

1.2.2) Comparative Static analysis in Math framework

H.W.

Having solved for the equilibrium solution, what economists usually ask is what would happen to the equilibrium if something, previously assumed to be fixed, has changed.

Example 1.C (cont.): National income model

Due date this weekend

- From the example 1.B, it is straightforward to solve for all the endogenous equilibrium solutions, Y^*, C^*, Y_d^* .

$$Y^* \rightarrow Y_d^* = Y^* - T_0$$

$$\rightarrow C^* = a + b Y_d^*$$

$$Y = C + I + G$$

$$Y = a + b(Y - T) + I_0 + G_0$$

$$Y = a + bY - bT + I_0 + G_0$$

$$(1-b)Y = a - bT + I_0 + G_0$$

$$Y^* = \frac{1}{1-b} (a + bT + I_0 + G_0)$$

- Numerically, if $a = 1$, $T_0 = \$0$, $I_0 = \$1$, $G_0 = \$1$ and $b = 0.5$, this yields us,

$$Y^* = \frac{1}{1-0.5} (1 - 0.5(0) + 1 + 1)$$

$$Y^* = 2(3)$$

$$Y^* = 6$$

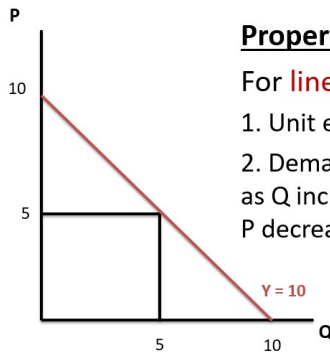
$$C^* = 1 + 0.5(6 - 0)$$

$$C^* = 4$$

$$Y_d^* = 6 - 0$$

$$Y_d^* = 6$$

Linear function: slope v.s. elasticity

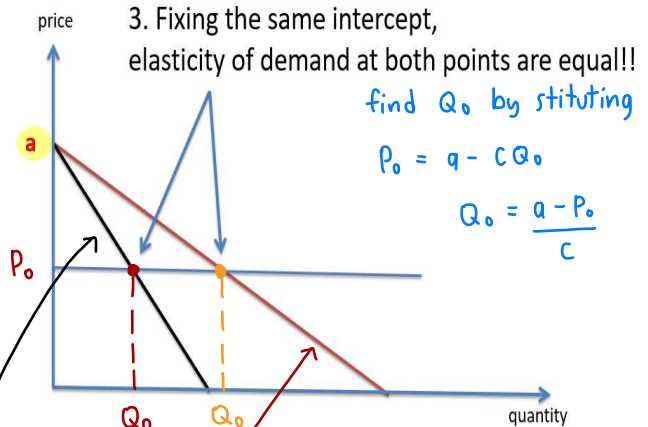


Property:

For **linear** demand curve:

1. Unit elasticity at the mid point.
2. Demand becomes more **inelastic** as Q increases, (correspondingly to P decreases)

Prove PED



find Q_0 by substituting

$$P_0 = a - cQ_0$$

$$Q_0 = \frac{a - P_0}{c}$$

Exercise 2.A:

$$P = a - bQ$$

use $\frac{\Delta Q}{\Delta P} \times \frac{P_0}{Q_0}$

2.A.1) Given a demand function by $p = a - bQ$, derive the formula for the elasticity of demand, and show that the third property holds

2.A.2) Given the market supply $p = c + dQ$ where $d \geq 0$, show that

- (i) elasticity of supply is always greater than 1 if $c > 0$,
- (ii) elasticity of supply is always equal to 1 if $c = 0$,
- (iii) elasticity of supply is always less to 1 if $c < 0$.

& Point PES $\rightarrow$ ใช้มันไหนก็ได้

Hint:

$$\text{use } \frac{\% \Delta Q_s}{\% \Delta P} = \frac{\frac{Q_1 - Q_0}{Q_0}}{\frac{P_1 - P_0}{P_0}}$$

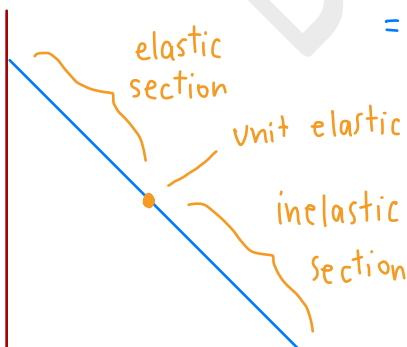
$$\begin{aligned} PED &= \frac{\Delta Q}{\Delta P} \times \frac{P_0}{Q_0} \\ &= -\frac{1}{b} \times \left(\frac{a + bQ}{Q} \right) \\ &= -\frac{a + bQ}{bQ} \\ &= 1 - \frac{a}{bQ} \end{aligned}$$

$$\begin{aligned} 2.2) PES &= \frac{1}{d} \times \left(\frac{c + dQ}{Q} \right) \\ &= \frac{c + dQ}{dQ} \\ &= 1 + \frac{c}{dQ} \end{aligned}$$

$$\text{if } c > 0, PES = 1 + \frac{c}{dQ} > 1$$

$$\text{if } c = 0, PES = 1 + \frac{c}{dQ} = 1$$

$$\text{if } c < 0, PES = 1 + \frac{c}{dQ} < 1$$



$$MC = \frac{\Delta \pi}{\Delta Q} = 4$$

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Due 22 sat

Example 2.1: A monopolist firm faces the market demand given by $P = 10 - Q$. Consider the following questions if the cost function $C(Q) = 4Q$.

$P(Q)$

- What is the revenue-maximizing level of output?

revenue function ; $TR(Q) = P(Q) \times Q$

$$= (10 - Q) \times Q$$

$$= 10Q - Q^2$$

At $Q = 5$, TR is max

$$TR = 25$$

$$\text{slope} = \frac{\partial TR}{\partial Q} = 10 - 2Q$$

maximum occurs when

$$\frac{\partial TR}{\partial Q} = 0$$

$$-10 - 2Q = 0$$

$$Q = 5$$

- What is the break-even output?

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$$\pi = 0 \rightarrow TR = TC$$

$$4Q = 10Q - Q^2$$

$$Q^2 - 6Q = 0$$

$$Q = 0, 6$$

$$\therefore Q = 6$$

- What is the profit-maximizing level of output?

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① $MR = MC$

② $\pi = TR - TC$ $\frac{d\pi}{dQ} = 0 \rightarrow Q^*$

$$= 10Q - Q^2 - 4Q$$

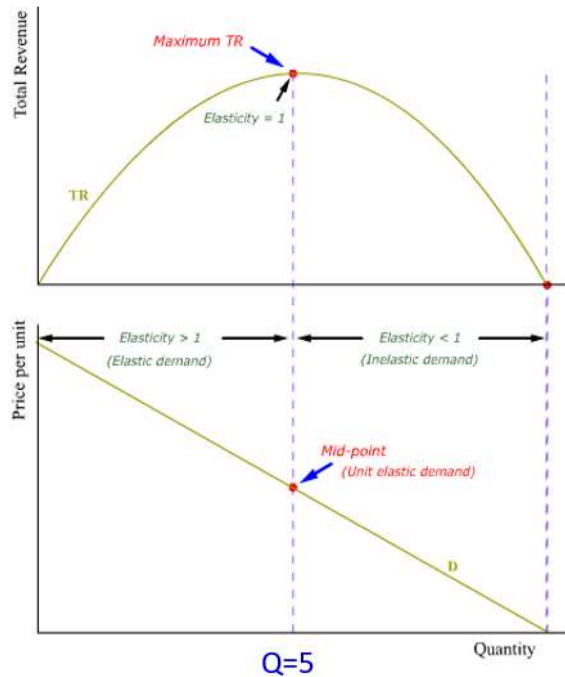
$$\frac{d\pi}{dQ} = 10 - 2Q - 4 = 0$$

$$6 - 2Q = 0$$

$$Q = 3$$

prove PED PUE Monday

Quadratic function: revenue function/break even analysis



$$\left| \frac{\% \Delta Q}{\% \Delta P} \right| : \begin{array}{ll} > 1 ; & \text{elastic} \\ = 1 ; & \text{unit elastic} \\ < 1 ; & \text{inelastic} \end{array}$$

Revenue maximizing output:
unit elasticity

Profit-maximizing output:
under region with **elastic** demand

Laffer Curve

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Exercise 2B. Consider a function that relates tax revenues R , in billions of dollars, to the average tax rate t such that $R = 350t - 500t^2$.

- (a) What tax rate(s) is consistent with raising tax revenues equal to \$60 billion?
 t_1 & t_2
- (b) What tax rate(s) is consistent with raising tax revenues equal to \$61.25 billion?
- (c) What tax rate is consistent with the maximum tax revenue? t^*

