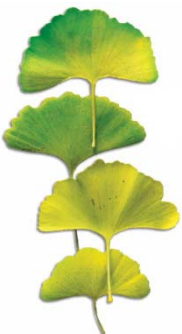


Chapter 8

Production

Chapter Outline

- The Input-Output Relationship of The Production Function
- Production In The Short Run
- Total, Marginal, and Average Products
- Production In The Long Run
- Returns To Scale



The Production Function

- ***Production function:*** the relationship that describes how inputs like capital and labor are transformed into output.
- Mathematically, $Q = F(K, L)$
 - K = Capital
 - L = Labor

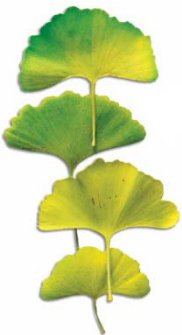
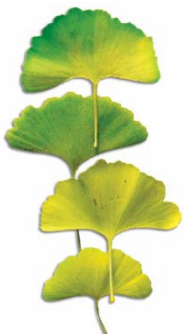
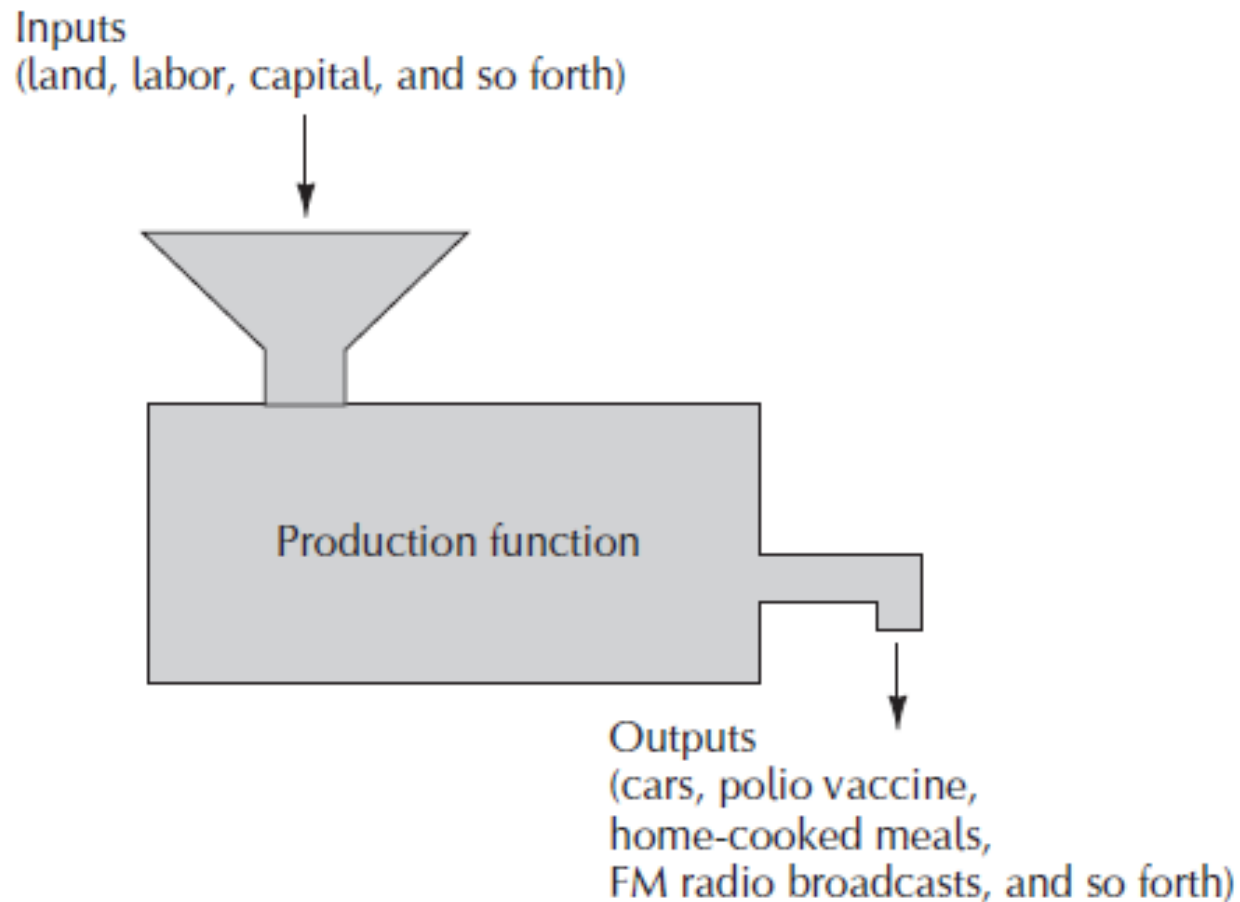
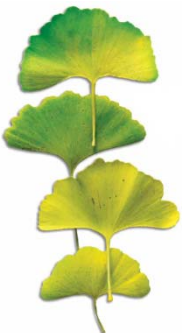


Figure 8.1: The Production Function



Fixed and Variable Inputs

- ***Long run:*** the shortest period of time required to alter the amounts of all inputs used in a production process.
- ***Short run:*** the longest period of time during which at least one of the inputs used in a production process cannot be varied.
- ***Variable input:*** an input that can be varied in the short run.
- ***Fixed input:*** an input that cannot vary in the short run.



Production in the Short Run

- Three properties:
 1. It passes through the origin
 2. Initially the addition of variable inputs augments output at an increasing rate
 3. beyond some point additional units of the variable input give rise to smaller and smaller increments in output.

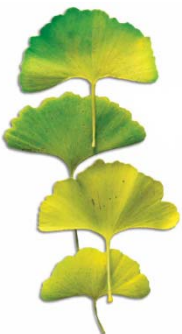


Figure 8.2: A Specific Short-Run Production Function

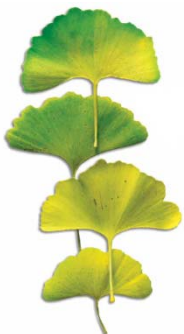
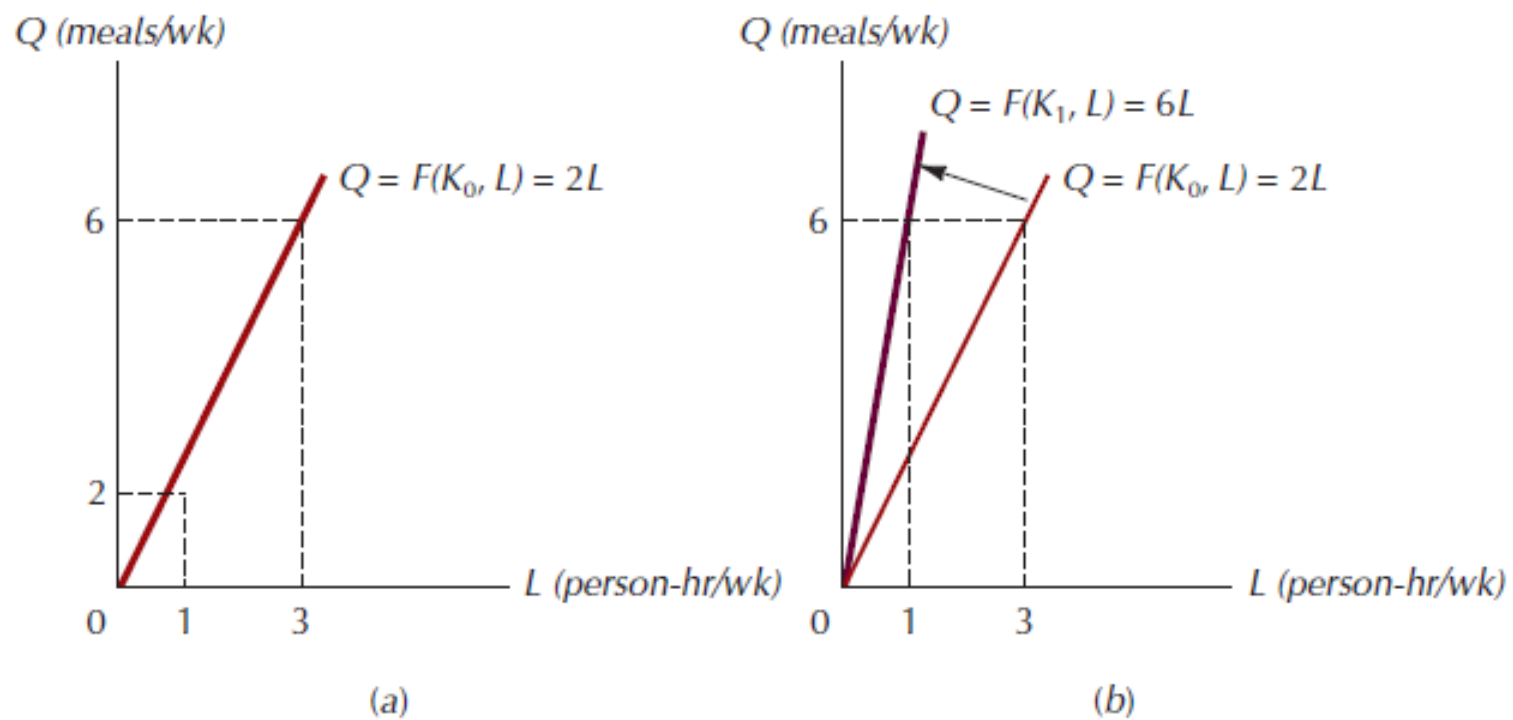
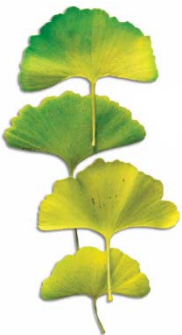
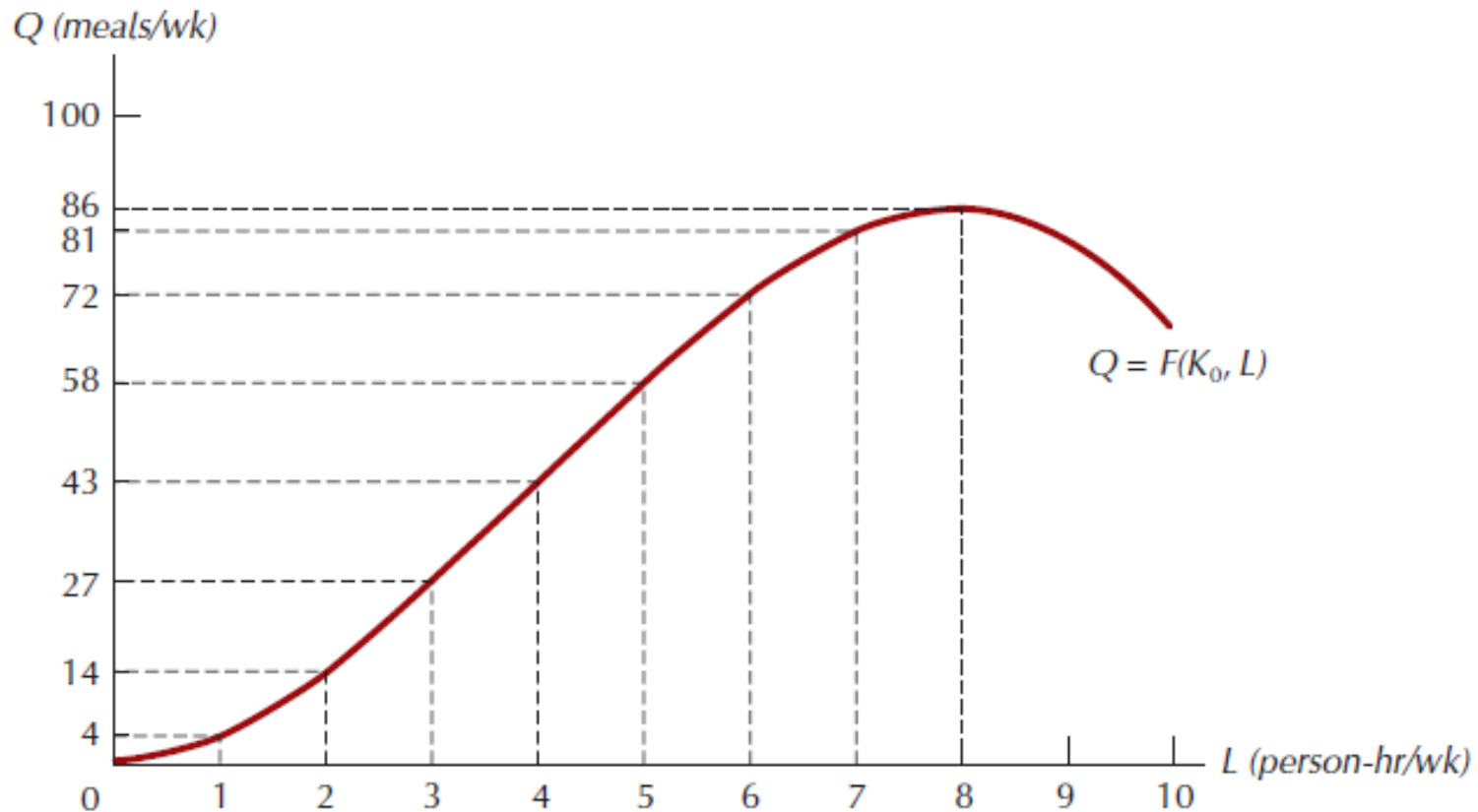


Figure 8.3: Another Short-Run Production Function



An Important Characteristic of Short-Run Production Functions

- ***Law of diminishing returns:*** if other inputs are fixed, the increase in output from an increase in the variable input must eventually decline.

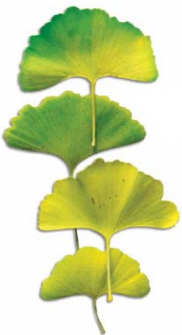
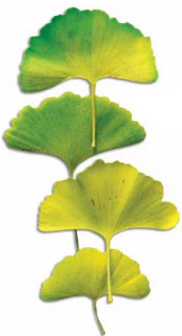
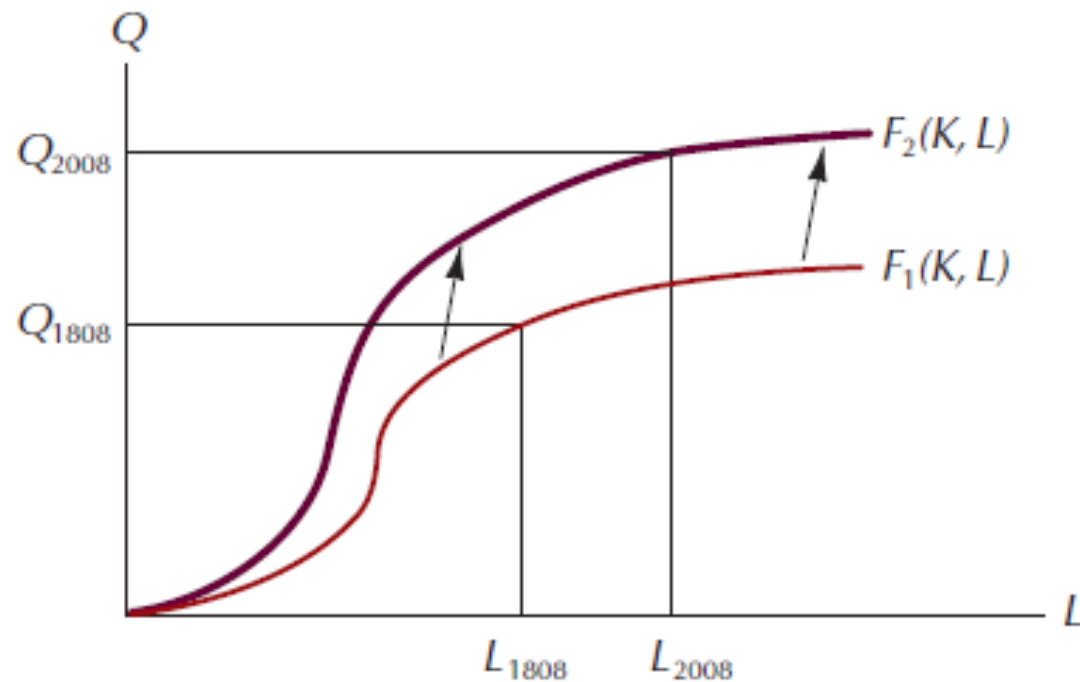


Figure 8.4: The Effect of Technological Progress in Food Production



Short-Run Production Function Components

- ***Total product curve:*** a curve showing the amount of output as a function of the amount of variable input.
- ***Marginal product:*** change in total product due to a 1-unit change in the variable input.
- ***Average product:*** total output divided by the quantity of the variable input.

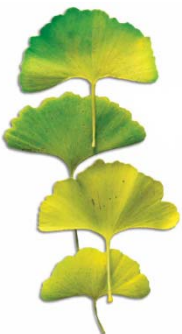
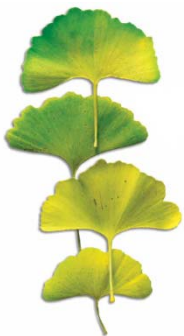
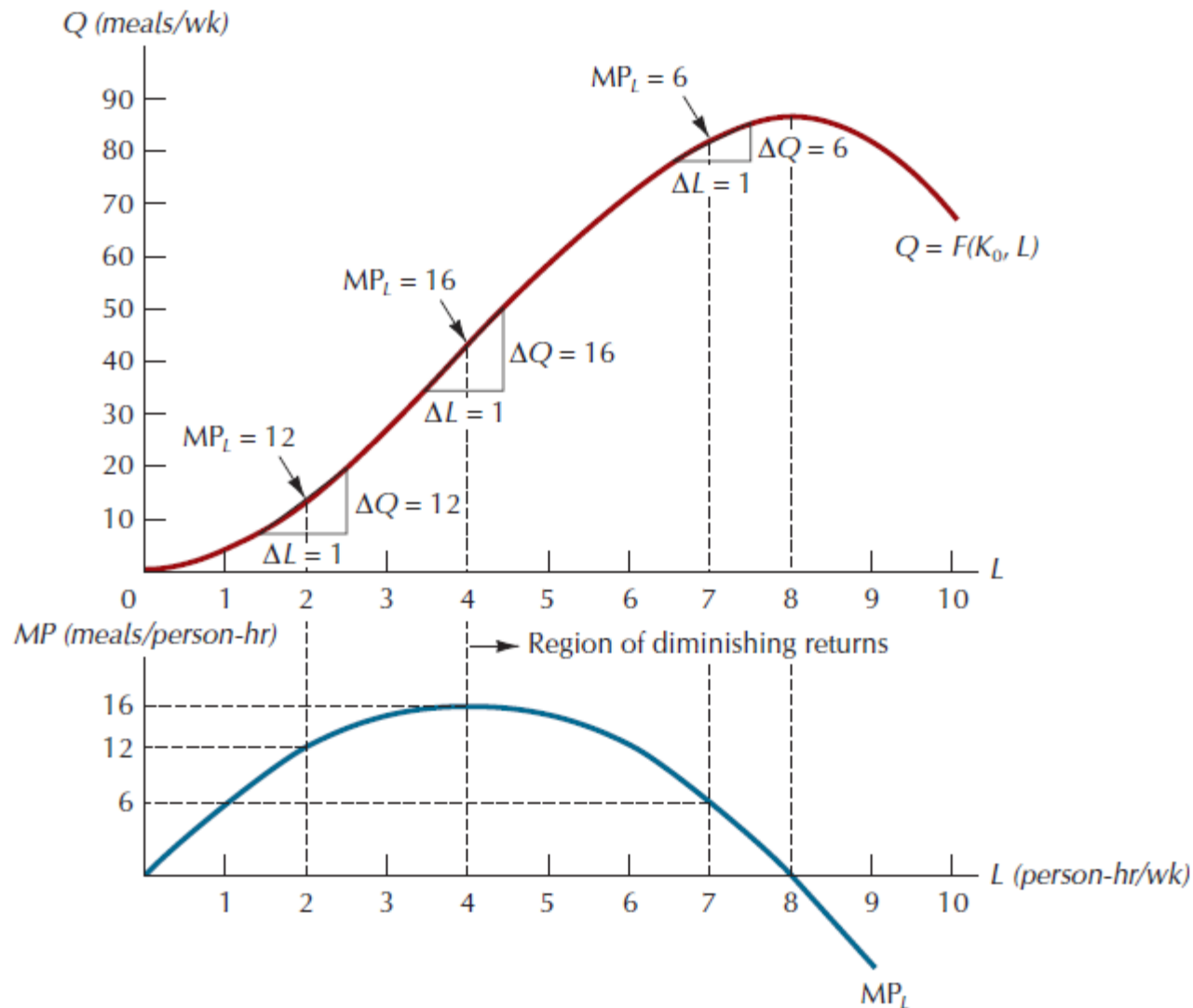


Figure 8.5: The Marginal Product of a Variable Input



Relationships Among Total, Marginal and Average Product Curves

- *When the marginal product curve lies above the average product curve, the average product curve must be rising*
- *When the marginal product curve lies below the average product curve, the average product curve must be falling.*
- *The two curves intersect at the maximum value of the average product curve.*

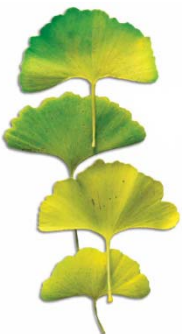
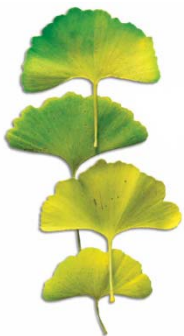
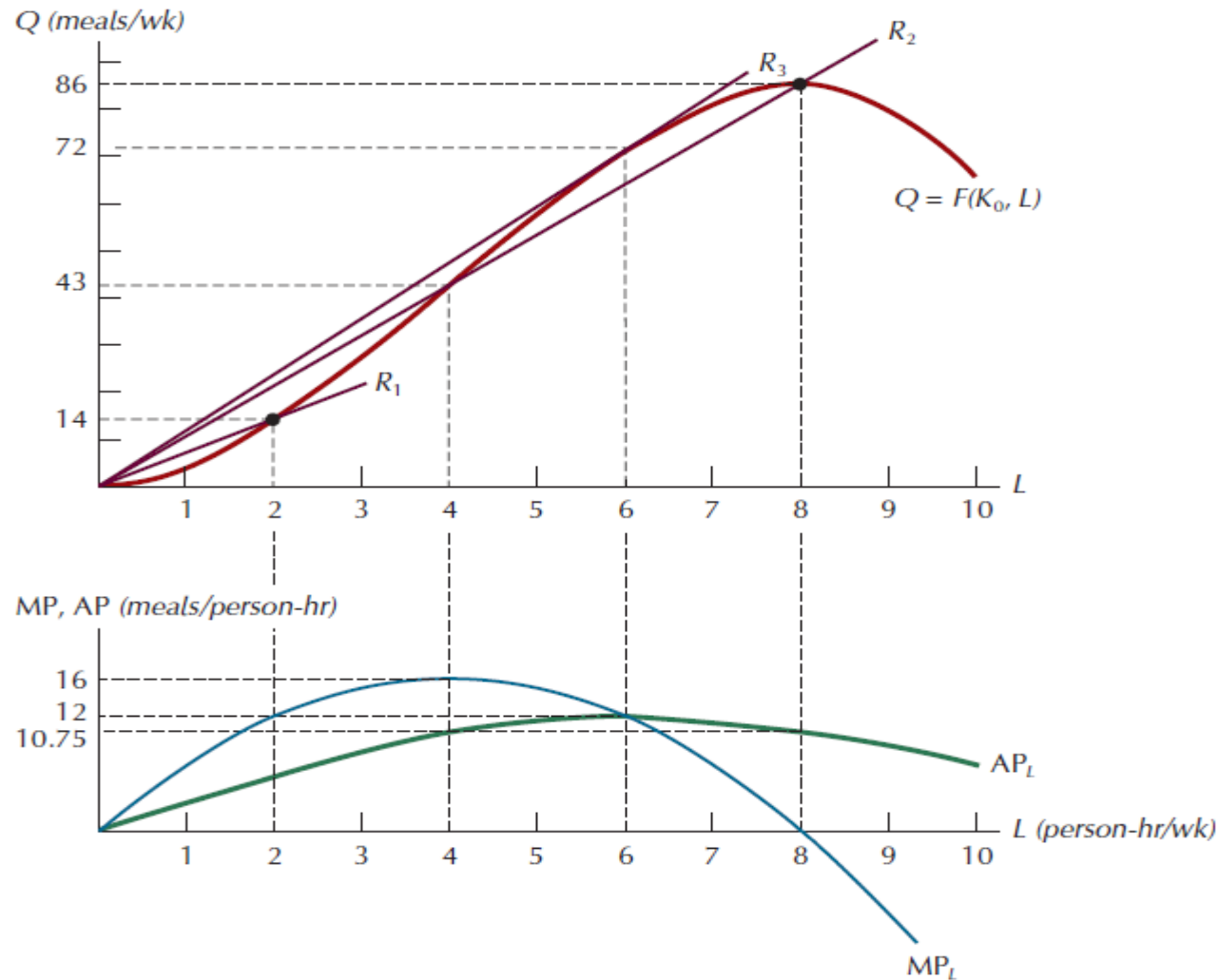
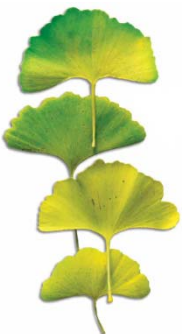


Figure 8.6: Total, Marginal, and Average Product Curves



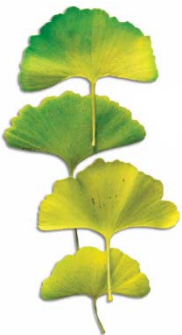
The Practical Significance of the Average Marginal Distinction

- Suppose you own a fishing fleet consisting of a given number of boats, and can send your boats in whatever numbers you wish to either of two ends of an extremely wide lake, east or west. Under your current allocation of boats, the ones fishing at the east end return daily with 100 pounds of fish each, while those in the west return daily with 120 pounds each. The fish populations at each end of the lake are completely independent, and your current yields can be sustained indefinitely.
- ***Should you alter your current allocation of boats?***



The Practical Significance Of The Average Marginal Distinction

- The general rule for allocating an input efficiently in such cases is to allocate the next unit of the input to the production activity where its marginal product is highest.



Production In The Long Run

- ***Isoquant***: the set of all input combinations that yield a given level of output.
- ***Marginal rate of technical substitution (MRTS)***: the rate at which one input can be exchanged for another without altering the total level of output.

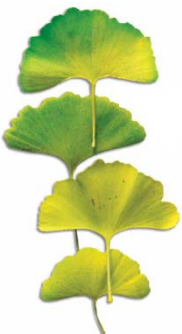


Figure 8.7: Part of an Isoquant Map for the Production Function

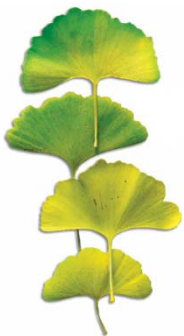
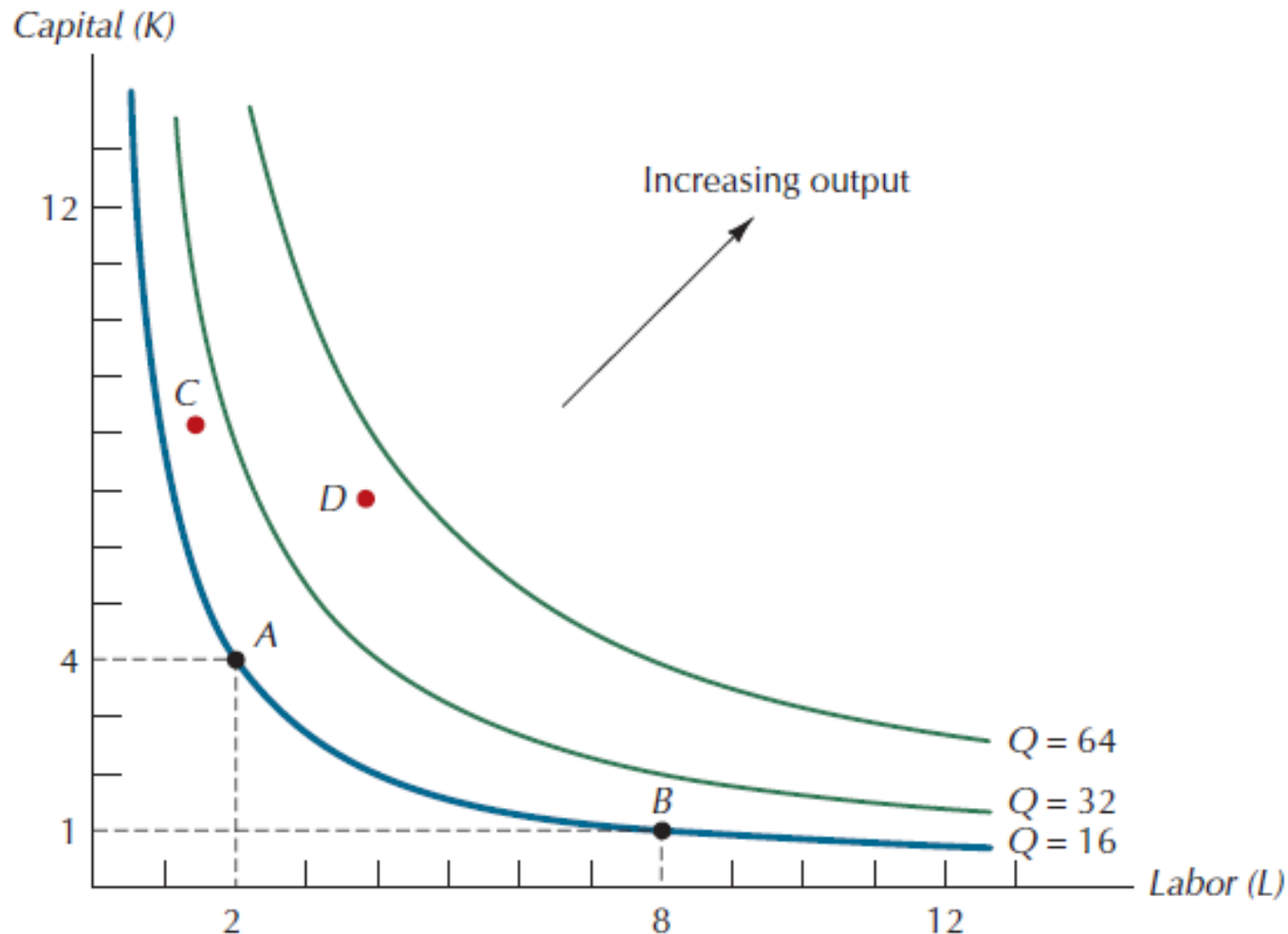


Figure 8.8: The Marginal Rate of Technical Substitution

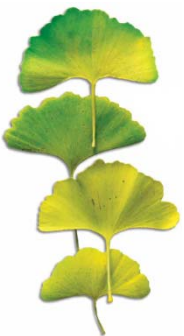
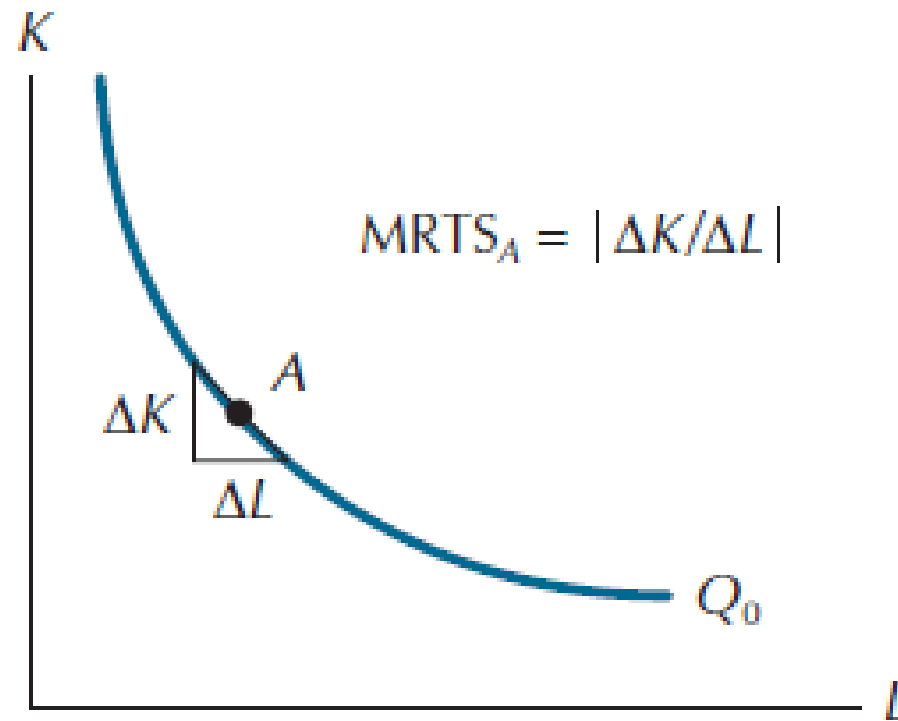
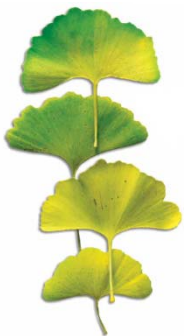
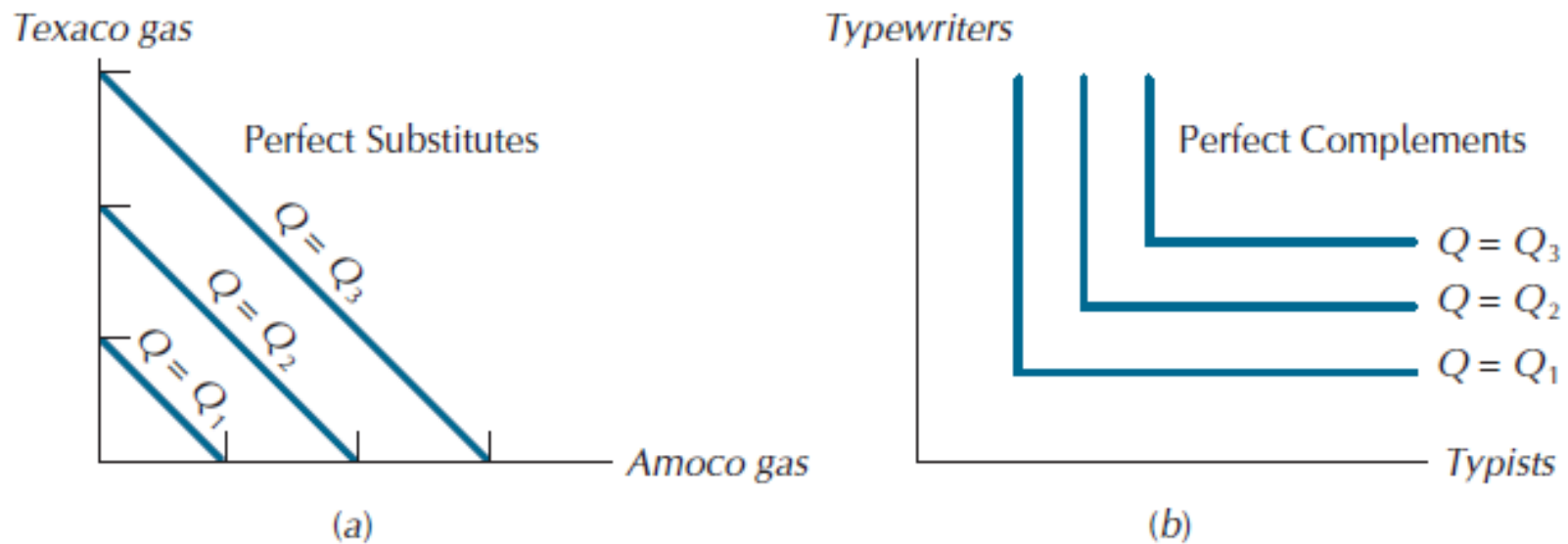


Figure 8.9: Isoquant Maps for Perfect Substitutes and Perfect Complements



Returns To Scale

- ***Increasing returns to scale:*** the property of a production process whereby a proportional increase in every input yields a more than proportional increase in output.
- ***Constant returns to scale:*** the property of a production process whereby a proportional increase in every input yields an equal proportional increase in output.
- ***Decreasing returns to scale:*** the property of a production process whereby a proportional increase in every input yields a less than proportional increase in output.

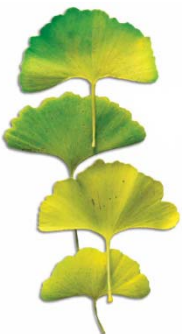


Figure 8.10: Returns to Scale Shown on the Isoquant Map

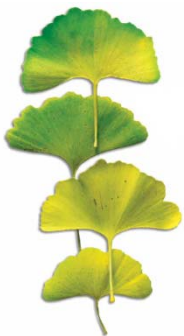
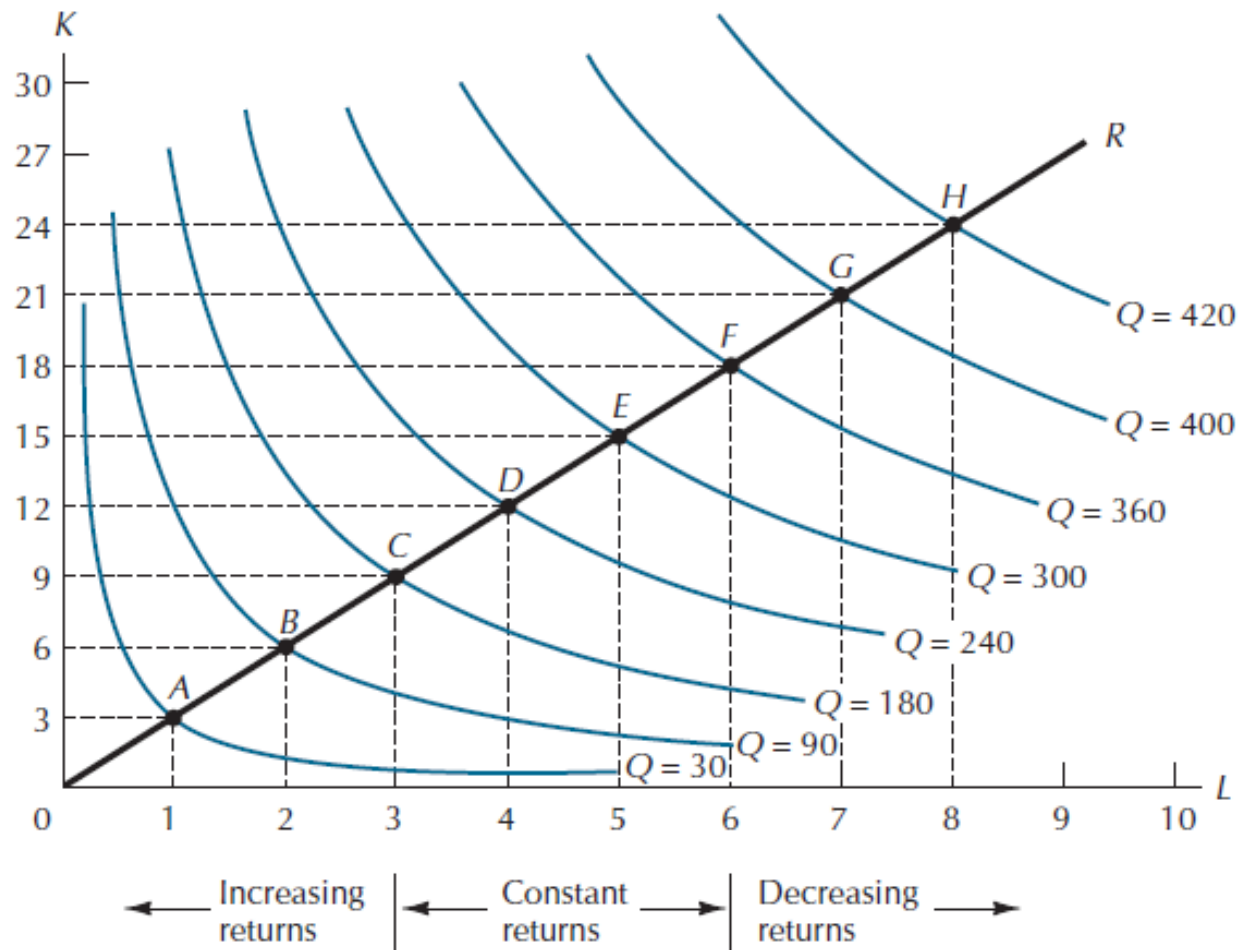


Figure 8.A.1: Effectiveness vs. Use: Lobs and Passing Shots

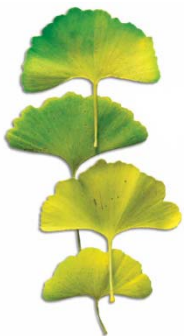
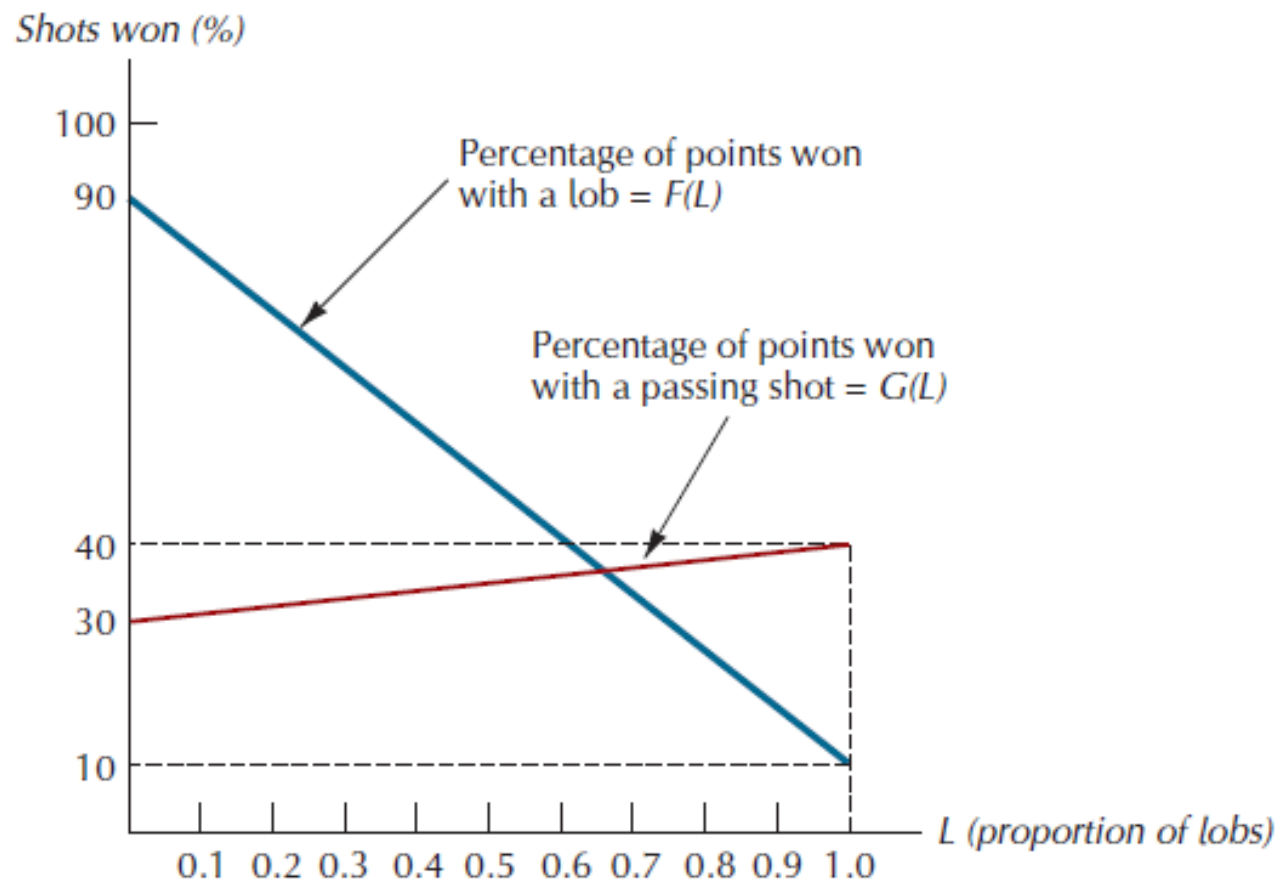


Figure 8A2: The Optimal Proportion of Lobs

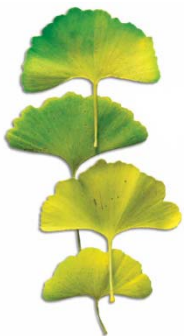
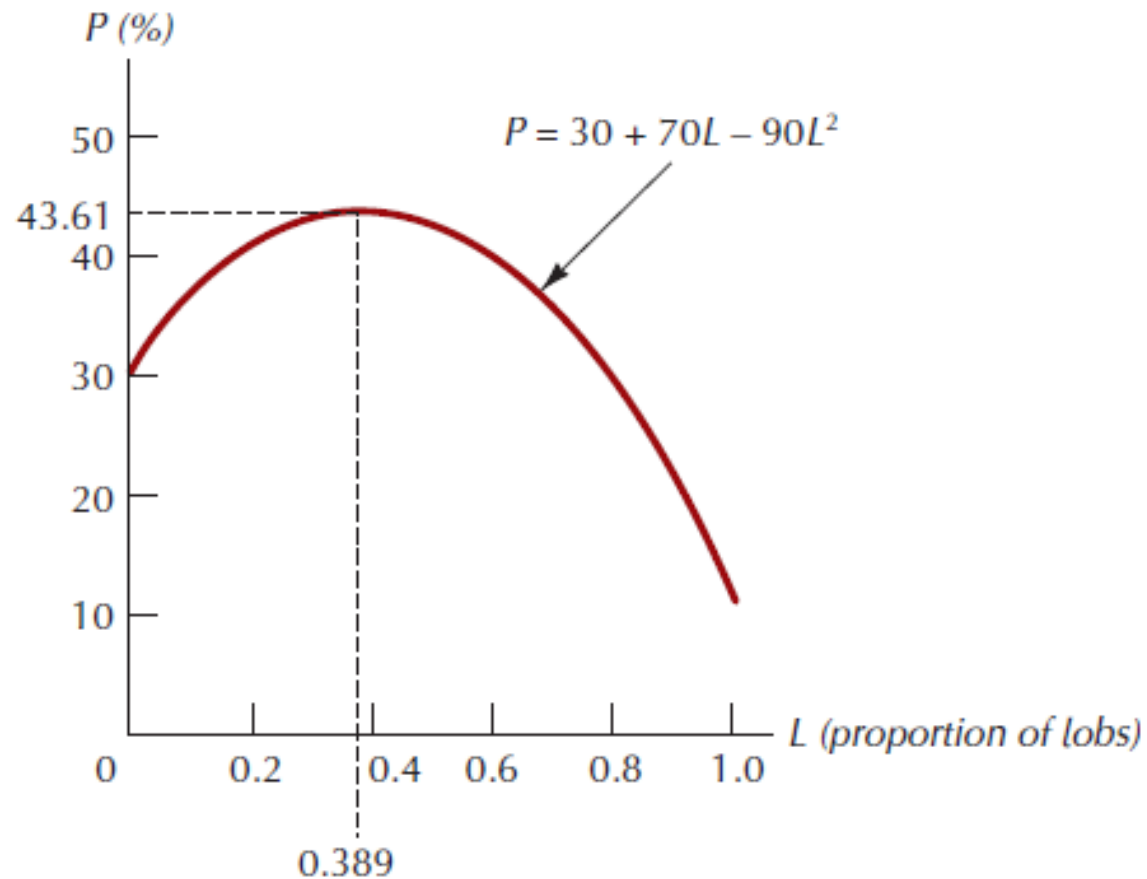


Figure 8A.3: At the Optimizing Point, the Likelihood of Winning with a Lob is Much Greater than of Winning with a Passing Shot

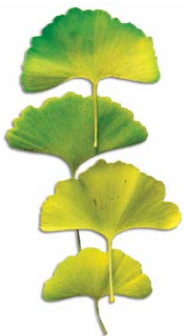
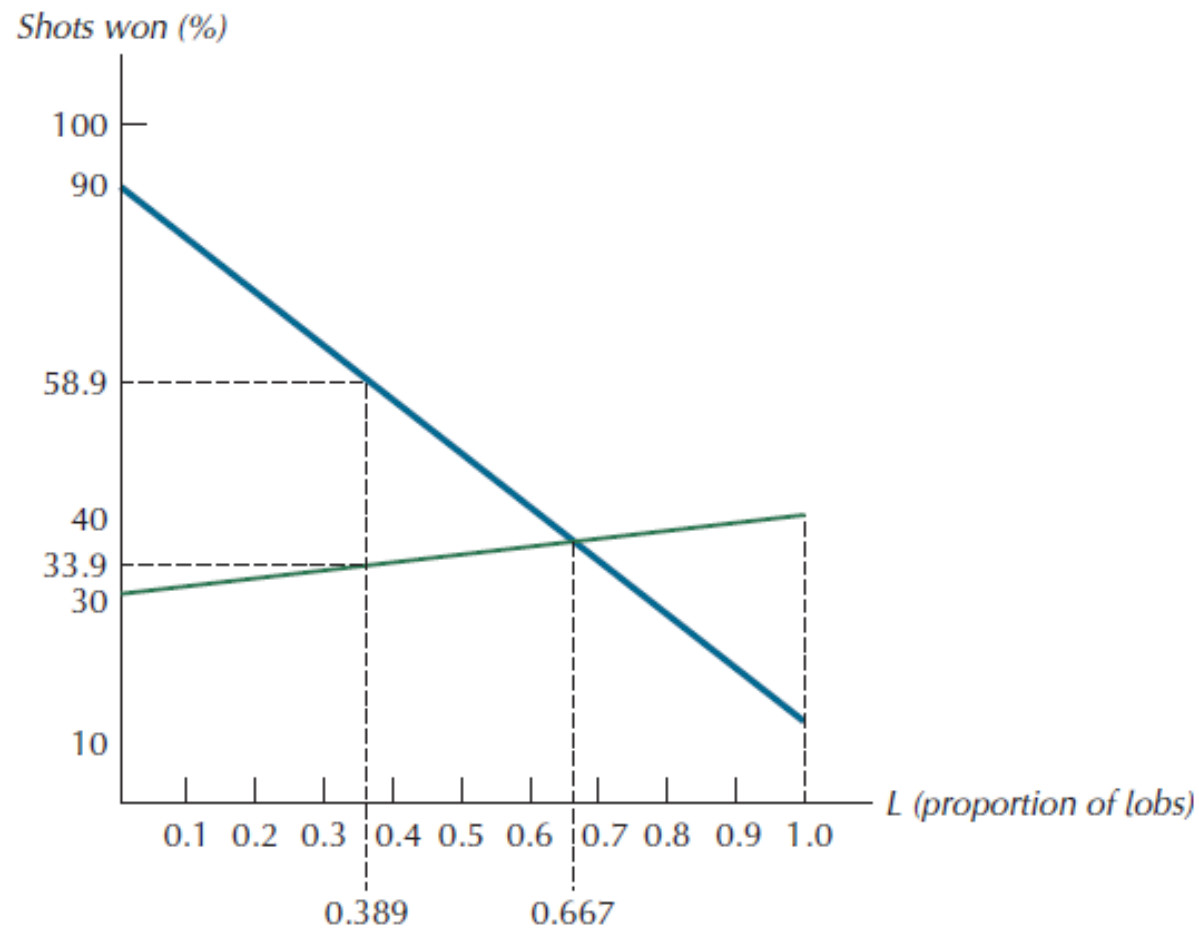


Figure 8A.4: The Production Mountain

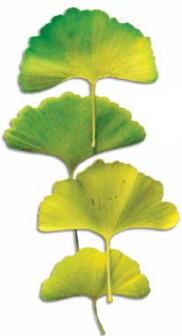
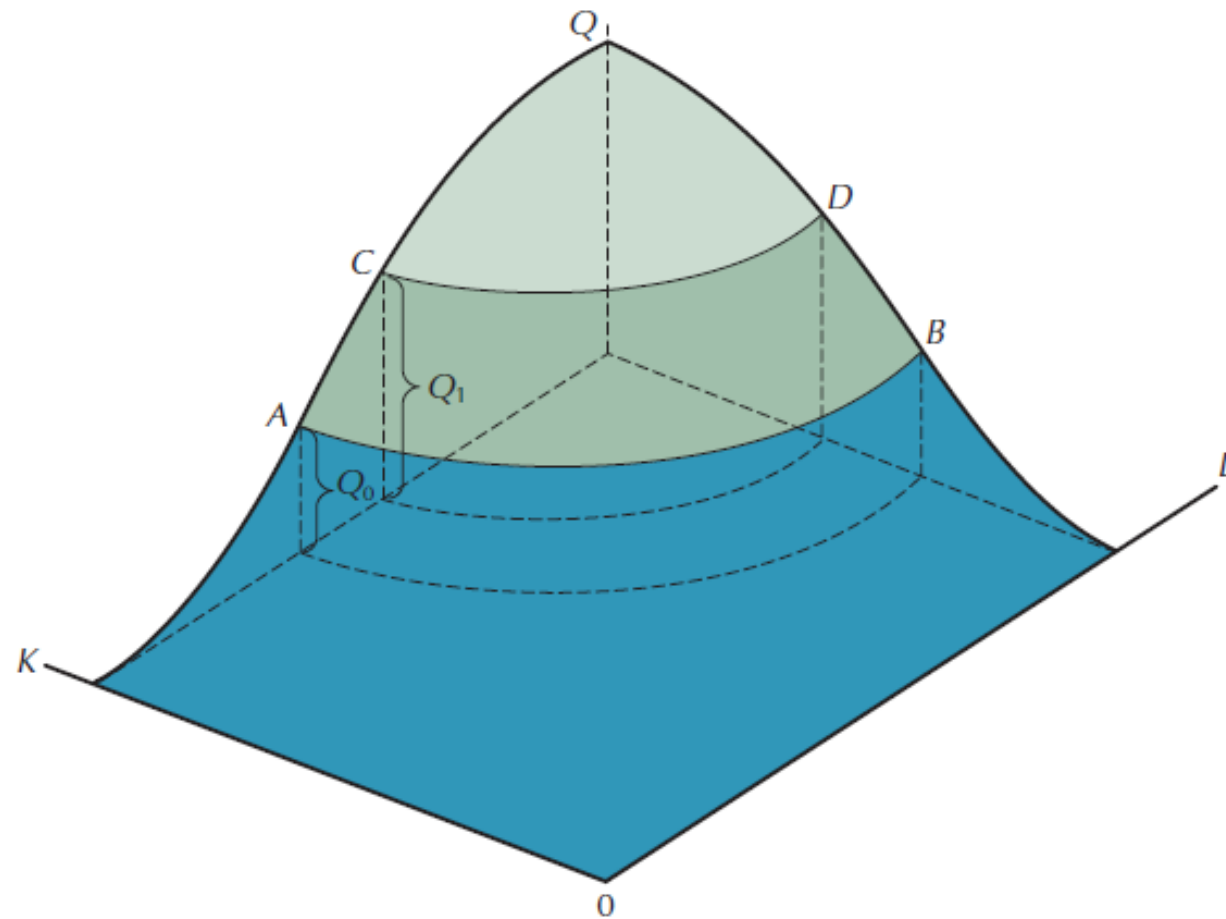
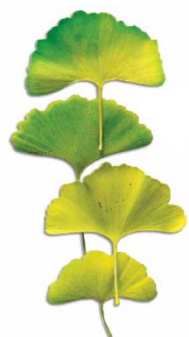
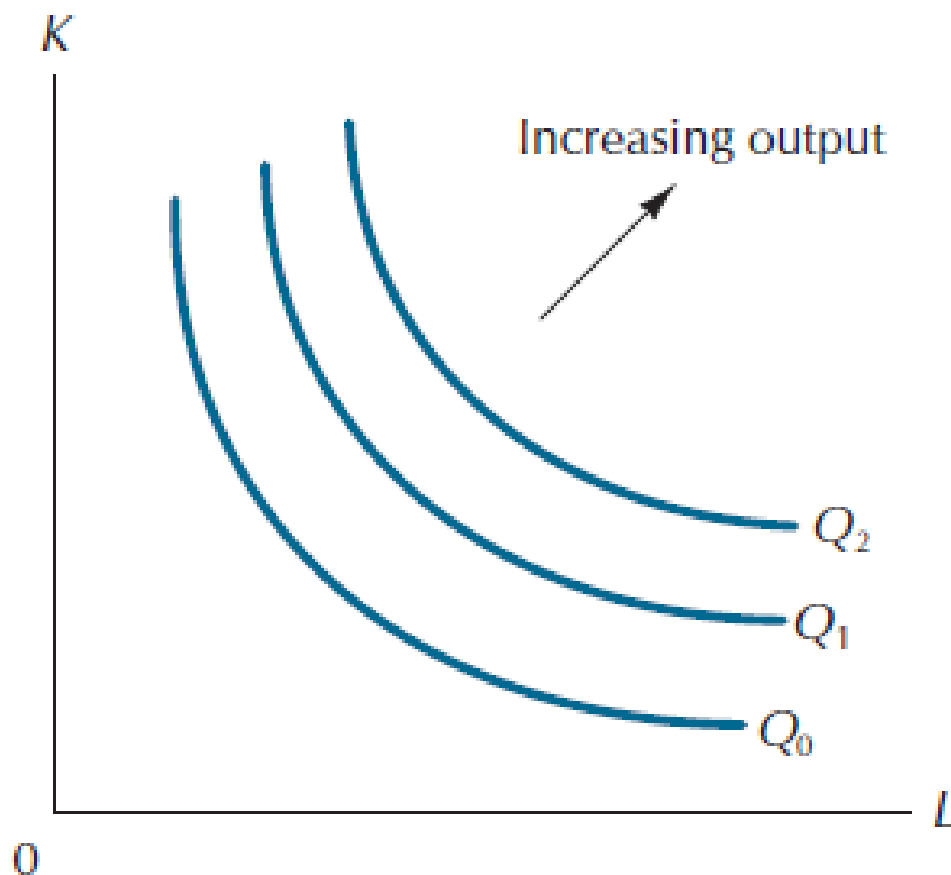


Figure 8A.5: The Isoquant Map Derived from the Production Mountain



Some Examples of Production Functions

- The Cobb-Douglas Production Function

which in the two-input case takes the form

$$Q = mK^{\alpha}L^{\beta}, \quad (8A.3)$$

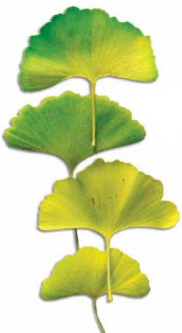
where α and β are numbers between zero and 1, and m can be any positive number.

To generate an equation for the Q_0 isoquant, we fix Q at Q_0 and then solve for K in terms of L . In the Cobb-Douglas case, this yields

$$K = \left(\frac{m}{Q_0}\right)^{-1/\alpha} (L)^{-\beta/\alpha}. \quad (8A.4)$$

For the particular Cobb-Douglas function $Q = K^{1/2}L^{1/2}$, the Q_0 isoquant will be

$$K = \frac{Q_0^2}{L}. \quad (8A.5)$$



Some Examples of Production Functions

- The Leontief, or Fixed-Proportions, Production Function

$$Q = \min(aK, bL).$$

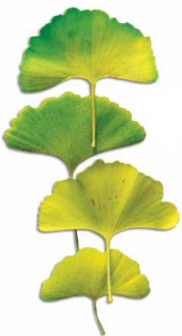


Figure 8A6: Isoquant Map for the Cobb-Douglas Production Function $Q = K^{1/2}L^{1/2}$

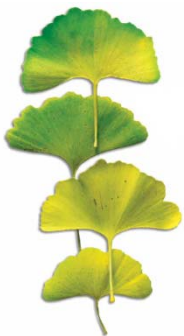
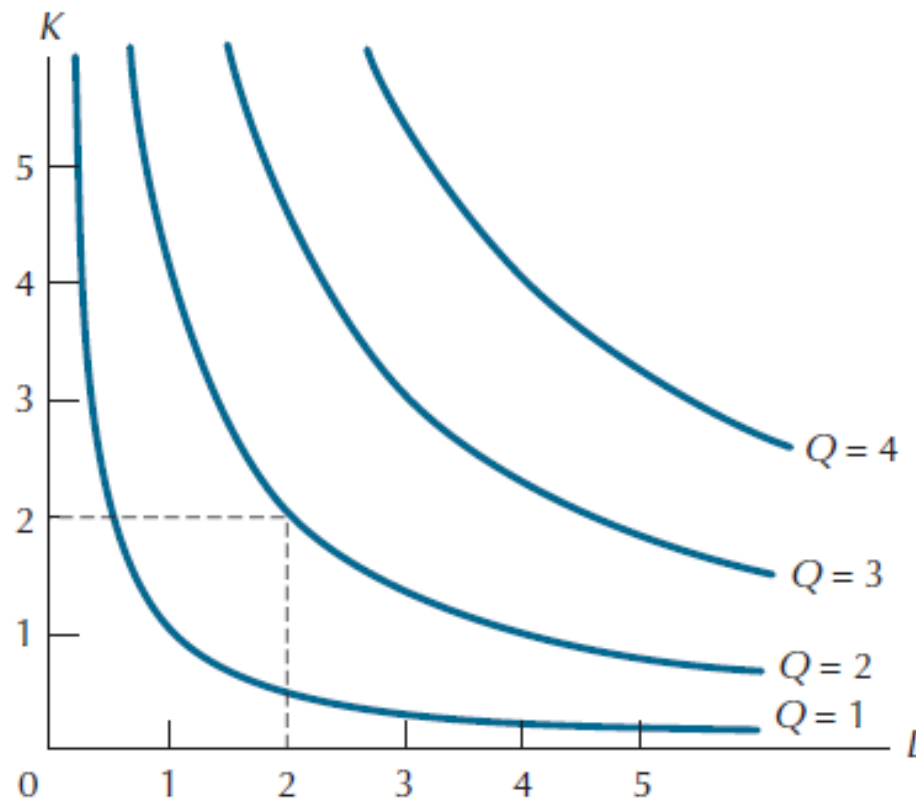
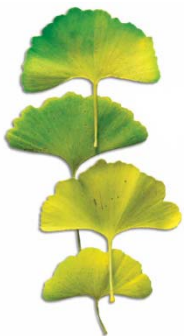
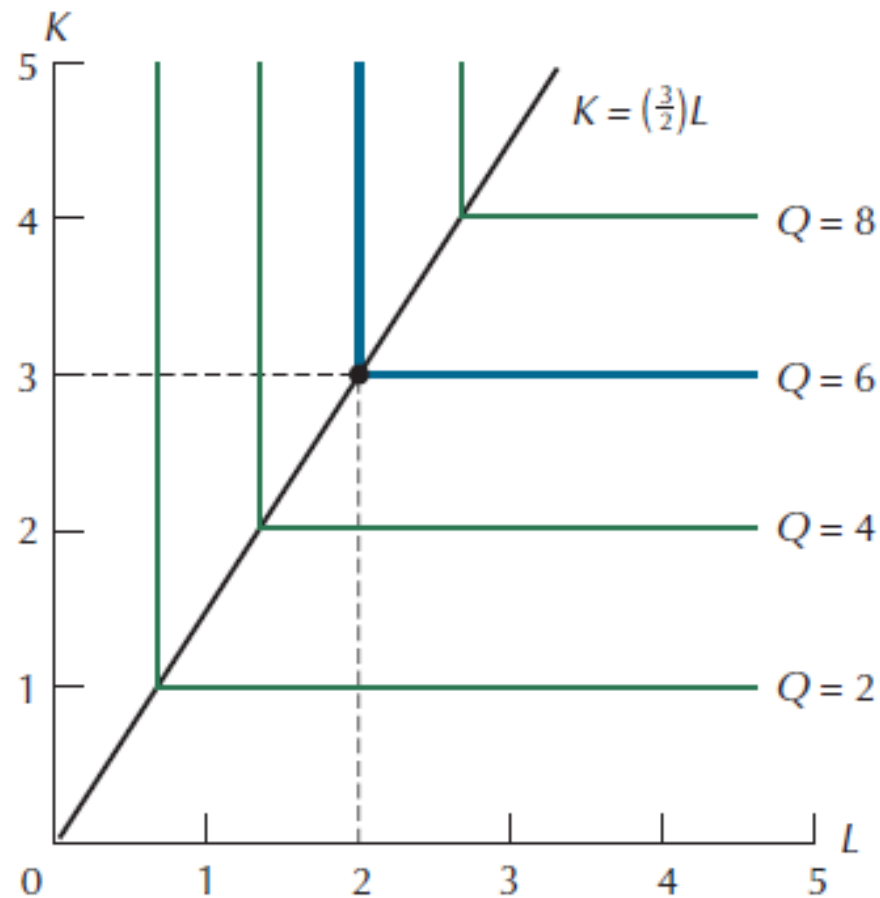


Figure 8A.7: Isoquant Map for the Leontief Production Function $Q = \min(2K, 3L)$



A Mathematical Definition of Returns to Scale

$$F(cK, cL) = 2(cK)(cL) = c^2 2KL = c^2 F(K, L)$$

Increasing returns: $F(cK, cL) > cF(K, L)$;

Constant returns: $F(cK, cL) = cF(K, L)$;

Decreasing returns: $F(cK, cL) < cF(K, L)$

