) From question 2, what happen if $N \to \infty$

—— " —— $N = 1$

In general,

- If $N \to \infty$, it means that the market output (Q) goes to a competitive level and the price (P^*) converges to marginal cost. The market is perfectly competitive market.
- In cournot model, there must be 2 or more firms competing in the market. So, if $n=1$, firm will becomes a monopolist.

if $n \to \alpha$;

- $q_i = \frac{a}{(n+1)b}$ will be nearly to 0 and each firm will sell at q nearly 0 unit
- $A = \frac{na}{(n+1)b}$ will be nearly to 0, q of every firms combined will be nearly α units
- $p = \frac{a}{n+1}$ will be nearly to 0, supply will increase as p decreases to nearly 0
- $\pi_i = \frac{a^2}{(n+1)^2 b} - c_i \rightarrow$ each firms will lose their profit.

If $n = 1$

- $q_i = \frac{a}{(n+1)b} = \frac{a}{2b} \rightarrow$ since $q = \frac{a}{2b} < q = \frac{na}{(n-1)b}$, monopoly will sell less.
- $A = \frac{na}{(n+1)b} = q$, it means than firm will become a monopolist.
- $p = \frac{a}{n+1} = \frac{a}{2} \rightarrow$ since $P_m = \frac{a}{2} > P_c = \frac{a}{n+1}$, monopoly will set higher price.
- $\pi_i = \frac{a^2}{(n+1)^2 b} - c_i = \frac{a^2}{4b} - c_i \rightarrow$ since $\pi_m > \pi_c$, monopolist will earn higher profit.