EE325 Section 2

HW 1 due September 5th, 2024, by email (kaewkwan325@gmail.com)

In order to receive HW points from the following questions, students must show all your work fully.

1. Find the answers following questions

   a) $\sum_{i=1}^{6}(a + bx_i)$

   b) $\sum_{y=0}^{3} f(x + y)$

   c) $\sum_{i=1}^{8} i^2$

   d) $\sum_{x=2}^{3}\sum_{y=3}^{4}(3x + 2y)$

2. Given X is discrete random variable. The probability distribution function (PDF) of this variable is shown in the table

X	-3	-2	-1	0	1	2	3
$f(x)$	0.5b	b	2.25b	2b	1.5b	0.5b	0.25b

   when b is constant number

   a) Find the value of b.
   b) Find the answer for $P(X \leq 3)$.
   c) Find the answer for $P(-3 \leq X \leq 3)$.
   d) Find the answer for $P(X \geq 1)$.

3. Given X is continuous random variable. The probability distribution function (PDF) of this variable is

$$f(x) = -\frac{1}{9}x + \frac{6}{9}, 0 \leq x \leq 3$$

   a) Find the value of $f(x)$.
   b) Find the answer for $P(1 \leq X \leq 3)$.
   c) Find the answer for $P(X \geq 2)$.
   d) Find the expected value of X.

4. Let random variable X be the outcome of throwing one dice and random variable Y be the outcome of tossing one coin. Coin has two sided that has valued 1 and 0.
   a) Construct the joint probability distribution function (PDF) table of X and Y.
   b) Find the marginal probability distribution function (PDF) of X.
   c) Find the marginal probability distribution function (PDF) of Y.
   d) Find the conditional probability distribution function (PDF) of X given Y is equal to 1.
   e) Find the expected value of X given Y is equal to 1.
   f) Find the variance of X given Y is equal to 1.

5. If X_1, X_2, X_3 is a random sample from a population with mean u and variance σ^2. X_1, X_2, X_3 are not independent

   $$Cov(X_1, X_2) = Cov(X_1, X_3) = Cov(X_2, X_3) = \frac{1}{4}\sigma^2$$

   $\bar{X}$ is estimator used to estimate mean value. $\bar{X} = \frac{1}{3}(X_1 + X_2 + X_3)$

   Find $E(\bar{X})$ and $var(\bar{X})$.

6. Given X_1, X_2, X_3, X_4 are independent identically distributed random variables from population with mean μ and variance σ^2. $\bar{X}$ is estimator used to estimate mean value. $\bar{X} = \frac{1}{4}(X_1 + X_2 + X_3 + X_4)$

   a) Find $E(\bar{X})$ and $var(\bar{X})$ in term of μ and σ.

   b) Given $\tilde{X} = \frac{1}{8}X_1 + \frac{1}{4}X_2 + \frac{1}{8}X_3 + \frac{1}{2}X_4$ is another estimator of μ. Show that $\tilde{X}$ is an unbiased estimator of u.

   c) Between $\bar{X}$ and $\tilde{X}$, which one is the better estimator for μ? Why?