Assignment #1

Instructions:

- For all questions, answer up to 4 decimal places.
- This assignment is due on **Tuesday, Mar 9, 2021 before class (11.00).**
- Write your answer in either digital or ordinary paper. For digital paper, export pages into a single PDF file. For ordinary paper, take photos of your writing and convert them into a single PDF file as well.
- There is no need to rewrite the question. Assign number item, ie. 1 a., clearly before your answer is sufficient.
- Submit your assignment into Moodle.
- Name your file as StudentID_Nickname (in Thai) such as 123456789_โบ้

1. Given this information

$n = 30$	$\sum_{i=1}^{n} X_i = 366$	$\sum_{i=1}^{n} Y_i = 631$	$\overline{X} = 12.20$	$\overline{Y} = 21.03$
$\sum_{i=1}^{n}(X_i)^2 = 5{,}564$	$\sum_{i=1}^{n} X_i Y_i = 7{,}524$	$\sum_{i=1}^{n}\left(X_i - \overline{X}\right)^2 = 1098.8$	$\sum_{i=1}^{n}(Y_i - \overline{Y})^2 = 882.97$	
$\sum_{i=1}^{n}(X_i - \overline{X})(Y_i - \overline{Y}) = -174.20$		$\sum_{i=1}^{n}\hat{u}_i^2 = 873.14$		

Answer the following questions. Show your work.

a) From regression model: $Y_i = \beta_1 + \beta_2 X_i + u_i$, $u_i \sim NIID\left(0, \sigma^2\right)$ (normally, identically and independently distributed), find the estimators of β_1 and β_2 with OLS method and explain the meaning of the model.

b) Find r^2 and explain its meaning.

c) If $X_i = 5$, estimate the value of $\hat{Y}_i$ and explain its meaning.

d) Find the estimators of $\mathrm{var}(u_i)$, $\mathrm{var}\left(\hat{\beta}_1\right)$ and $\mathrm{var}\left(\hat{\beta}_2\right)$

e) Test the hypothesis whether coefficients are different from zero at 0.05 level of significance.

f) Test the hypothesis whether coefficients are less than zero at 0.01 level of significance.

a) From regression model: $Y_i = \beta_1 + \beta_2 X_i + u_i$, $u_i \sim NIID(0, \sigma^2)$ (normally, identically and independently distributed), find the estimators of β_1 and β_2 with OLS method and explain the meaning of the model.

$$\beta_1 = \bar{Y} - \beta_2 \bar{X} \quad , \quad \beta_2 = \frac{\Sigma(X_i - \bar{X})(Y_i - \bar{Y})}{\Sigma(X_i - \bar{X})^2}$$

$$\beta_2 = \frac{-174.20}{1098.8} = -0.1585$$

$$\beta_1 = 21.03 - (-0.1585)(12.20) = 22.9637$$

$$\therefore \beta_1 = 22.9637 \quad , \beta_2 = -0.1585$$

$\beta_1 = 22.9637$ means that when $X = 0$, on average, Y is predicted to be 22.9637 unit

$\beta_2 = -0.1585$ holding other thing constant, if X increase by 1 unit, Y will decrease by 0.1585 unit

b) Find r^2 and explain its meaning.

$$r^2 = 1 - \frac{RSS}{TSS} = 1 - \frac{\Sigma \hat{u}_i^2}{\Sigma(Y_i - \bar{Y})^2}$$

$$r^2 = 1 - \frac{873.14}{882.97} = 0.01113$$

$\therefore$ X can explain Y at 0.01113 (or 1.113 %) which is the explained part from regression

c) If $X_i = 5$, estimate the value of $\hat{Y}_i$ and explain its meaning.

$Y_i = 22.9637 - 0.1585 X_p$

$Y_i = 22.9637 - 0.1585(5)$

$Y_p = 22.1712$

$\therefore$ when $X_p = 5$, Y_p is predict to be 22.1712 units.

d) Find the estimators of $\text{var}(u_i)$, $\text{var}(\hat{\beta}_1)$ and $\text{var}(\hat{\beta}_2)$

From the assumption, NIID $(0, \sigma^2)$; $\sum x_p^2 = \sum (X_i - \bar{X})^2$ where $x_i^2 = (X_i - \bar{X})^2$

$$\text{var}(U_i) = \hat{\sigma}^2 = \frac{\sum \hat{U}_i^2}{n-k} = \frac{873.14}{30-2} = 31.1836$$

$$\text{var}(\hat{\beta}_1) = \frac{\sum X_i^2}{n \sum x_i^2} \hat{\sigma}^2 = \frac{5564}{30(1098.8)}(31.1836) = 5.2635$$

$$\text{var}(\hat{\beta}_2) = \frac{\hat{\sigma}^2}{\sum x_i^2} = \frac{31.1836}{1098.8} = 0.02838$$

e) Test the hypothesis whether coefficients are different from zero at 0.05 level of significance.

1. $H_0: \beta_2 = 0$ $\quad\quad \hat{\sigma}_{\hat{\beta}_2} = \sqrt{0.02838} = 0.1685$

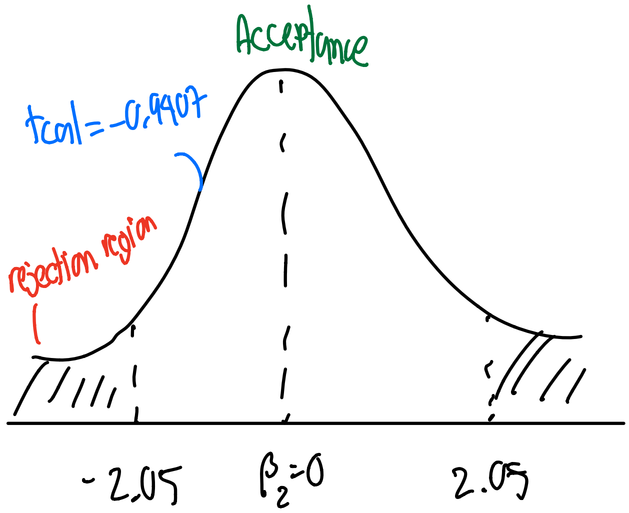
$H_1: \beta_2 \neq 0$

2. $t_{cal} = \dfrac{\hat{\beta}_2 - \beta_2}{\hat{\sigma}_{\hat{\beta}_2}} = \dfrac{-0.1585 - 0}{0.1685} = -0.9407$

3. $\alpha = 0.05$, $n = 30$, $n-k = 28$

lower bound: $t_{\frac{0.05}{2}} = -2.05$

Upper bound: $t_{\frac{0.05}{2}} = 2.05$



∴ Tcal lies within acceptance region, so we cannot reject the null hypothesis (H_0), at the significance level of 95%.

f) Test the hypothesis whether coefficients are less than zero at 0.01 level of significance.

1. $H_0 : \beta_2 \geq 0$ $\qquad \hat{\sigma}_{\hat{\beta}_2} = \sqrt{0.02838} = 0.1685$

$H_1 : \beta_2 < 0$

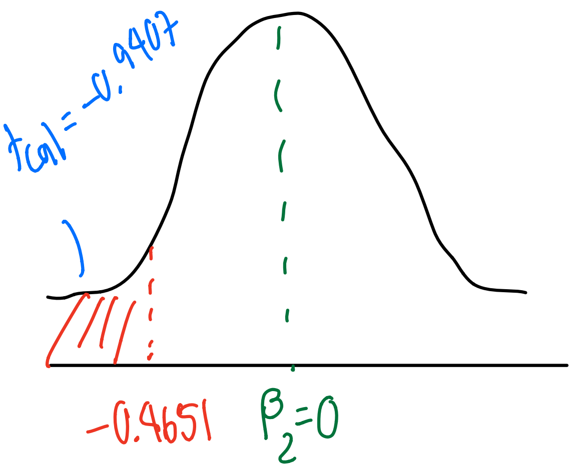
2. $t_{cal} = \dfrac{\hat{\beta}_2 - \beta_2}{\sigma_{\beta_2}} = \dfrac{-0.1585 - 0}{0.1685} = -0.9407$

3. $\alpha = 0.01$, $n = 30$, $n-k = 28$

specific value $\geq$, acceptance region right, use lower bound

lower bound : $\beta_2 - t_{\frac{\alpha}{2}} \cdot \hat{\sigma}_{\hat{\beta}_2}$

$= 0 - 2.76 \cdot 0.1685 = -0.4651$



$\therefore$ T_{cal} lies beyond rejection region

So, we can reject null hypothesis (H_0)

at the significance level of 99%. ✗

2. Given that Y is market price of a car (USD) while X is how long a car aged (years), results of the regression are as follows.

$$\hat{Y}_i = 7{,}836 - 502.4X_i$$
$$se = (52) \quad (411.8)$$

Given that u_i is normally, identically and independently distributed with zero mean and σ^2 variance, total number of observations is 11,

$\bar{X} = 7.45,$

$\hat{\sigma}^2 = 212{,}877,$

$\sum(X_i - \bar{X})^2 = 78.73,$

Answer the following questions. Show your work.

a) Does the sign of $\hat{\beta}_2$ make economic sense? Provide your explanation.

b) If you are a car expert ($X_0 = 5$ and someone asks you to estimate how much his car will be **averagely** priced at when his car is 5 years old, how much is the market price range that you would estimate that you can make sure that for 95% of the time, market price will be within the specific range?

c) If you multiply all the X with 10, report the new SRF with the standard error resulted from the multiplication.

d) Calculate the elasticity of market price when a car is 10 years old.

a. yes, Because the slope of regression model is negative. it shows relationship between price of a car and how long car used. So it illustrate that the higher number of used car in years, the lower the market price.

b. prediction $\hat{Y}_0 = 7{,}836 - 502.4(5) = 5{,}324$

1. $var(\hat{Y}_0) = \hat{\sigma}^2\left[\frac{1}{n} + \frac{(X_0 - \bar{X})^2}{\Sigma(X_i - \bar{X})^2}\right]$

$= 212{,}877\left[\frac{1}{11} + \frac{(5 - 7.45)^2}{78.73}\right]$

$var(\hat{Y}_0) = 35582.5345$

2. $\hat{\sigma}_{\hat{Y}_0} = \sqrt{var(\hat{Y}_0)}$

$= \sqrt{35582.5345}$

$\hat{\sigma}_{\hat{Y}_0} = 188.63$

3. 95% CI $\hat{Y}_0 = 5{,}324$

- upper: $\hat{Y}_0 + (t_{\frac{\alpha}{2}} \cdot \sigma\hat{Y}_0) = 5{,}324 + (2.262 \cdot 188.63) = 5750.6810$
- lower: $\hat{Y}_0 - (t_{\frac{\alpha}{2}} \cdot \sigma\hat{Y}_0) = 5{,}324 - (2.262 \cdot 188.63) = 4897.3189$

∴ 95 out of 100 time. The market price will be between 4897.3189 and 5750.6810

c) If you multiply all the X with 10, report the new SRF with the standard error resulted from the multiplication.

New SRF when $X \cdot 10$

$$\hat{Y}_p = 7836 - 502.4(10)(X)$$
$$(52) \qquad (411.87)(10)$$

$$\hat{Y}_p = 7836 - 5024(X)$$
$$(52 \qquad (4118)$$

$\cancel{A}$

d) Calculate the elasticity of market price when a car is 10 years old.

$$\hat{Y}_i = 7836 - 502.4X_i$$

linear model elasticity $= \beta_2\left(\frac{X_i}{Y_i}\right)$

10 years old

$$\hat{Y}_i = 7836 - 502.4(10)$$

$$\hat{Y}_i = 2812$$

$$\text{elasticity} = -502.4 \times \frac{10}{2812}$$

elasticity $= -1.7866$ $\cancel{A}$

∴ elasticity of market price when a car is 10 years old is -1.7866