Extra Note

We would like our utility function to have 2 properties:

1. Monotonicity
2. Diminishing Marginal Utility

To show that Utility Function has "Monotonicity",

you need to show that $\frac{dU(x,y)}{dx} > 0 \ \ and \ \ \frac{dU(x,y)}{dy} > 0$, i.e. $MUx > 0 \ and \ MUy > 0$.

That is, you find the **first derivative** of "utility function" and check if it is positive.

Note

1. Monotonicity means that higher x and/or y gives **higher "utility"**.
2. "How much" utility increases is given by the marginal utility.
   e.g. If MUx = 3, this means one more x will give 3 units of utility.

Read Carefully

To prove that Utility Function has "Diminishing Marginal Utility",

you need to show that $\frac{d^2U(x,y)}{dx^2} < 0 \ \ and \ \ \frac{d^2U(x,y)}{dy^2} < 0$, i.e. $\frac{dMUx}{dx} < 0 \ \ and \ \ \frac{dMUy}{dy} < 0$.

That is, you find the **second derivative** of "utility function" and check if it is negative.

Note

1. Diminishing MU means that higher x and/or y gives **lower "MU"**.
2. This means that the additional happiness from consuming one more unit gradually diminishes.
   e.g. At x = 1, suppose MUx = 3. Diminishing MU implies that when x increases from 1 to 2, MUx will be less than 3.

Lastly, to show that MRSxy is diminishing (i.e. slope of IC curves gradually decreases),

you need to show that as x increases, MRSxy falls, i.e. you need to show that $\frac{dMRS}{dx} < 0$.

Alternatively, you can "observe". For example, let MRSxy = y/x.

Here, as x increases, MRSxy falls, i.e. MRSxy is diminishing.

Example

Let $U = \sqrt{xy}$. Answer the following questions.

1. **Write down the equation of the IC showing bundles of goods that give U = 4.**

$$4 = \sqrt{xy}$$

$$16 = xy$$

$$IC\ equation\ (U = 4):\ \ y = \frac{16}{x}$$

2. **Find the slope of this IC curve by using derivative.**

$$IC\ equation\ (U = 4):\ \ y = \frac{16}{x} = 16x^{-1}$$

$$Slope\ of\ IC = \frac{dy}{dx} = -16x^{-2}$$

Note: we have shown that IC has a negative slope.

3. **Determine whether the utility function has monotonicity for both goods.**

To show monotonicity, we need to show $\frac{dU(x,y)}{dx} > 0\ \ and\ \ \frac{dU(x,y)}{dy} > 0.$

$$U = \sqrt{xy} = x^{1/2}y^{1/2}$$

$$\frac{dU}{dx} = MUx = \left(\frac{1}{2}y^{1/2}\right)x^{-1/2}\ \ which\ is\ positive\ for\ all\ x$$

$$\frac{dU}{dy} = MUy = \left(\frac{1}{2}x^{1/2}\right)y^{-1/2}\ \ which\ is\ positive\ for\ all\ y$$

Hence, we have shown that utility function has monotonicity.

4. **Interpret your result.**

Monotonicity or positive MU means that higher consumption of good X and/or good Y gives higher utility.

5. **Determine whether the utility function has diminishing MU for both goods.**

To show diminishing MU, we need to show $\frac{d^2U(x,y)}{dx^2} < 0\ \ and\ \ \frac{d^2U(x,y)}{dy^2} < 0.$

$$From\ \frac{dU}{dx} = \left(\frac{1}{2}y^{1/2}\right)x^{-1/2}\ \ and\ \ \frac{dU}{dy} = \left(\frac{1}{2}x^{1/2}\right)y^{-1/2}\ \ we\ get$$

$$\frac{d^2U}{dx^2} = \frac{dMUx}{dx} = \left(-\frac{1}{2}\right)\left(\frac{1}{2}y^{1/2}\right)x^{-3/2}\ \ which\ is\ negative\ for\ all\ x$$

$$\frac{d^2U}{dy^2} = \frac{dMUy}{dy} = \left(-\frac{1}{2}\right)\left(\frac{1}{2}x^{1/2}\right)y^{-3/2}\ \ which\ is\ negative\ for\ all\ y$$

Hence, we have shown that utility function has diminishing MU.

6. Interpret your result.

Consuming one more unit of good will make you happier, as implied from monotonicity. But the "size" of the happiness you get will become smaller and smaller as more good is consumed. This is what diminishing MU means.

7. Derive MRSxy.

$$MRSxy = \frac{MUx}{MUy}$$

$$MUx = \left(\frac{1}{2}y^{1/2}\right)x^{-1/2}\ \ and\ \ MUy = \left(\frac{1}{2}x^{1/2}\right)y^{-1/2}$$

$$MRSxy = \frac{y}{x}$$

8. Does the utility function have diminishing MRSxy?

Yes, MRSxy is decreasing in X. That is, as X increases, MRSxy falls.

9. What is the MRSxy of the IC showing bundles of goods that give U = 4?

$$IC\ equation\ (U = 4):\ \ y = \frac{16}{x}$$

$$MRSxy = \frac{y}{x} = \frac{16}{x^2} = 16x^{-2}$$

10. Compare your answer from Question 9 and Question 2.

$$We\ observe\ that\ \ -MRSxy = \frac{dy}{dx} = Slope\ of\ IC\ (U = 4)$$

11. Find the value of the slope at X = 1 of the IC showing bundles of goods that give U = 4.

$$-MRSxy = \frac{dy}{dx} = Slope\ of\ IC\ (U = 4) = -\frac{16}{x^2}$$

$$At\ X = 1, \qquad \frac{dy}{dx} = Slope\ of\ IC\ (U = 4) = -16$$

12. Interpret this result.

Note that MRSxy = 16 at X = 1. This means the consumer is willing to give up 16 units of Y for one more unit of X.