Numerical Solution MA217 Midterm 2013 (1.30 hour exam)

1. (a) (0,0) saddle point, (-3, 0) saddle point, $\left(0,\frac{3}{2}\right)$ saddle point and

   $\left(-1,\frac{1}{2}\right)$ relative minimum

   (b) $f(x,y,a)=x^2y-2xy^2+axy+4$, $\Delta a=03.1-3=0.1$

   $f^*{}_{new}=3.45$

2. From Q1, critical points from $f(x,y)$ are (0,0), (-3, 0), $\left(0,\frac{3}{2}\right)$ and $\left(-1,\frac{1}{2}\right)$ and all are outside the constraint. You should not do this calculation for critical points again. Just take the results from Q1.

   Critical points from boundaries:-

   boundary $x=-2$, invalid critical point is $\left(-2,\frac{1}{4}\right)$ outside domain

   boundary $y=1$, critical point is $\left(-\frac{1}{2},1\right)$

   boundary $y=1-4x$, critical point is $(-0.06,1.24)$ and invalid critical point is $(0.15,0.4)$ outside domain

   end points are (0,1), (-2,1), (-2,9)

   Absolute maximum is (-2, 9) and absolute minimum is $\left(-\frac{1}{2},1\right)$

3. (a) Critical points are $(0.15,0.4)$ and $(0.15,0.4)$. Absolute maximum value is $f(0.15,0.4)=4.14$ and absolute minimum value is $f(-0.06,1.24)=3.9658$.

   (b) $\lambda=0.1212, \Delta C=0.9-1=-0.1,\quad f^*{}_{new}=3.9536$

4. Case I $y+4x=1$

   Give critical point $(0,1)$

   Case II $y+4x<1$

   Give critical point $\left(-\frac{1}{2},1\right)$

   Absolute maximum is $(0,1)$ and absolute minimum is $\left(-\frac{1}{2},1\right)$

5. Note that the constraints are $\geq$, you need to multiply -1 to both sides of the constraint equations.

   Case I $y=1$ and $x=-2$

   Give invalid critical point $(1,-2)$ and $\mu_2=3$ because $\mu_1=-6$

   Case II $y=1$ and $x>-2$

   Give invalid critical point $\left(-\frac{1}{2},1\right)$ because $\mu_1=-\frac{3}{4}$

Case III $y > 1$ and $x = -2$

Give invalid critical point $\left(-2, \frac{1}{4}\right)$ but violate $y > 1$

(A valid critical point needs to satisfy all starting questions.)

Case IV $y > 1$ and $x > -2$

Note can use what we did from Q1 and we get 3 invalid critical points (0,0) violate $y > 1$, (-3, 0) violate $x > -2$ and $\left(-1, \frac{1}{2}\right)$ violate $y > 1$ and 1 valid critical point $\left(0, \frac{3}{2}\right)$.

Hence, there is only 1 valid critical point $\left(0, \frac{3}{2}\right)$ and $f^* = 4$ but because there is only 1 point, we can only say that this is the critical point but cannot determine if it is the absolute maximum or minimum.